The least open quantum system

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[Experiment at ETH Zurich]

LETTER

doi:10.1038/nature17409

Quantum phases from competing short- and long-range interactions in an optical lattice

Renate Landig[1], Lorenz Hruby[1], Nishant Dogra[1], Manuele Landini[1], Rafael Mottl[1], Tobias Donner[1] & Tilman Esslinger[1]

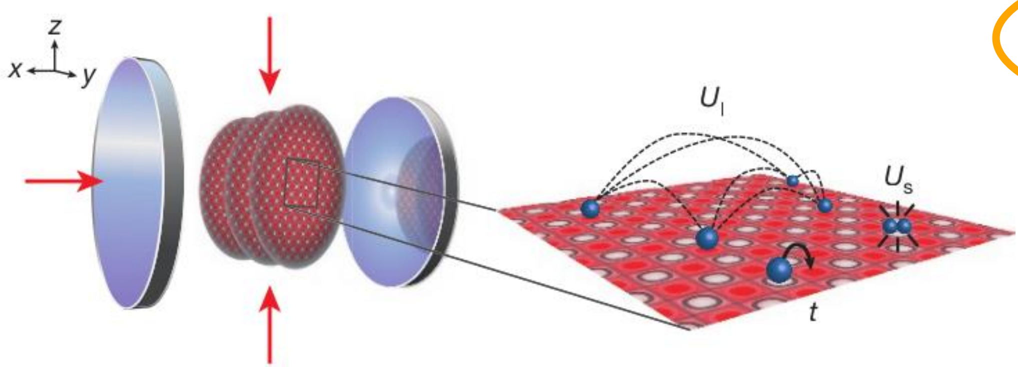


Figure 1 | Illustration of the experimental scheme that realizes a lattice model with on-site and infinite-range interactions. Left, a stack of 2D systems along the y axis is loaded into a 2D optical lattice (red arrows) between two mirrors (shown grey). The cavity induces atom–atom interactions of infinite range. Right, illustration of the competing energy scales: tunnelling t, on-site interactions U_s and long-range interactions U_l.

$$\widehat{H} = \widehat{H}_b + \widehat{H}_c + \widehat{H}_{bc}$$

$$\widehat{H}_b = -J \sum_{\langle i,j \rangle} \widehat{b}_i^\dagger \widehat{b}_j + \frac{U}{2} \sum_j \widehat{n}_j (\widehat{n}_j - 1)$$

$$\widehat{H}_c = \delta\, \widehat{a}^\dagger \widehat{a}$$

$$\widehat{H}_{bc} = -\frac{\Omega}{\sqrt{N}} \left(\widehat{a} + \widehat{a}^\dagger\right) \widehat{\Delta}$$

checkerboard order parameter $\widehat{\Delta} = \sum_j f_j \widehat{n}_j$

$$f_j = (-1)^{j_x + j_y}$$

photon mode configuration relative to lattice (here:checkerboard)

losses only in the cavity

$$\frac{d\widehat{\rho}}{dt} = -i \left[\widehat{H}, \widehat{\rho}\right] + \kappa \left[2\widehat{a}\widehat{\rho}\widehat{a}^\dagger - \widehat{a}^\dagger\widehat{a}\widehat{\rho} - \widehat{\rho}\widehat{a}^\dagger\widehat{a}\right]$$

Landig, Hruby, Dogra, Landini, Mottl, Donner, Esslinger, Nature 532, 476 (2016)

Adiabatic elimination – extended Bose-Hubbard model

Assumes cavity dissipation is the fastest → cavity mode adapts to instantaneous steady state

$$\hat{H} = \hat{H}_b + \underline{\hat{H}_c + \hat{H}_{bc}}$$

$$\hat{H}_c = \delta\, \hat{a}^\dagger \hat{a}$$

$$\boxed{a_{\text{est}} = \frac{\Omega}{\sqrt{N}}\left(\frac{\Delta}{\delta - i\kappa}\right)}$$

$$\hat{H}_{bc} + \hat{H}_c = -\frac{\Omega^2}{N}\frac{\delta}{\delta^2+\kappa^2}\hat{\Delta}^2$$

$$\boxed{\hat{\Delta} = \sum_j f_j \hat{n}_j}$$

checkerboard order parameter

<u>Effective ALL-TO-ALL interaction</u> for atoms mediated by cavity mode

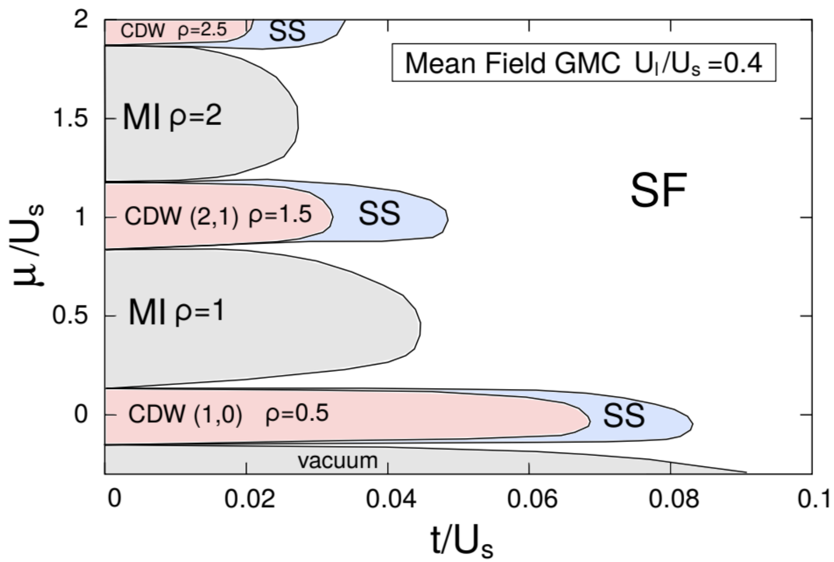
(Extended Bose-Hubbard)

$$f_j = (-1)^{j_x + j_y}$$

Many studies of the EBH, predict e.g.:

- true long-range order in 2d
- 2nd order phase-transition of
- supersolid/charge-density wave

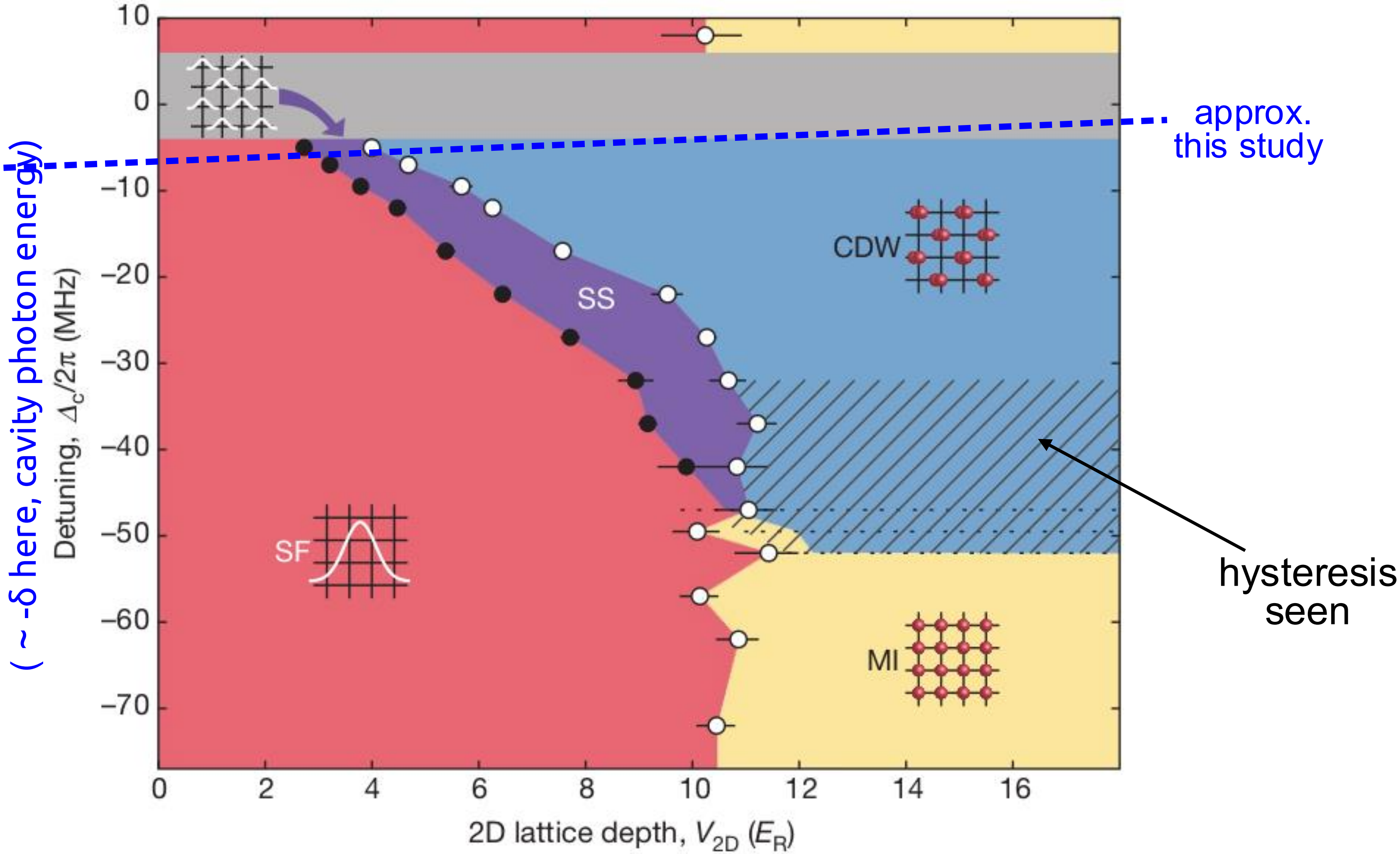
Li,He, Hofstetter, PRA 87, 051604® (2013)
Nogra, Brennecke, Huber, Donner, PRA 94, 023632 (2016)
Sundar, Mueller, PRA 94, 033631 (2016)
Himbert, Cormick, Kraus, Sharma, Morigi, PRA 99, 043633 (2019)
Flottat, deParny, Hebert, Rousseau, Batrouni, PRB 95, 144501 (2017)
Sharma, Jaeger, Kraus, Roscilde, Morigi, PRL 129, 143001 (2022)
and many others



