Bosonic Dynamics in Optical Lattices: Effects of Driving and Dissipation

Ivana Vasić
Scientific Computing Laboratory, Center for the Study of Complex Systems,
Institute of Physics Belgrade, University of Belgrade, Serbia

Center for the Study of Complex Systems

INSTITUTE OF PHYSICS BELGRADE

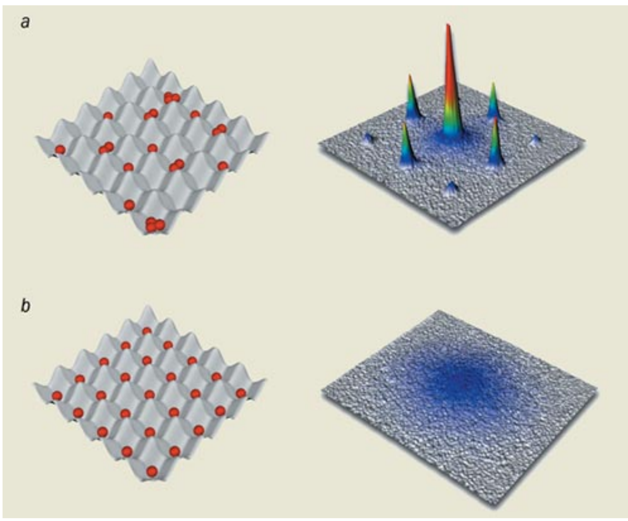
Motivation:

- Cold atoms are versatile quantum simulators
- Optical lattices loaded with cold atoms provide a clean realization of the Hubbard model [1]
- Driven optical lattices enrich the set of quantum models that can be simulated in cold-atom experiments [2]
- Localised dissipation is used as a way of probing properties of a bosons in a lattice [3]
- We study dynamical regimes of strongly correlated bosonic atoms under driving and dissipation

I) Bose-Hubbard model

- Optical lattices - standing waves of laser light loaded with cold atoms
- Parameters: tunneling amplitude J, local repulsive interaction U

$$H_{\rm BH} = -J\sum_{<m,n>}\left(b_m^\dagger b_n + \text{h. c.}\right) + \frac{U}{2}\sum_m n_m(n_m-1)$$

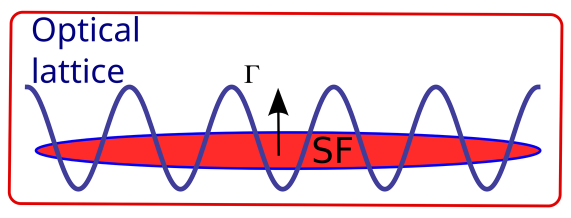


- The ground-state phase diagram at integer filling : Mott-insulator to superfluid transition [1]
- Quantum phase transition at zero temperature driven by J/U
- Numerical methods: exact diagonalization for small lattice sizes, finite-temperature Lanczos system for larger lattice sizes [2], the Gutzwiller mean-field theory [1]
- We analyze dynamics induced by a dissipative local defect
  I. Vidanović, D. Cocks, and W. Hofstetter, Phys. Rev. A 89, 053614 (2014).
- Driven optical lattices feature synthetic magnetic fields. We investigate the stability of fractional Hall states in a few-particle sample in a driven optical lattice
  A. Hudomal, N. Regnault, and I. Vasić, Phys. Rev. A 100, 053624 (2019).
- Recently, quantum transport within the Fermi-Hubbard model has been investigated in cold-atom setup
  The onset of bad-metal behavior, characterized by the resistivity linear in temperature, has been established.
  We investigate the conductivity in the strongly-interacting regime of the Bose-Hubbard model
  I. Vasić and J. Vučičević, Phys. Rev. B 110, 064501 (2024).

