Nonstabilizerness in open XXZ spin chains

D. Sticlet[1], B. Dóra[2], D. Szombathy[2], G. Zaránd[2], C. P. Moca[2,3]

[1] National Institute for R&D of Isotopic and Molecular Technologies, Romania
[2] Budapest University of Technology and Economics, Hungary
[3] University of Oradea, Romania

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Nonstabilizerness and the quantum advantage

Why do we need non-stabilizer states?

- **Stabilizer states**: prepared by Clifford operations (CNOT, Pauli, Hadamard, ...) from $|0\rangle^{\otimes L}$.
- Stabilizer states are classically simulable in polynomial time (Gottesman–Knill theorem [1]).
- Computational hardness necessary to achieve quantum advantage requires "magic states" (non-stabilizer states) to obtain a universal gate set [2].

How to quantify the degree to which a state is non-stabilizer?
Stabilizer Rényi Entropy (SRE) measures the distance of a state from the set of efficiently simulable stabilizer states [3].
Let L be the number of qubits and the set $\mathcal{P}_L$ of Pauli strings P,

$$\mathcal{P}_L = \{\bigotimes_{j=1}^{L} P_j \mid P_j \in \{I, X, Y, Z\}\}, \quad |\mathcal{P}_L| = 4^L.$$

The expectation values $\langle\psi|P|\psi\rangle^2$ form a probability distribution. For stabilizer states, the distribution is peaked: $\langle\psi|P|\psi\rangle^2 = 1$ (2^L values), and $\langle\psi|P|\psi\rangle^2 = 0$ ($4^L - 2^L$ values). Non-stabilizer states have a more spread-out distribution measured by the SRE:

$$M_n(|\psi\rangle) = \frac{1}{1-n}\log_2 \sum_{P\in\mathcal{P}_L} \frac{|\langle\psi|P|\psi\rangle|^{2n}}{2^L}, \quad \text{(here } n = 2\text{)}.$$

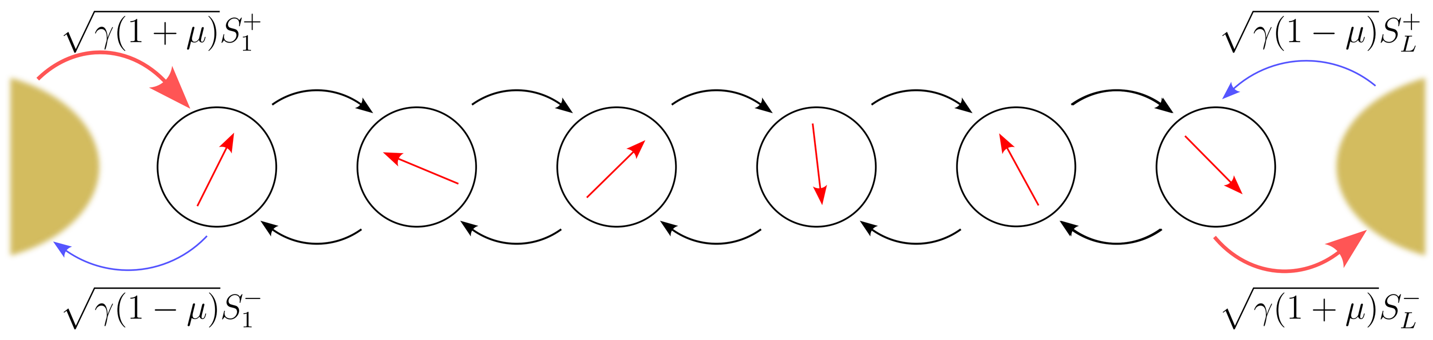
$M_2(|\psi\rangle) = 0$ iff $|\psi\rangle$ is a stabilizer state and $M_2(|\psi\rangle) > 0$ for non-stabilizer states.

Questions

- Is nonstabilizerness preserved under system evolution with environmental interaction?
- Are there non-trivial SRE NESSs in open quantum systems?
- Can SRE dynamics reveal transport properties of the system?
- Can we distinguish the contribution of quantum correlations to SRE?