Phase diagram – cavity mediated long range interaction

- Unusual open collective system - only one out of many modes is dissipative (the cavity mode)
- Some similarity to quantum measuring devices, weak measurements
- Full system description has remained somewhat elusive when the lattice is large

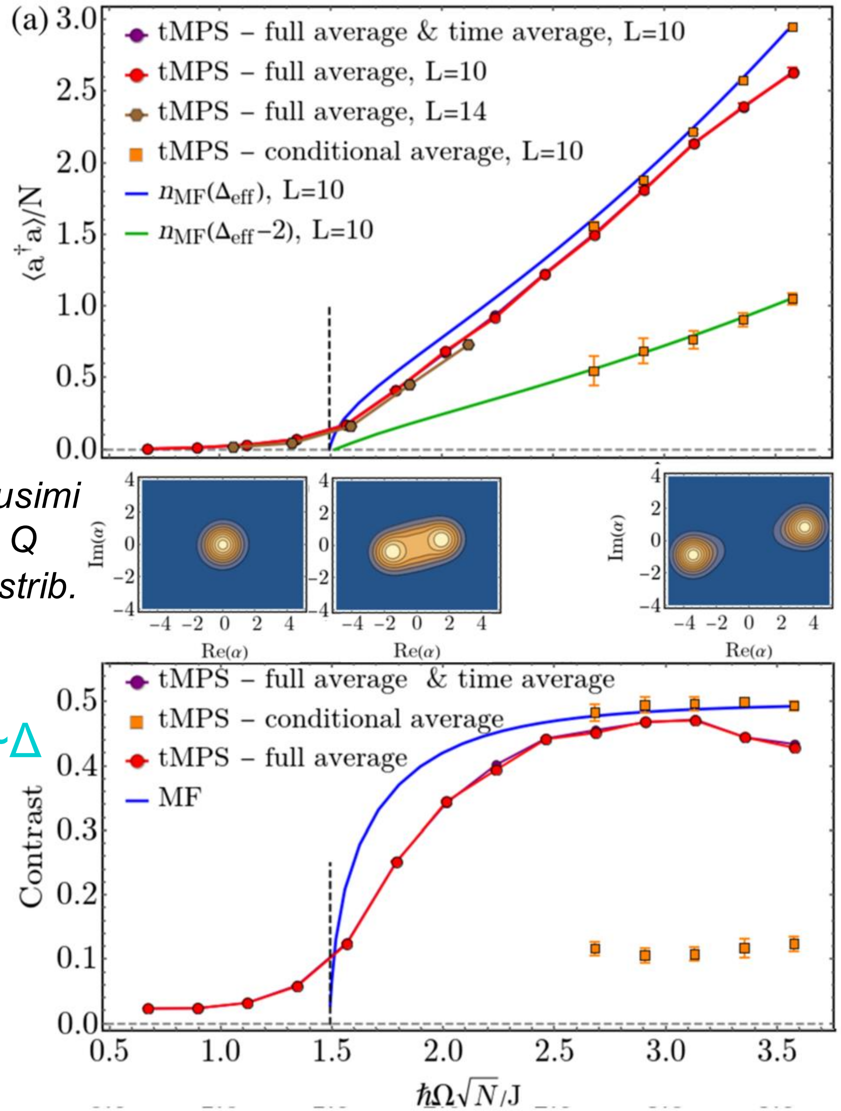
Landig, Hruby, Dogra, Landini, Mottl, Donner, Esslinger, Nature 532, 476 (2016)



(~ Ω^2/N here – coupling strength)

Beyond the EBH – Monte Carlo wavefunction+MPS approach

Halati, Sheikhan, Ritsch, Kollath, PRL 125,093604 (2020)
Halati, Sheikhan, Kollath, PRR 2, 043255 (2020)

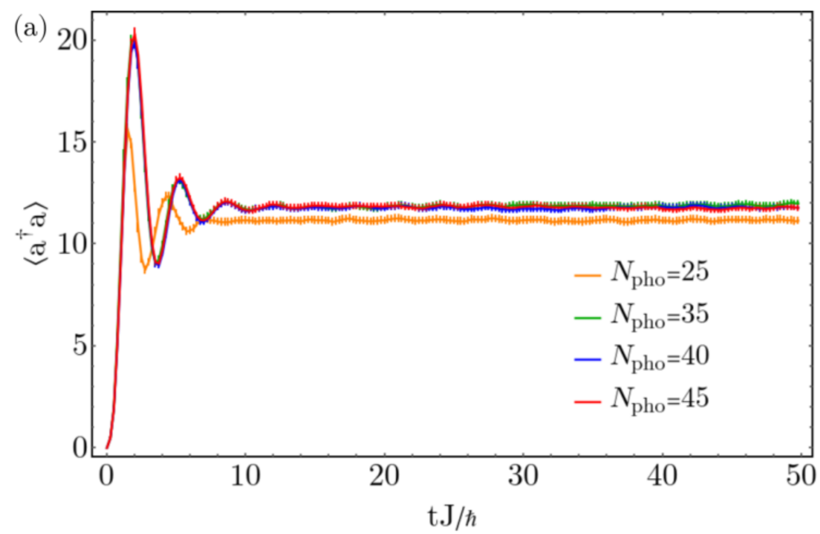


Husimi Q distrib.

~Δ

limited to small atom occupations ~1/2
while experiment has occupations 2-3 usually

seems to reach a nice steady state, though…..



although experimental preparation times can be much longer… tJ~10^4 - 10^5

Can we use other, approximate methods to treat 2D systems of reasonable size?

Phase space representations – tractable quantum mechanics?

Example: positive-P representation

M subsystems (modes, sites, *Δv*) *1, …, j*

"ket" amplitude α_j

"bra" amplitude β_j^*

Coherent state basis, *local*

$$|\alpha_j\rangle_j = e^{-|\alpha_j|^2/2}\, e^{\alpha_j \widehat{a}_j^\dagger}|\text{vac}\rangle$$

Local operator (*"kernel"*)

$$\widehat{\Lambda}(\boldsymbol{\lambda}) = \bigotimes_j \frac{|\alpha_j\rangle_j\langle\beta_j^*|_j}{\langle\beta_j^*|_j|\alpha_j\rangle_j}$$

Full system configuration:
scales as the number of modes ONLY

$$\boldsymbol{\lambda} = \{\alpha_1,\ldots,\alpha_M,\beta_1,\ldots,\beta_M\}$$

Full density matrix

$$\widehat{\rho} = \int d^{4M}\boldsymbol{\lambda}\; P_+(\boldsymbol{\lambda})\,\widehat{\Lambda}(\boldsymbol{\lambda})$$

Correlations between subsystems are in the distribution $P_+(\boldsymbol{\lambda})$

distribution $P_+(\boldsymbol{\lambda})$ is positive and real $\longrightarrow$ can be sampled randomly

Scalable quantum dynamics?