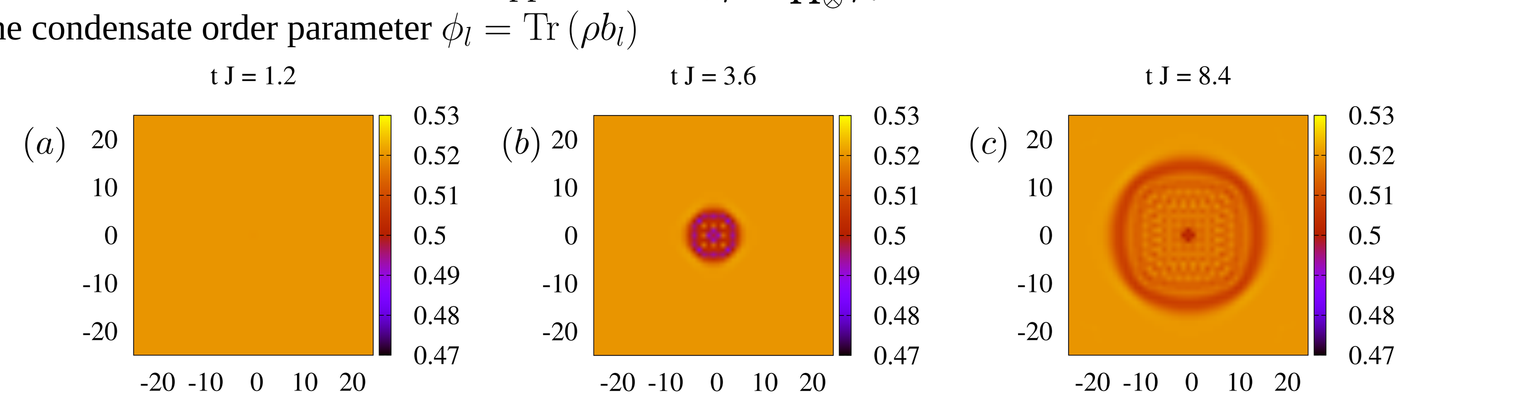
II) Dissipation through localised loss

- Localised dissipation as a way of probing properties of a quantum system [4]

$$\frac{d\rho}{dt} = -i\,[H,\rho] + \frac{\Gamma}{2}\left(2a_l\rho a_l^\dagger - \{n_l,\rho\}\right)$$



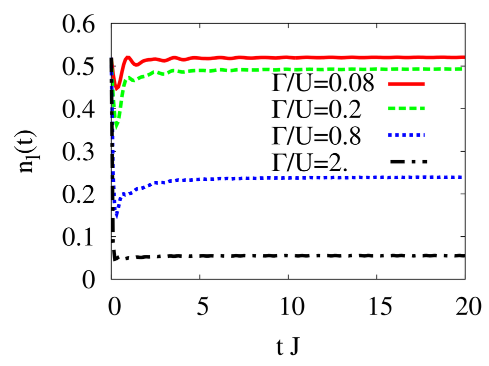
- Method - the Gutzwiller mean-field approximation, $\rho = \prod_\otimes \rho_l$.
- The condensate order parameter $\phi_l = \mathrm{Tr}\,(\rho b_l)$

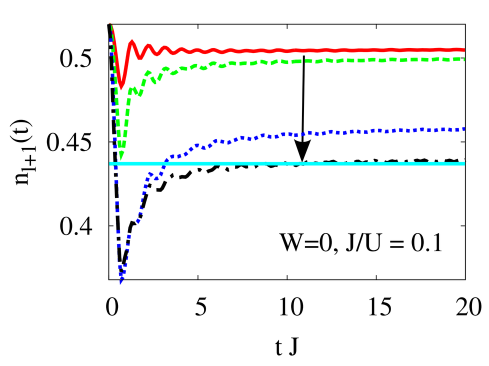


- Experimentally accessible quantity:

$$\frac{dN(t)}{dt} = \Gamma n_l(t)$$

rate of expelled atoms





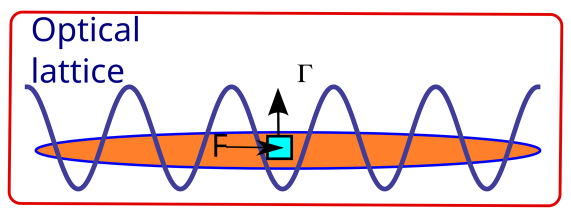
- Lossy site: initial fast exponential decay of the local density followed by a saturation
- The properties of the neighbouring site do not change so rapidly with Γ

- In the quasi-steady state, when the dissipation is balanced by hopping, a constant current of expelled particles develops
- Analytical result for the steady state of an effective single cavity [5]

$$n_l = \langle b_l^\dagger b_l\rangle = \left|\frac{2F}{U}\right|^2 \frac{1}{|c|^2} \times \frac{\mathcal{F}(1+c,1+c^*,8|F/U|^2)}{\mathcal{F}(c,c^*,8|F/U|^2)}$$

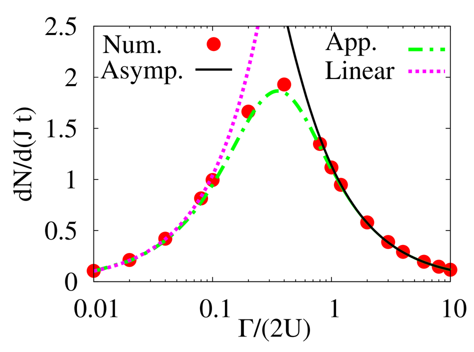
$$c = 2(-\mu - i\Gamma/2)/U, F = 4J\phi_{l+1}$$

$\mathcal{F}(c,d,z)$ is a generalized hypergeometric function.