Model: Boundary-driven XXZ chain

Infinite-temperature SRE dynamics in the open XXZ spin-$\frac{1}{2}$ chain:



$$H = \frac{J}{4}\sum_{j=-L/2}^{L/2-1}(X_jX_{j+1} + Y_jY_{j+1} + \Delta Z_jZ_{j+1}),$$

with hopping $J > 0$ and anisotropy Δ.
The chain is driven at its boundaries by Lindblad operators injecting a spin-↑ current from left to right:

$$F_{1,1} = \sqrt{\gamma(1-\mu)}\,S_1^-, \qquad F_{1,2} = \sqrt{\gamma(1+\mu)}\,S_1^+,$$
$$F_{L,1} = \sqrt{\gamma(1+\mu)}\,S_L^-, \qquad F_{L,2} = \sqrt{\gamma(1-\mu)}\,S_L^+,$$

with driving parameter $\mu \in [0, 1]$.
The system evolves under the Lindblad master equation:

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_j\Big(F_j\rho F_j^\dagger - \tfrac{1}{2}\{F_j^\dagger F_j, \rho\}\Big).$$

The initial state is maximally disordered: $\rho(t=0) \propto \mathbb{1}$.
Transport regimes at $T = \infty$ characterized by the *dynamical exponent z* for the steady-state spin current $j \sim L^{1-z}$ [4]:

Regime	Condition	z
Ballistic	$\Delta < 1$	1
Superdiffusive	$\Delta = 1$	3/2
Diffusive	$\Delta > 1$	2

How does the SRE generation reflect these distinct transport regimes?

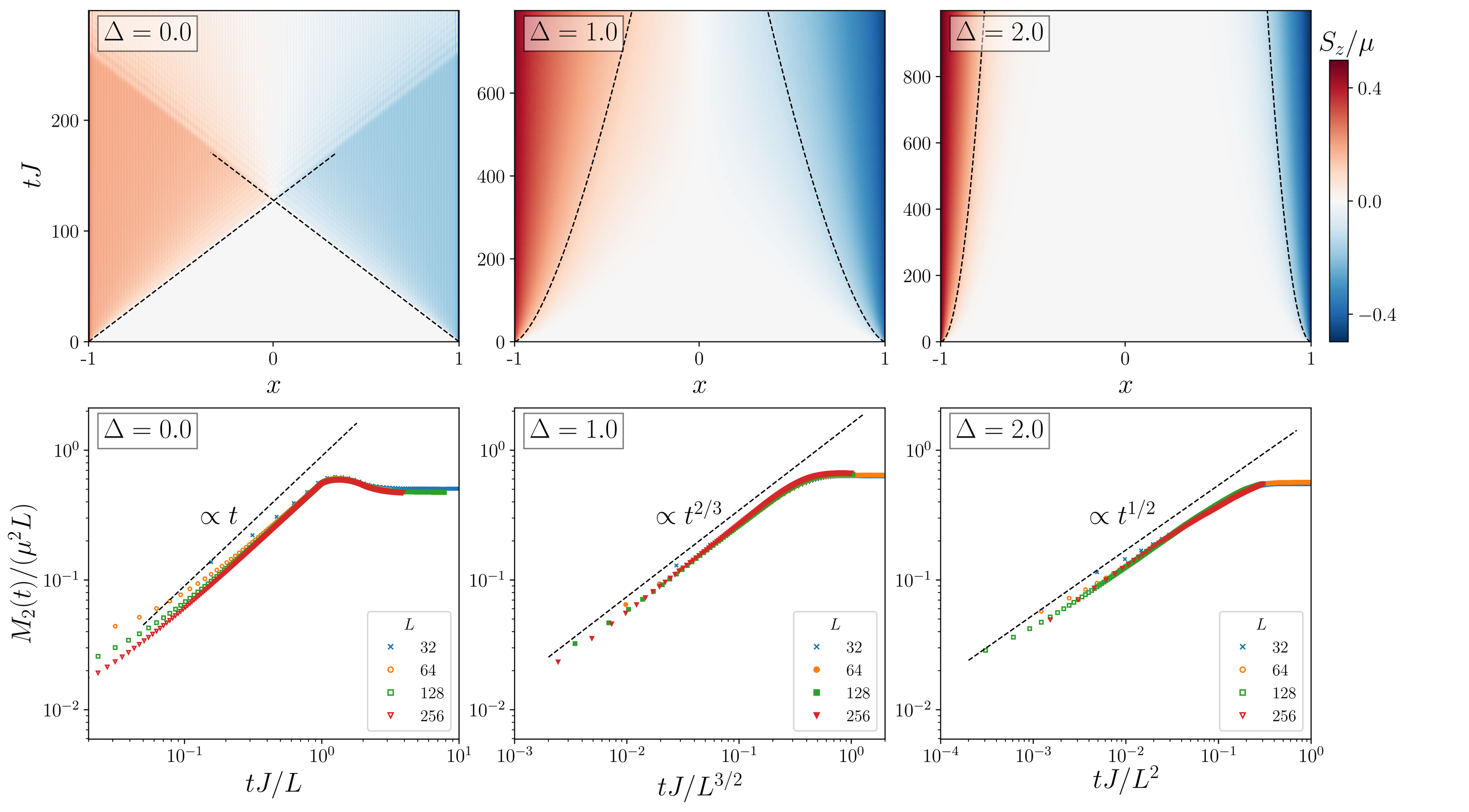
Methods

Measuring nonstabilizerness using the stabilizer 2-Rényi entropy for open systems:

$$M_2(\rho) = -\log_2 \frac{\sum_{P\in\mathcal{P}_L}\mathrm{tr}(P\rho)^4}{\sum_{P\in\mathcal{P}_L}\mathrm{tr}(P\rho)^2},$$

Time evolution performed using TEBD algorithm for the vectorized $\rho(t) \mapsto \vec{\rho}(t)$ using ITensor library [5]. SRE extracted from the MPS norm [6].

SRE dynamics and universal scaling



(Top) Magnetization and (bottom) SRE evolution for $\Delta \in \{0, 1, 2\}$ at $\mu = 0.05$.

SRE generation is sensitive to the transport regime:

- Initial growth depending on the transport regime, similar to magnetization dynamics.
- Universal scaling law for different system sizes and driving parameters.
- The exact scaling is preserved for small driving $\mu \lesssim 0.1$ [linear response].

Universal scaling law for SRE:

$$M_2(L,t) \sim \begin{cases} t^{1/z} & t \ll L^z, \\ L & t \gg L^z. \end{cases}$$

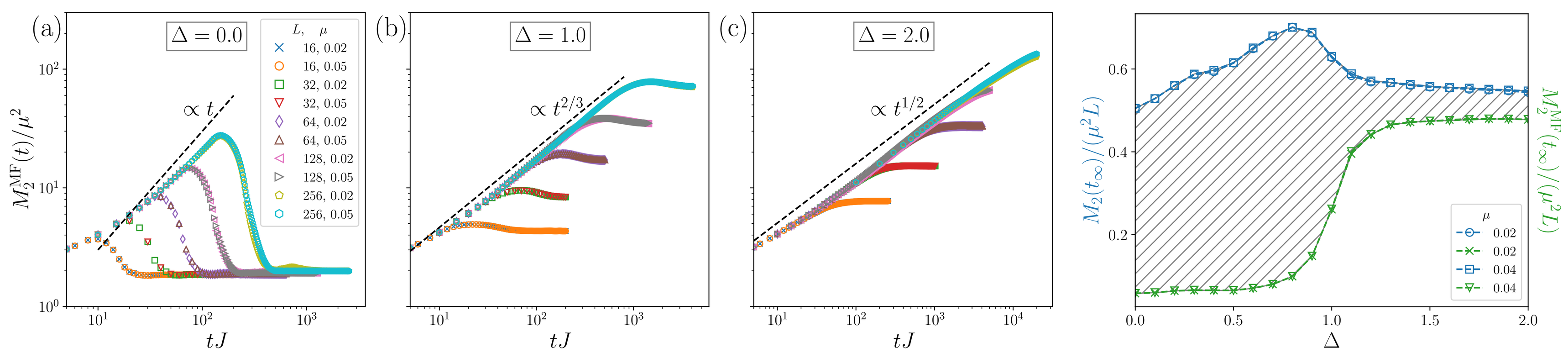
The pre-NESS growth exponent z directly encodes the transport regime.

Mean-field SRE

What is the contribution of quantum correlations to SRE?
Eliminating correlations by performing a mean-field (MF) approximation on Pauli strings

$$\langle\bigotimes_{i=1}^{L} P_i\rangle \mapsto \bigotimes_{i=1}^{L}\langle P_i\rangle, \qquad P_i \in \{I_i, X_i, Y_i, Z_i\}.$$

SRE computed from local magnetization information.



(Left) SRE MF dynamics in the three regimes and (right) total vs. MF NESS SRE.

MF SRE yields the same scaling as the total SRE.
The difference between total and MF SRE is maximal in the regime $\Delta \lesssim 1$ where quantum fluctuations are strongest.

Summary

- SRE generation in the boundary-driven XXZ chain encodes the **transport regime** through the universal growth law $M_2 \sim t^{1/z}$.
- The NESS magic is extensive ($M_2 \sim L$).
- M_2 scales quadratically with the driving parameter ($M_2 \sim \mu^2$).
- Open quantum systems can host non-stabilizer NESSs.

References

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