Density matrix $\hat{\rho}$ $\leftrightarrow$ distribution P_+ for the fields $\leftrightarrow$ random samples of the fields α β

Master equation:

dissipation γ

$$\frac{\partial\hat{\rho}}{\partial t} = -i\left[\hat{H},\hat{\rho}\right] + \frac{\gamma}{2}\left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}\right)$$

Bose-Hubbard site

$$\boxed{\hat{H} = \frac{U}{2}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} - \Delta\hat{a}^\dagger\hat{a}}$$

apply operator identities

Fokker Planck equation

$$\frac{\partial P_+}{\partial t} = \left\{-\frac{\partial}{\partial\alpha}\left(-iU\alpha\beta + i\Delta - \frac{\gamma}{2}\right)\alpha - \frac{\partial}{\partial\beta}\left(iU\alpha\beta - i\Delta - \frac{\gamma}{2}\right)\beta + \frac{\partial^2}{\partial\alpha^2}\left(\frac{-iU}{2}\right)\alpha^2 + \frac{\partial^2}{\partial\beta^2}\left(\frac{iU}{2}\right)\beta^2\right\}P_+$$

deterministic GPE (ket) deterministic (bra) diffusion (quantum noise)

stochastic correspondence

Stochastic (Langevin) equations:

$$\frac{d\alpha}{dt} = \left(-iU\alpha\beta + i\Delta - \frac{\gamma}{2}\right)\alpha + \sqrt{-iU}\,\alpha\xi(t)$$

$$\frac{d\beta}{dt} = \left(+iU\alpha\beta - i\Delta - \frac{\gamma}{2}\right)\beta + \sqrt{+iU}\,\beta\widetilde{\xi}(t)$$

semiclassical part quantum noise part

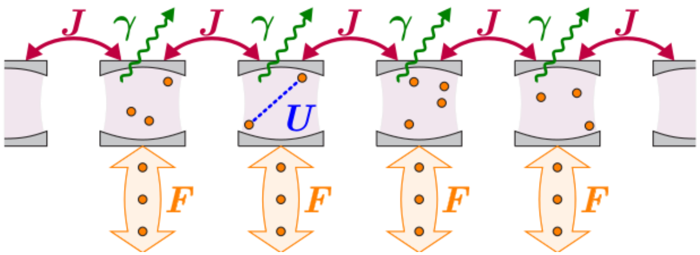
different noises

White noise

$$\langle\xi(t)\xi(t')\rangle_{\mathrm{stoch}} = \delta(t-t')$$

Driven dissipative Bose-Hubbard model

$$\hat{H} = \sum_j \hat{H}_j - \sum_{\text{connections } i,j} \left[J_{ij} \hat{a}_j^\dagger \hat{a}_i + J_{ij}^* \hat{a}_i^\dagger \hat{a}_j \right]$$



Vincentini, Minganti, Rota, Orso, Ciuti, PRA **97**, 013853 (2018)

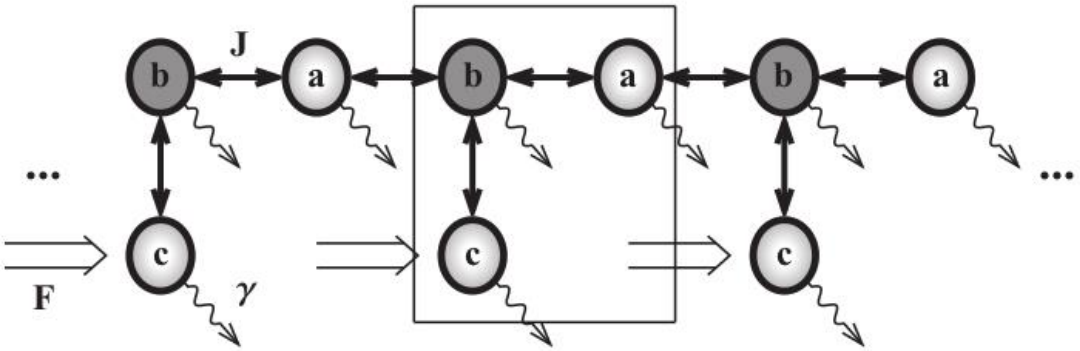
$$\hat{H}_j = -\Delta_j \hat{a}_j^\dagger \hat{a}_j + \frac{U_j}{2} \hat{a}_j^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_j + F_j \hat{a}_j^\dagger + F_j^* \hat{a}_j$$

coherent driving

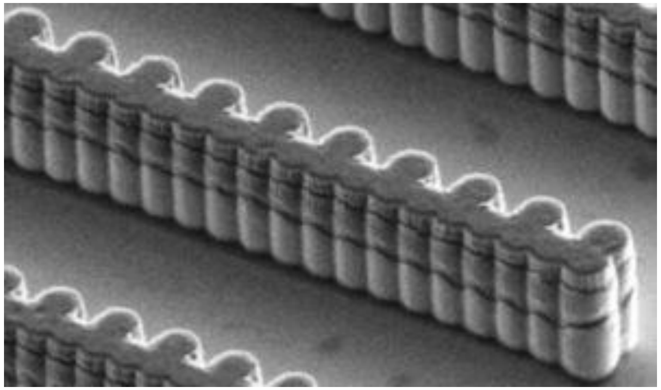
$$\frac{\partial \hat{\rho}}{\partial t} = -i \left[\hat{H}, \hat{\rho} \right] + \sum_j \frac{\gamma_j}{2} \left[2 \hat{a}_j \hat{\rho} \hat{a}_j^\dagger - \hat{a}_j^\dagger \hat{a}_j \hat{\rho} - \hat{\rho} \hat{a}_j^\dagger \hat{a}_j \right]$$

dissipation

Also structured lattices – e.g. Lieb lattice



Casteels, Rota, Storme, Ciuti, PRA **93**, 043833 (2016)



Baboux, Ge, Jacqmin, Biondi, Galopin, Lemaitre, Le Gratiet, Sagnes, Schmidt, Tureci, Amo, Bloch, PRL **116**, 066402 (2016)

The catch and a hopeful suspicion

positive-P representation:

- Evolution equations unstable
- Noise amplified → useful simulation time short

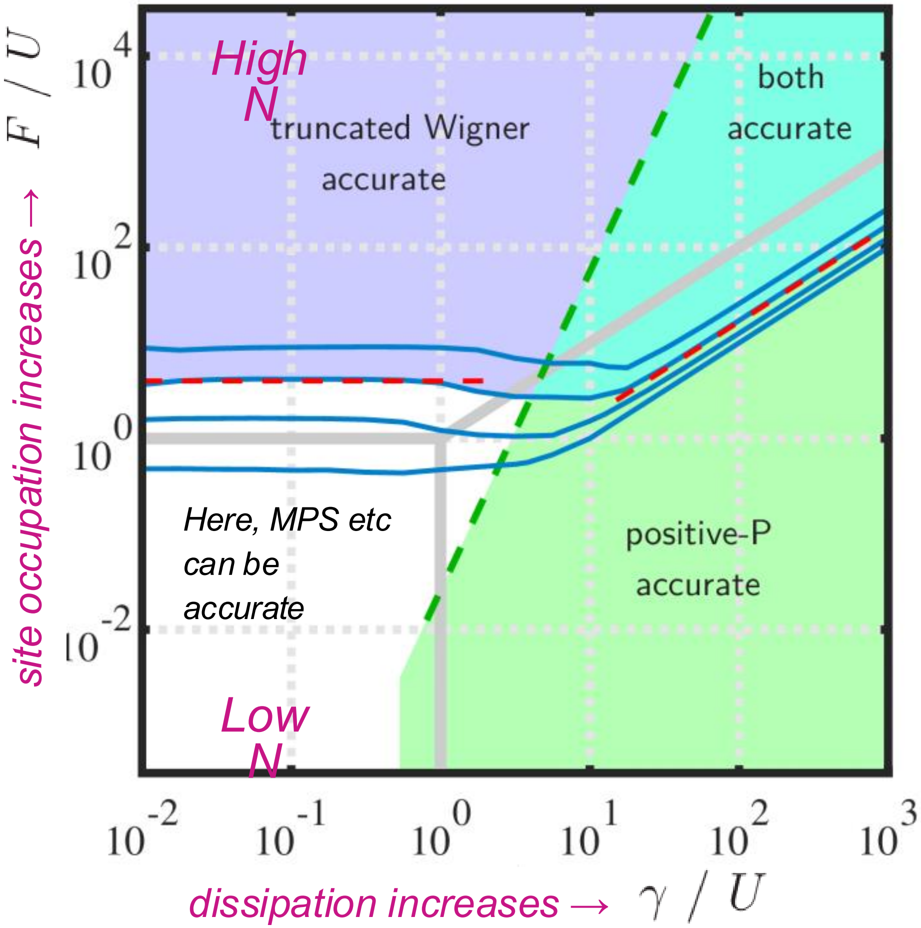
truncated Wigner representation:

- equations stable
- not necessarily trustworthy at long times because of truncation

open systems

- suspicion that stability is improved (positive-P)
- suspicion that long-time behaviour is better constrained (truncated Wigner)

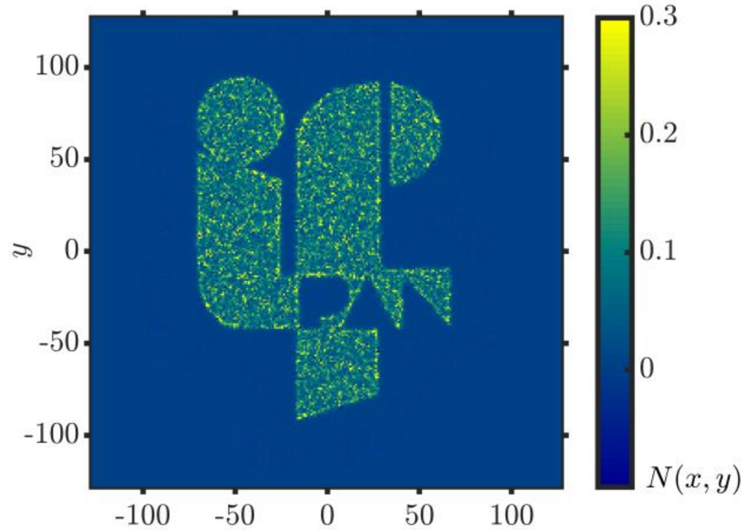
Phase space methods - dissipative Bose-Hubbard model



- Computational effort scales *linearly* with the size of the system.
  → BIG SYSTEMS, DYNAMICS
- Computational effort is independent of dimensionality.
- Time-dependent system parameters and non-uniformity are trivial to implement.
- trajectories ≈ SINGLE RUNS

256 x 256 system dynamics→steady state equiv to density matrix elements:

$3^{2\times256\times256} \sim 10^{62\,000}$



- Deuar, Ferrier, Matuszewski, Orso, Szymańska, PRX Quantum **2**, 010319 (2021).

Simulations in experimental regime using truncated Wigner

G. Orso, J. Z., P Deuar, PRL **134**, 183405 (2025)

cavity photon mode

noise due to losses

$$i d\alpha = \left[(\delta - i\kappa)\alpha - \frac{\Omega}{\sqrt{N}} \sum_j f_j \left(|\beta_j|^2 - \tfrac{1}{2}\right) \right] dt + i\sqrt{\kappa}\, dW_c$$

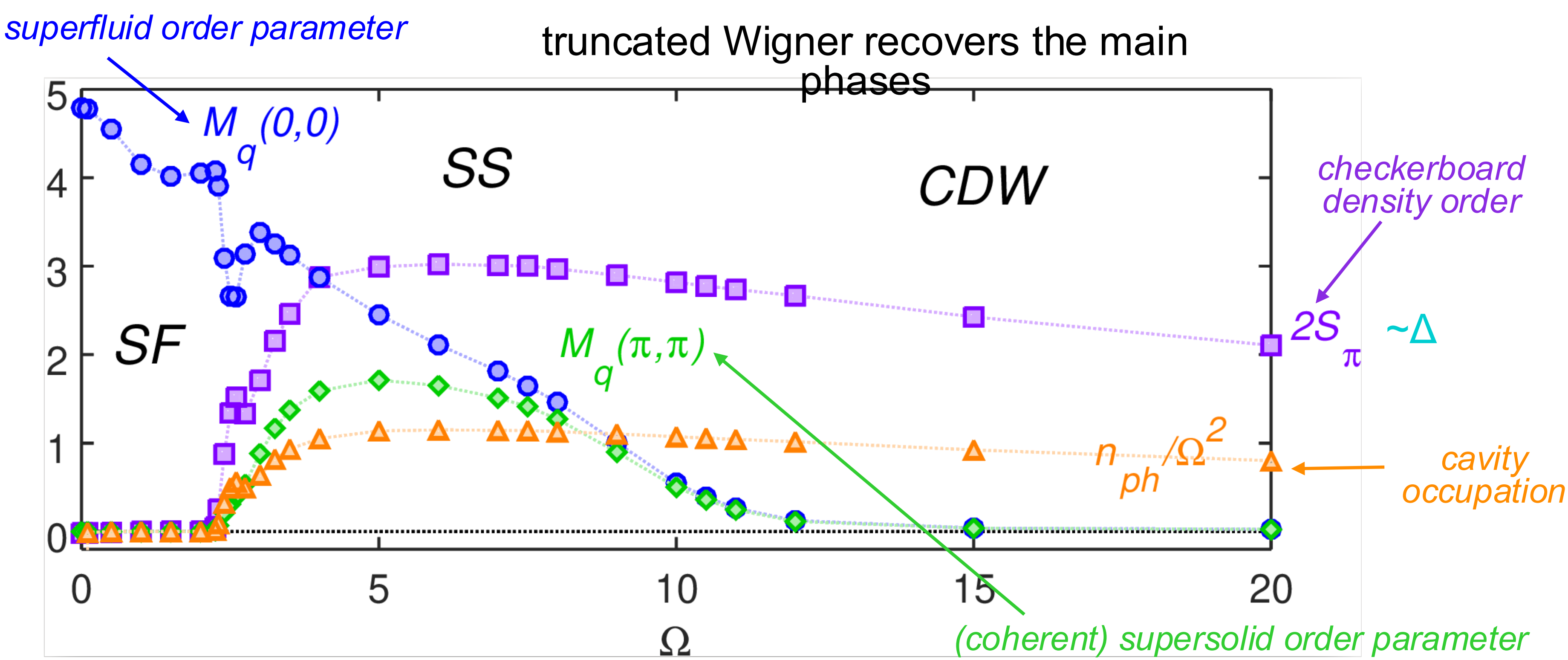
atoms in optical lattice

semiclassical part

$$i d\beta_j = \left[U\left(|\beta_j|^2 - 1\right)\beta_j - \frac{\Omega}{\sqrt{N}}(\alpha+\alpha^*) f_j \beta_j \; - J \sum_{r\in X(j)} \beta_r \right] dt$$

nearest neighbours

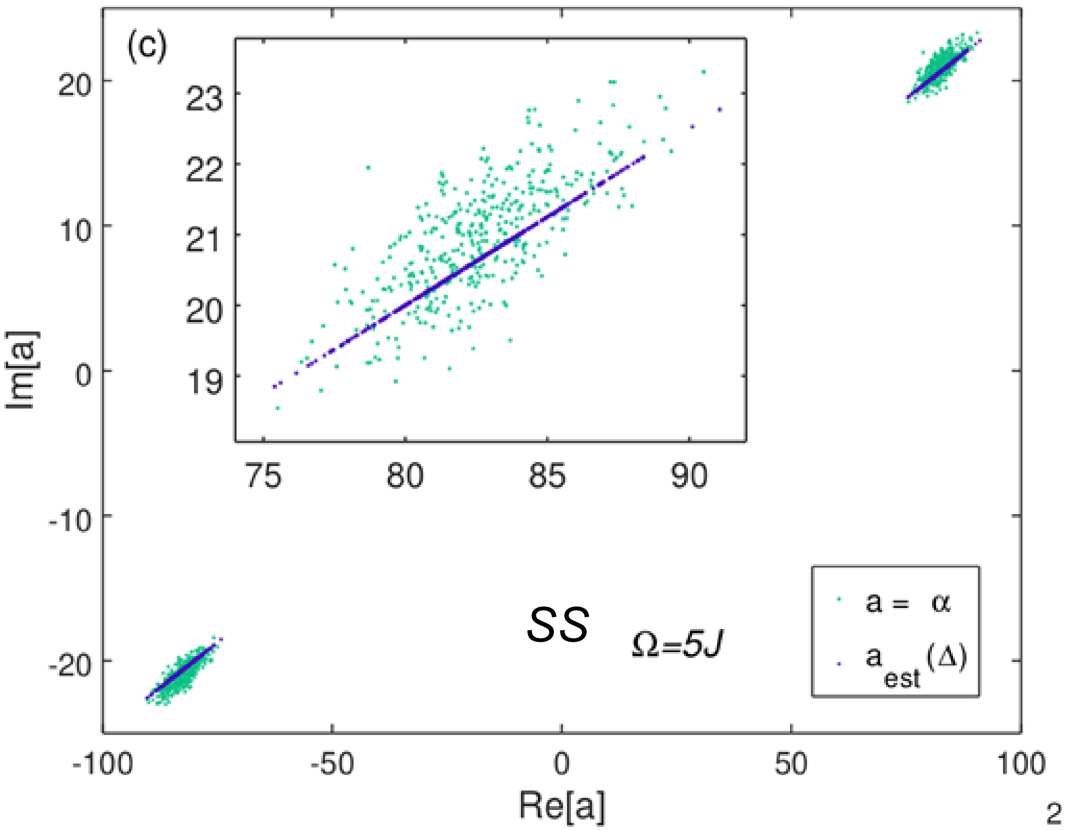
truncated Wigner recovers the main phases