Quantum Zeno effect: $\gamma \gg 1$

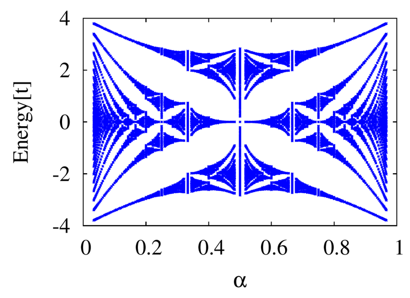
$$\frac{dN}{dt} \approx 4z^2|\phi_{l+1}^{\rm imp}|^2\frac{J^2}{\Gamma}\left(1+\frac{4\mu^2}{\Gamma^2}\right)^{-1}$$



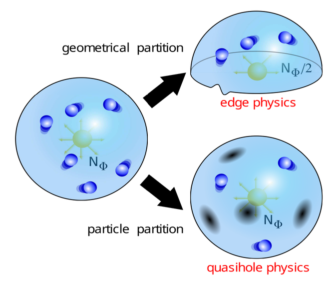
III) Probing fractional Hall states in driven optical lattices

- The Harper-Hofstadter model: a two-dimensional lattice with uniform flux

$$\hat{H}_0 = -J\sum_{m,n}\left(\hat{a}_{m+1,n}^\dagger\hat{a}_{m,n} + \text{h. c.}\right) - J\sum_{m,n}\left(e^{i2\pi\alpha m}\hat{a}_{m,n+1}^\dagger\hat{a}_{m,n} + \text{h. c.}\right)$$



- The spectrum as a function of flux - the Hofstadter butterfly
- Strong repulsive interaction can stabilize fractional Hall states, for both fermions and bosons
- For $N_p/N_\phi = 1/2$, $N_\phi = \alpha\, L_x \times L_y$, the ground state corresponds to the Laughlin state $\psi(z_1,z_2,\ldots,z_{N_p}) \sim \prod_{i<j}(z_i-z_j)^2$



- Topological properties encoded in the entanglement $\rho_A = \mathrm{Tr}_B\rho$
  - Entanglement entropy $S = -\mathrm{Tr}\,(\rho_A\log\rho_A)$
  - Entanglement spectra [6] $\rho_A = e^{-H_A}$
  - Momentum resolved particle-entanglement spectrum – *counting*
- Driven Bose-Hubbard model [9] $\hat{\mathcal{H}}_{\rm lab}(t) = \hat{H}_{\rm BH} + \hat{H}_{\rm drive}(t) + \omega\sum_{mn} n\,\hat{n}_{m,n}$
- Linear potential inhibits hopping along one direction
- Tunneling is restored by a resonant drive $\hat{H}_{\rm drive}(t) = \frac{\kappa}{2}\sum_{m,n}\sin\left(\omega t - \phi_{m,n} + \frac{\phi}{2}\right)\hat{n}_{m,n}$, $\phi_{m,n} = \phi(m+n)$
- From the Floquet theorem, the time-evolution operator is given by

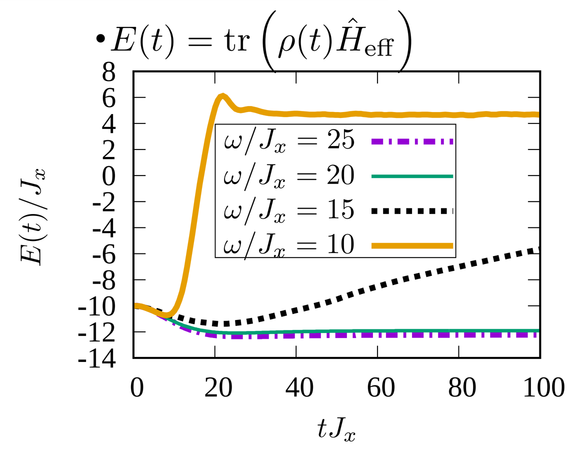
$$\hat{U}(t,t_0) = e^{-i\omega t\hat{V}}e^{-i\hat{K}(t)}e^{-i(t-t_0)\hat{\mathcal{H}}_{\rm eff}}e^{i\hat{K}(t)}e^{i\omega t_0\hat{V}}$$