Adiabatic elimination compared to simulation

full behaviour from our simulations

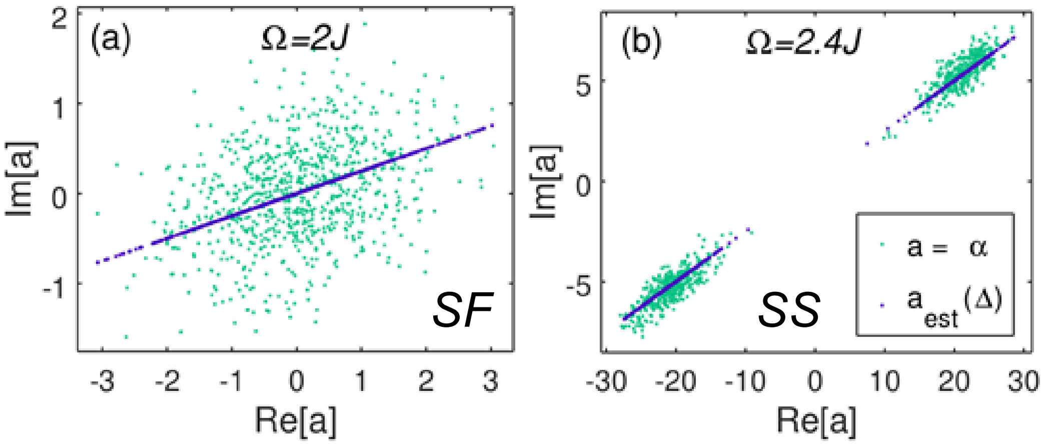
green: full QM, *blue: adiabatic EBH estimate*



$$a_{\mathrm{est}} = \frac{\Omega}{\sqrt{N}} \left(\frac{\Delta}{\delta - i\kappa} \right)$$

$$\hat{\Delta} = \sum_j f_j \hat{n}_j$$

checkerboard order parameter



G. Orso, J. Zakrzewski, PD, PRL **134**, 183405 (2025)

Metastability at long times (when adiabatic elimination relaxed)

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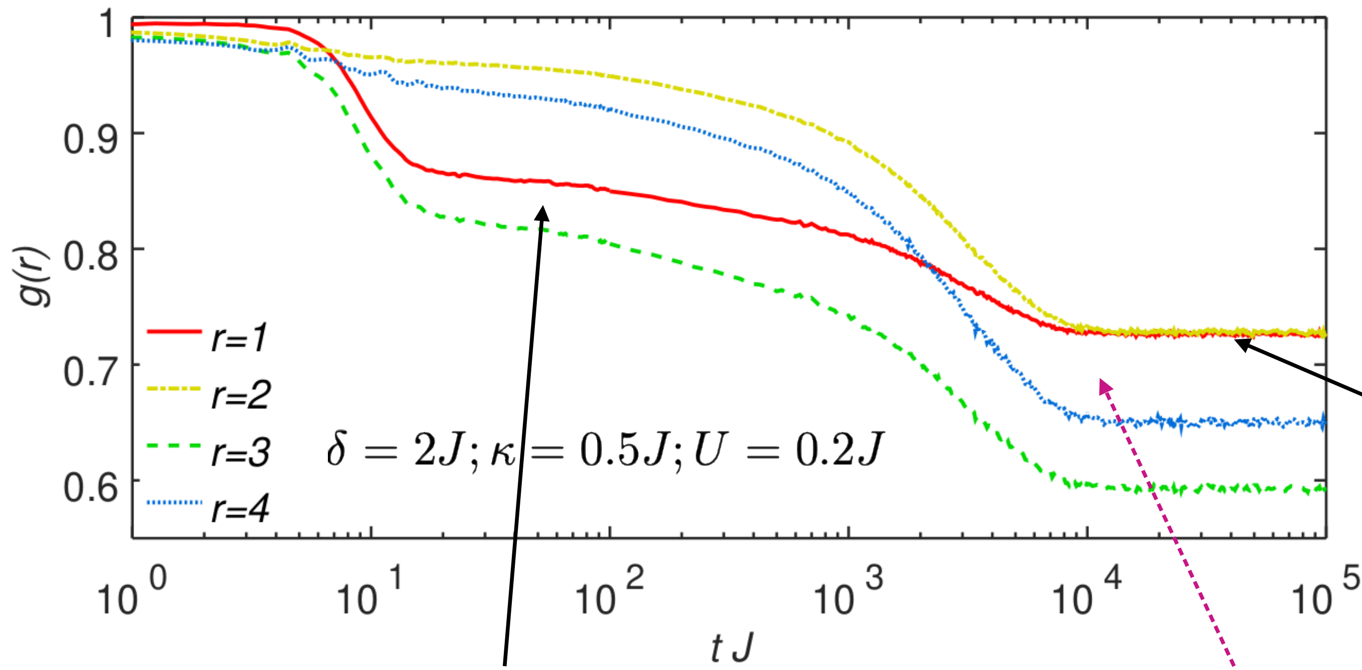


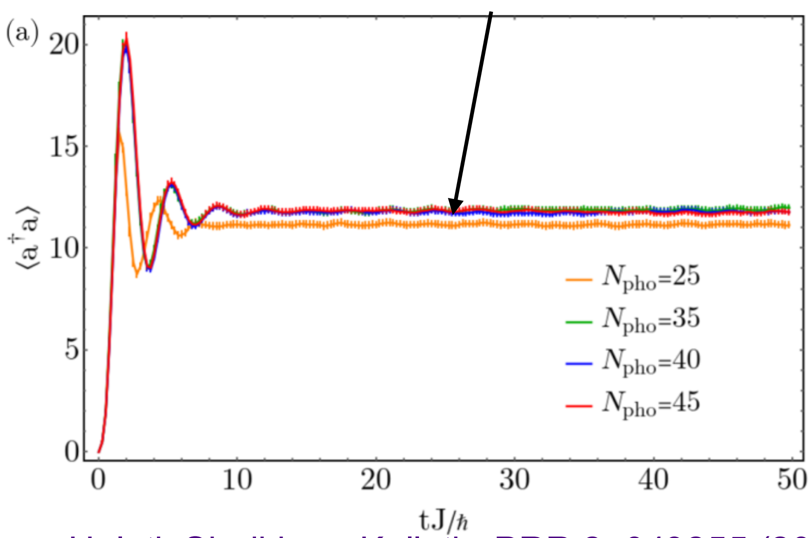
coherence at range

$$g(r) = \frac{1}{N}\sum_j^r \mathrm{Re}(\langle \hat{b}_j^\dagger \hat{b}_{j+r} \rangle)$$

real stationary state

stationary-looking state

experimental preparation timescales



from a Monte Carlo wavefunction + MPS approach

Halati, Sheikhan, Ritsch, Kollath, PRL 125,093604 (2020)