- Micromotion operator $\hat{K}(t)$, periodic
- For a finite-size system in the high-frequency regime
- Time-independent effective Hamiltonia $\hat{\mathcal{H}}_{\rm eff}$

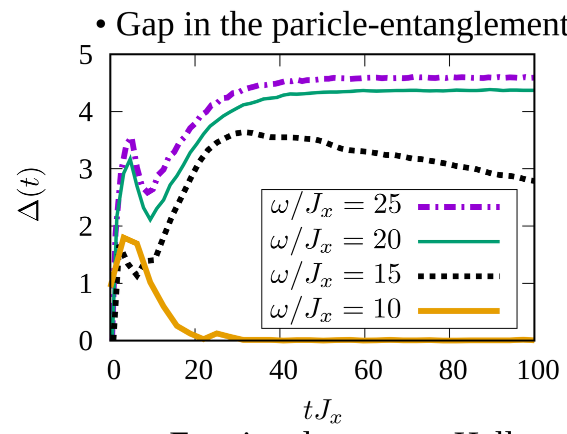
$$\hat{H} = \hat{H}_0 + \frac{U}{2}\sum_{m,n}\hat{n}_{m,n}(\hat{n}_{m,n}-1)$$

$\boxed{N_p = 5, U/J_x = 10, \eta/J_x = 0.05}$

- $E(t) = \mathrm{tr}\left(\rho(t)\hat{H}_{\rm eff}\right)$



- Gap in the paricle-entanglement spectrum

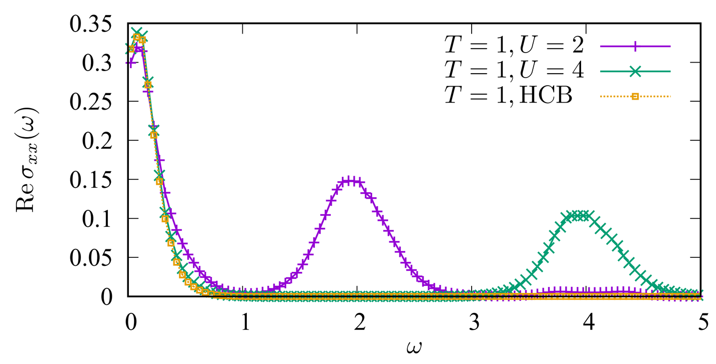


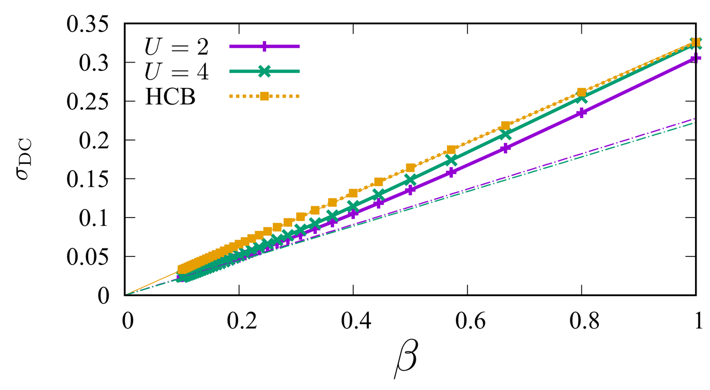
- We slowly restore the tunneling amplitude along the y direction, $J_y(t) = J_y\tanh(\eta t)$, and slowly ramp up the driving amplitude $\kappa(t) = \kappa\tanh(\eta t)$
- Fractional quantum Hall states down to $\omega/J_x \geq 20, U/J_x = 10$, and down to $\omega/J_x \geq 15, U/J_x = 1$

IV) Conductivity of lattice bosons at high temperatures

- The retarded current-current correlation function $C_{xx}(t) = -i\theta(t)\langle[J_x(t),J_x]\rangle_\beta, \beta = 1/T$
- Lattice current $J_x = -iJ\sum_{\langle ij\rangle_x}\left(b_i^\dagger b_j - H.\ c.\right)$
- Kubo formula for the optical conductivity [7] $\sigma_{xx}(\omega) = \frac{i}{\omega+i\eta}\left(\langle -E_{\rm kin}^x\rangle_\beta + C_{xx}(\omega)\right)$

$$\mathrm{Im}\,C_{xx}(\omega) = \frac{\pi}{Z(\beta)}\sum_{n,m}|\langle n|J_x|m\rangle|^2\left(e^{-\beta E_n} - e^{-\beta E_m}\right)\delta\left(\omega + E_n - E_m\right)$$