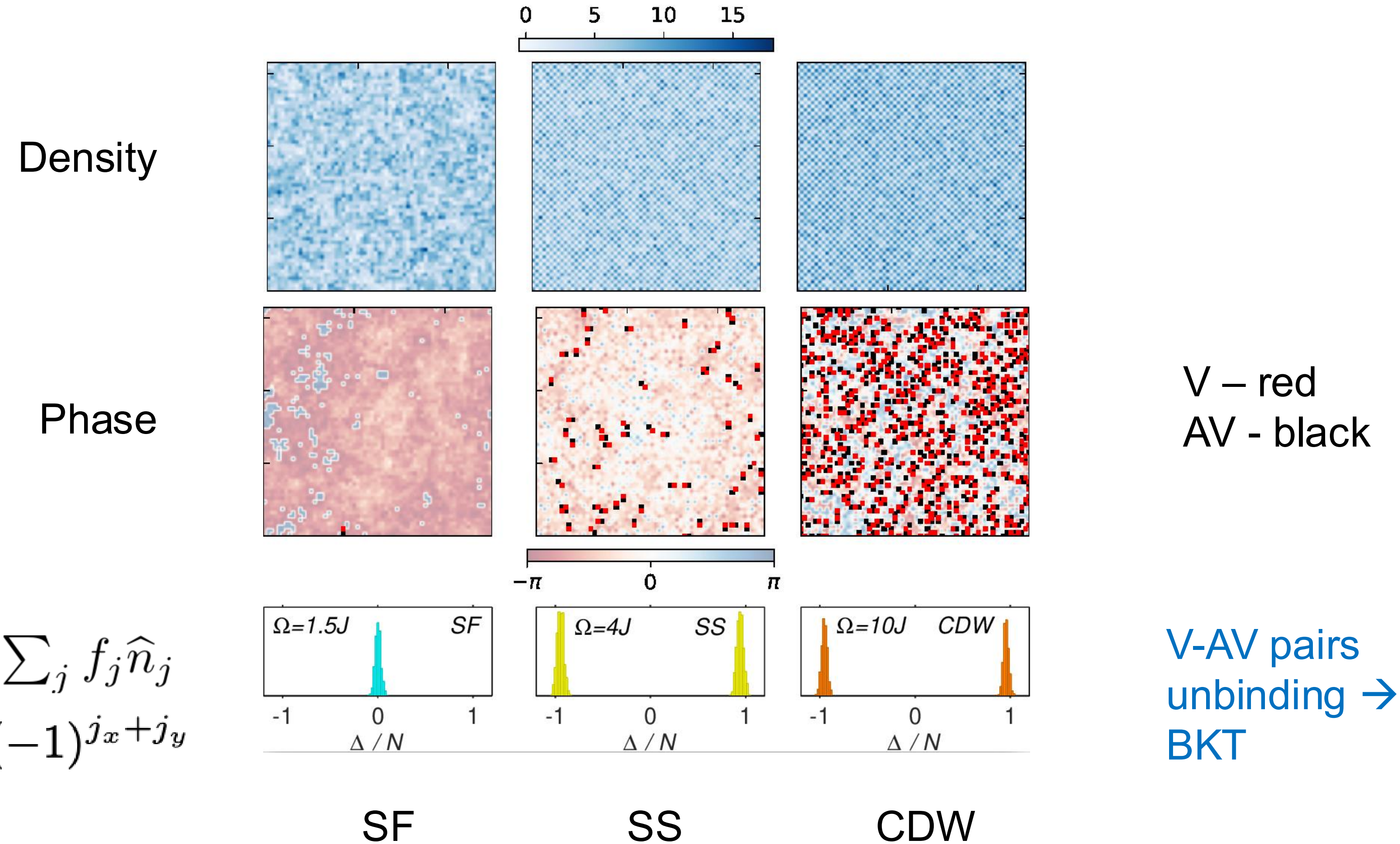
Halati, Sheikhan, Kollath, PRR 2, 043255 (2020)

Halati, Sheikhan, Kollath, PRR 2, 043255 (2020)

Phase transitions in the long-time steady state disagree with EBH

EBH: 2nd order phase transition SS/CDW here: BKT phase transition

64x64 lattice $tJ=10^4$, single realization



$$\widehat{\Delta} = \sum_j f_j \widehat{n}_j$$

$$f_j = (-1)^{j_x + j_y}$$

V-AV pairs unbinding → BKT

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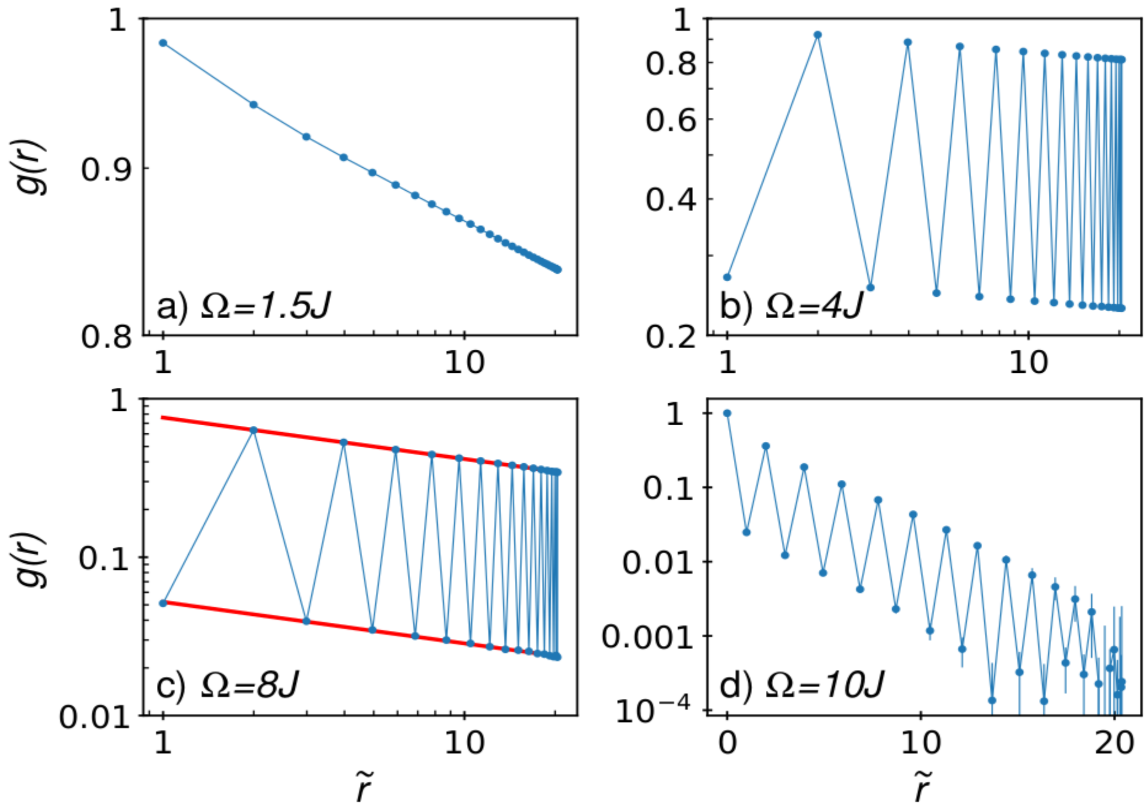
Phase transitions in the long-time steady state disagree with EBH

EBH: true long-range order in SF and SS here: quasi-long range order only; 64x64

$$g(r) = \frac{1}{N}\sum_j \mathrm{Re}(\langle \widehat{b}_j^\dagger \widehat{b}_{j+r}\rangle),$$

$$g(r) = (a + (-1)^r b)/\tilde{r}^\eta$$

$$\tilde{r} = (L/\pi)\sin(\pi r/L)$$



Exp decay in CDW

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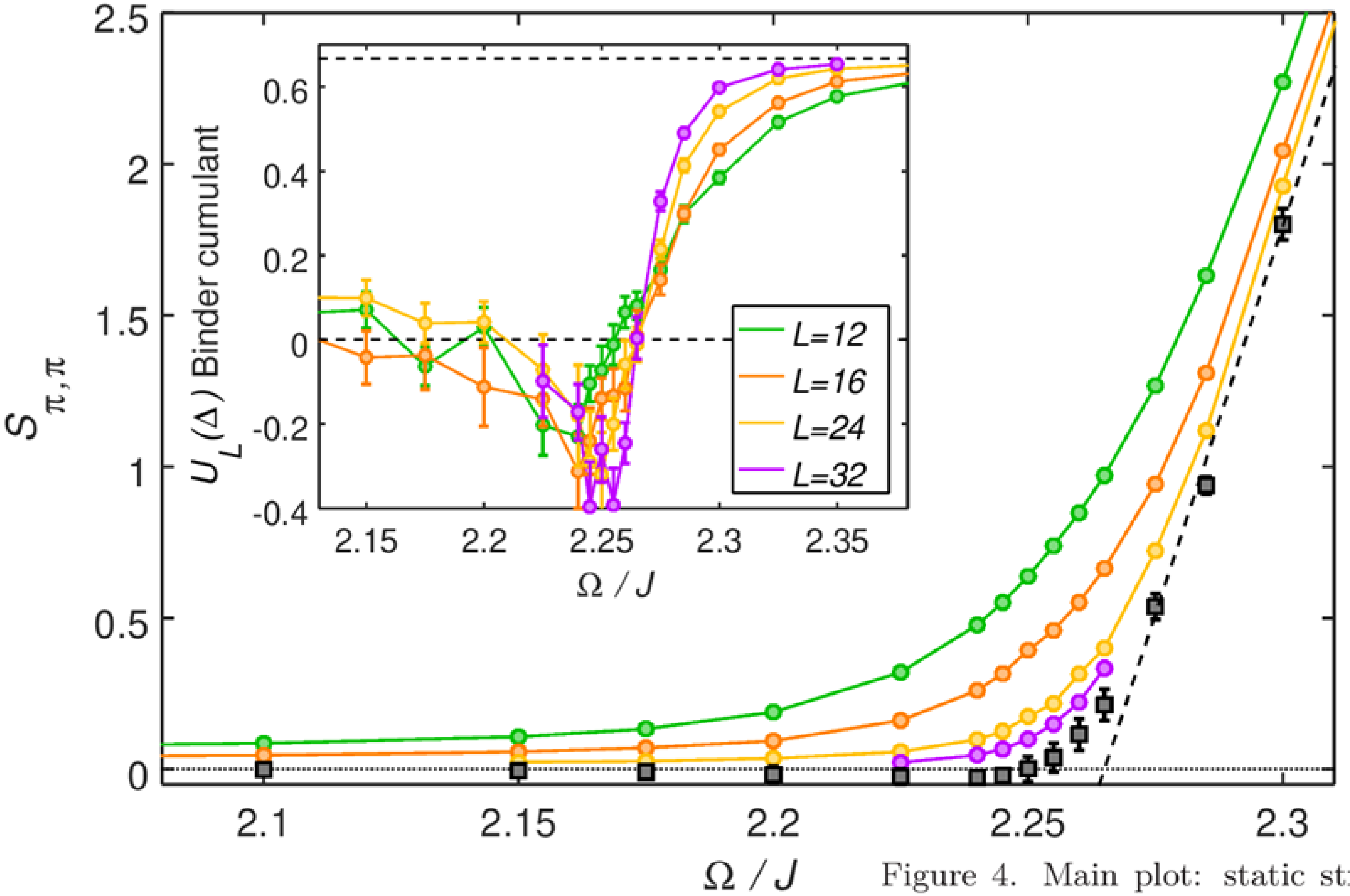
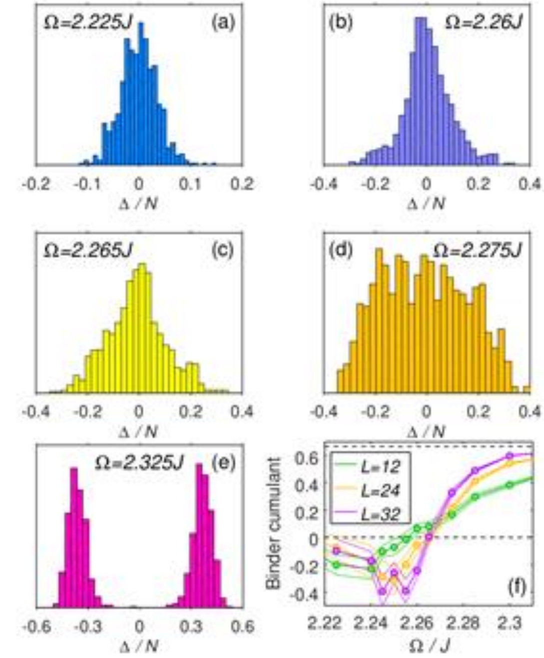
SF – SS 2nd order transition





checkerboard order parameter

$$\widehat{\Delta} = \sum_j f_j \widehat{n}_j$$

$$S_{\pi,\pi} = \sum_{ij} e^{i\pi(i-j)}(\langle n_i n_j\rangle - \langle n_i\rangle\langle n_j\rangle)/L^4$$

Binder cumulant

$$U_L(\Delta) = 1 - \langle\Delta^4\rangle/3\langle\Delta^2\rangle^2$$

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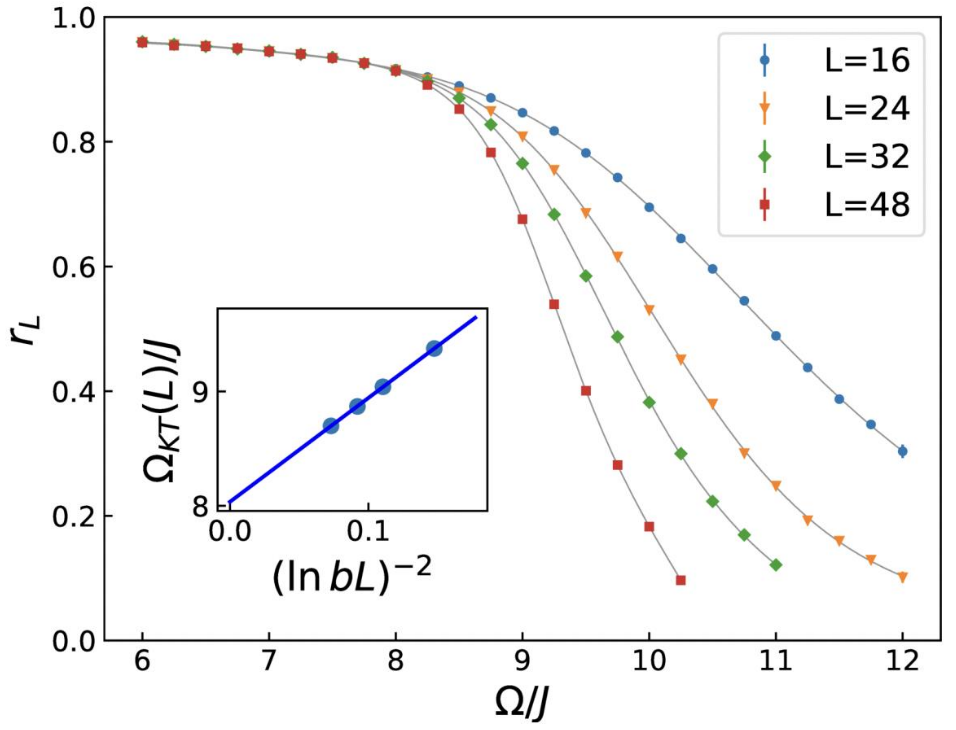
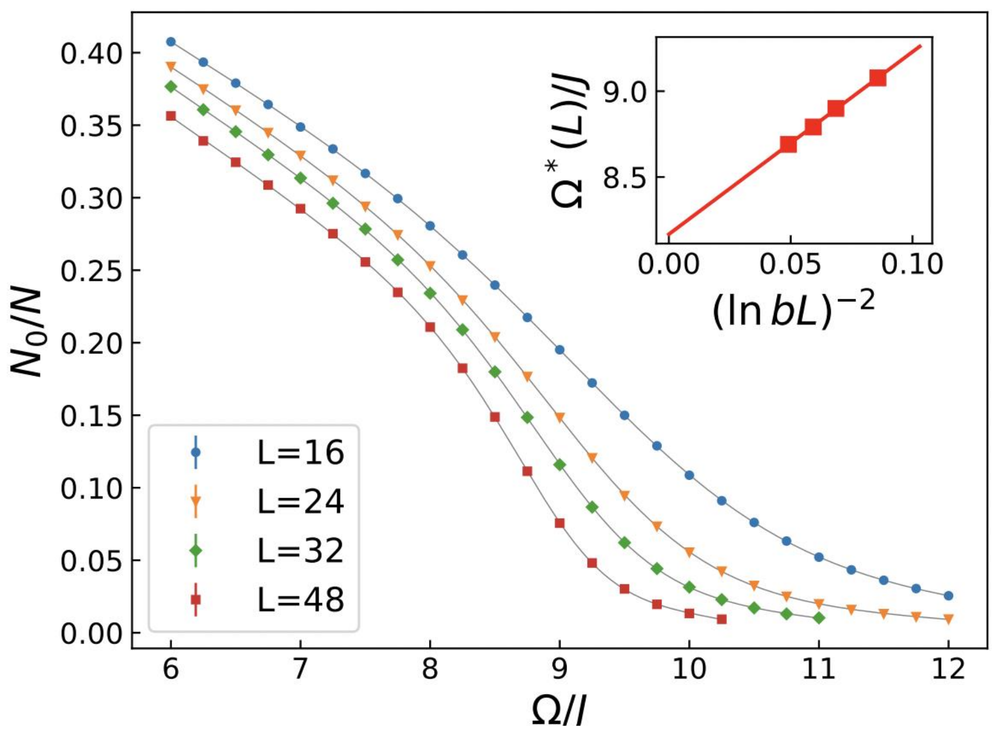
Figure 4. Main plot: static structure factor $S_{\pi,\pi}(L)$ versus Ω across the superfluid-supersolid phase transition, showing finite size scaling. Black points represent the $L \to \infty$ thermodynamic limit $A(\Omega)$, according to (6). The dashed black line is a linear fit of $A(\Omega)$ in the above threshold phase, giving the Dicke transition point as $\Omega_s = 2.265(5)$. The inset shows the Binder cumulant U_L for the "magnetization" Δ/N across the transition. Here the crossing of $U_L = 0$ occurs at $\Omega_s = 2.265(2)$. Data at $tJ = 10^4$, 800 trajectories.