- The analysis of the Kubo formula

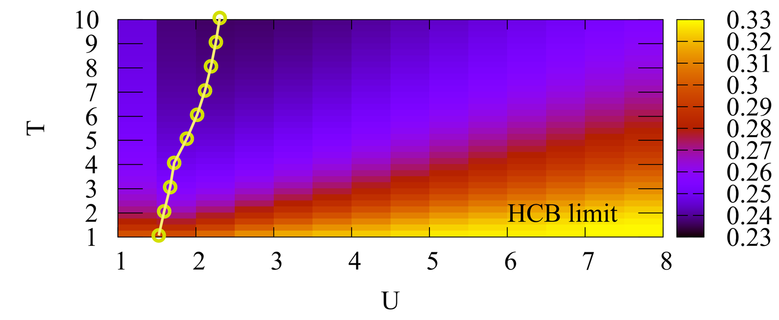
$$\sigma_{DC} = \frac{\beta}{Z(\beta)}\sum_n \lim_{\omega\to 0} f_n(\omega)e^{-\beta E_n} \qquad f_n(\omega) = \lim_{\omega\to 0,\Delta\omega\to 0}\frac{\pi}{\Delta\omega}\sum_{m:|E_n-E_m-\omega|<\Delta\omega/2}|\langle n|J_x|m\rangle|^2.$$

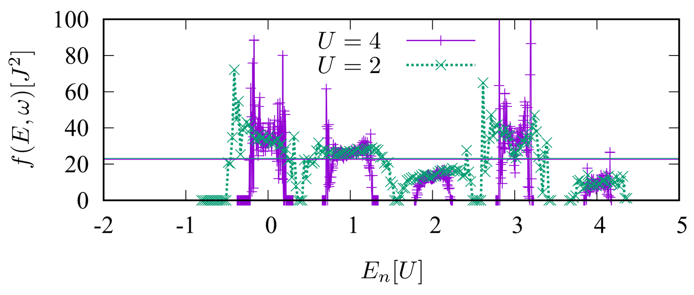
- We find the regime of resistivity linear in temperature

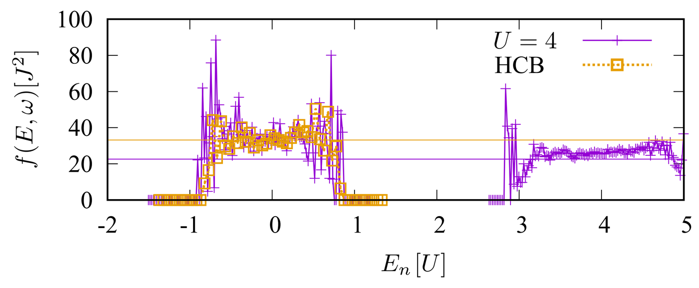
$$\sigma_{\rm DC}(\beta) \approx c_1\beta + c_2\beta^2$$

$$c_1 \approx \langle f_n(\omega)\rangle_n \equiv \frac{1}{\dim\mathcal{H}}\sum_n f_n(\omega)$$

$$c_2 \approx \langle f_n(\omega)\rangle_n\langle E_n\rangle_n - \langle f_n(\omega)E_n\rangle_n$$







- When the interactions are very strong $c_1 \sim J$
- As the tunneling becomes stronger, this dependence takes quadratic form $c_1 \sim J^2$
- At very strong coupling and half filling, we identify a separate linear resistivity regime at lower temperature, corresponding to the hard-core boson regime [8]

References

[1] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).
[2] A. Eckardt, Rev. Mod. Phys. 89, 011004 (2017).
[3] P. Prelovšek and J. Bonča, Springer Series in Solid–State Sciences 176 (2013).
[4] G. Barontini, R. Labouvie, F. Stubenrauch, A. Vogler, V. Guarrera, H. Ott, PRL 110, 035302 (2013).
[5] P. D. Drummond, D. F. Walls, JPA 13 725 (1980).
[6] A. Sterdyniak, N. Regnault, B. A. Bernevig, Phys. Rev. Lett. 106, 100405 (2011).
[7] G. D. Mahan, Many Particle Physics, Third Edition (Plenum, New York, 2000)
[8] N. H. Lindner and A. Auerbach, Phys. Rev. B 81, 054512 (2010).

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