BKT transition





Correction ratio $r_L = g(L/2)/g(L/4)$

$$r_L = f(L/\xi); \xi = \exp(c/\sqrt{\Omega/\Omega_c - 1})$$

$$\Omega_{KT}(L) = \Omega_c + \frac{c^2\Omega_c}{\ln^2 bL} = 8.03(4)$$

Condensate fraction – inflection point

$$\Omega^*(L) = 8.17(14)J$$

[approach from Y. Tomita & Y. Okabe (2002)]

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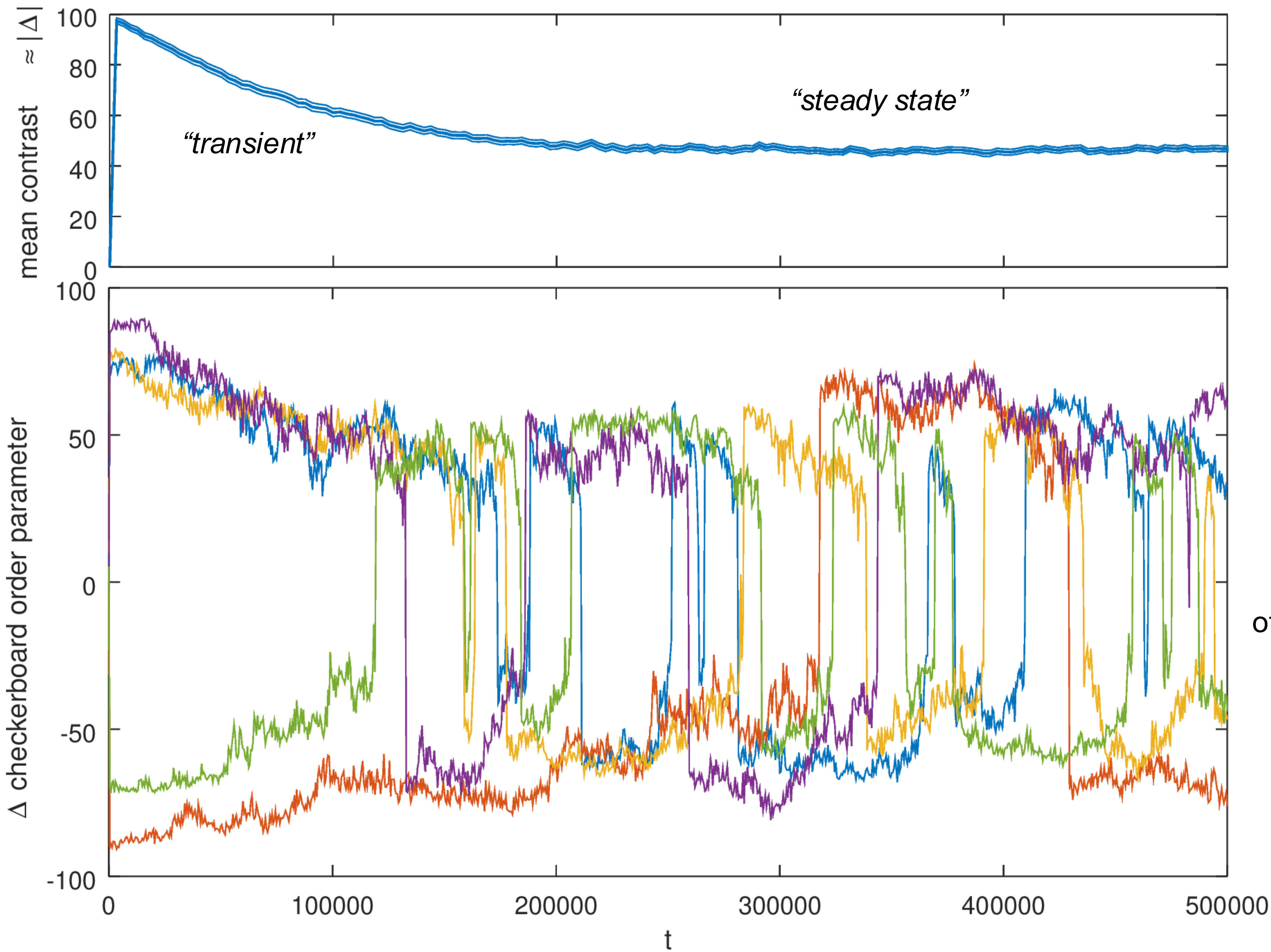
Metastability and switching

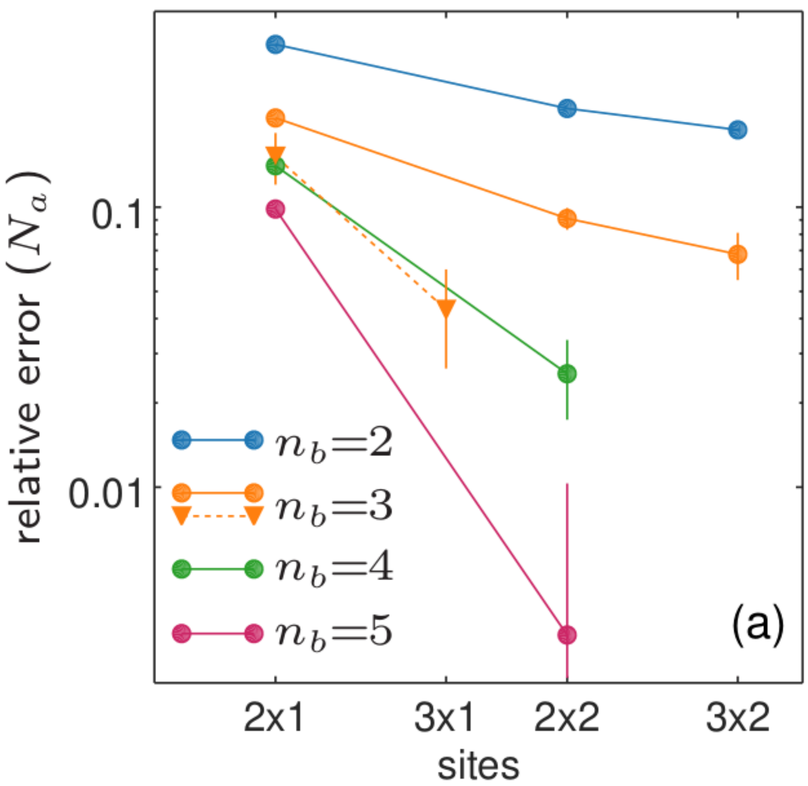
Time evolution of checkerboard order parameter Δ for mean and for single runs (supersolid regime)

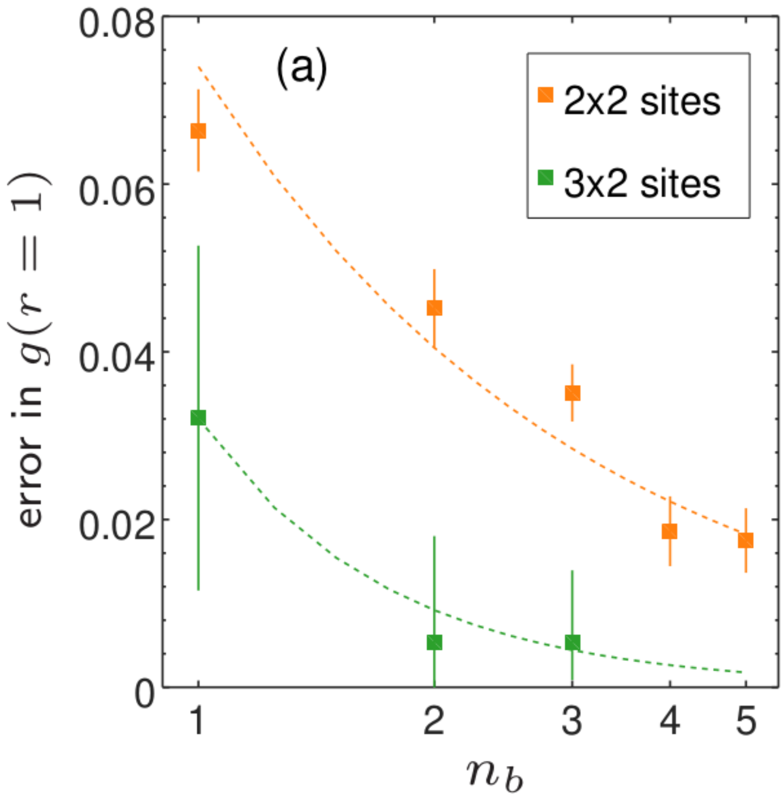


steady state
is actually
an ensemble
of metastable states

Big reveal – Improvement of accuracy with system size

in Truncated Wigner





Comparison to quantum trajectories calculations on small lattices

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Conclusions

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* We can successfully simulate such big systems with truncated Wigner when dissipative
* **Accuracy of truncated Wigner improves with system size** in a dissipative system
* Long time steady states are not the same as the medium time ones
    - nor like the adiabatically eliminated EBH states
* “steady state” is actually an ensemble of metastable states

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