Optimised Quantum Feedback Controls Protocols: Advantages and Limitations

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1. Poster Summary

Optimising feedback control involves engineering certain feedback protocols that produce system states with high fidelity relative to a desired target state. We first introduce the physics of continuous measurement which is fundamental for quantum feedback systems. Next we focus on the Quantum Fokker-Planck master equation that governs the unconditional evolution of these particular systems. Finally, we focus on a qubit coupled to thermal bath system and explore the possible feedback protocols, the systems steady state in certain parameter space and specific quantum trajectories that arise.

2. Continuous Measurement[1]

Projective measurement Vs Continuous measurement	
Strong	Weak
Single measurement	Sequence of weak measurements
Instantaneous	Measurement back-action
Single observation	Measurement record in time
Wavefunction collapse	Quantum trajectories
System strongly disturbed	System weakly disturbed
High information gain	Low information gain
Small measurement uncertainty	Large measurement uncertainty

Quantum trajectories

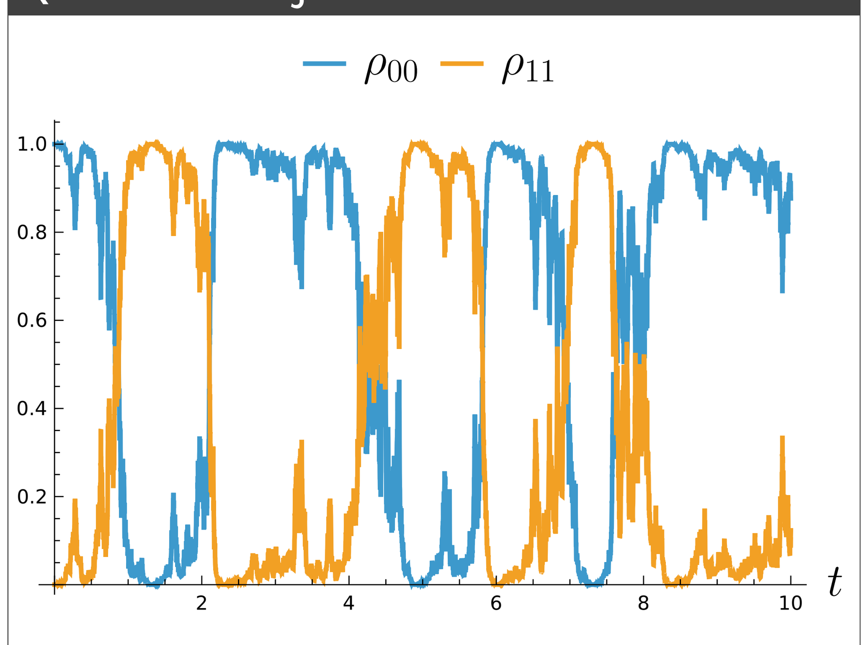

Figure 1: Populations against time for isolated driven qubit under homodyne detection, simulated using the Stochastic master equation with $H = \sigma_x$, $A = \sigma_z$, $\lambda = 1$, $\gamma = 2$, $\eta = 1$.

Quantum trajectories

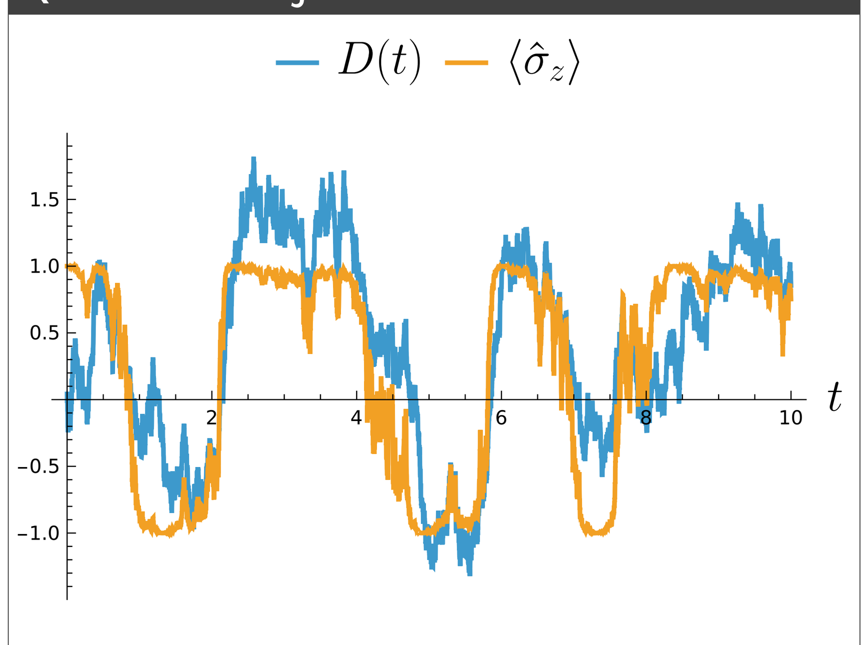

Figure 2: Corresponding measurement record from figure 1 comparing the expectation value of the measured observable against the filtered measurement record $D(t)$.

Belavkin Equation or Stochastic Master Equation

$$\frac{d\hat{\rho}_c}{dt} = \mathcal{L}(D)\hat{\rho}_c(t) + \lambda\,\mathcal{D}[\hat{A}]\hat{\rho}_c(t) + \sqrt{\lambda\eta}\,\frac{dW}{dt}\{\hat{A} - \langle\hat{A}\rangle,\, \hat{\rho}_c(t)\}$$

$$z(t) = \sqrt{\eta}\,\langle\hat{A}\rangle + \frac{1}{2\sqrt{\lambda}}\frac{dW}{dt}$$

3. Quantum Fokker Planck Master Equation[2]

$$\frac{\partial\hat{\rho}_t(D)}{\partial t} = \mathcal{L}(D)\hat{\rho}_t(D) + \lambda\mathcal{D}[\hat{A}]\hat{\rho}_t(D) - \gamma\partial_D\frac{1}{2}\left\{\sqrt{\eta}\hat{A} - D(t), \hat{\rho}_t(D)\right\} + \frac{\gamma^2}{8\lambda}\partial_D^2\hat{\rho}_t(D)$$

$$D(t) = \int_{-\infty}^{t} dt'\,\gamma\,e^{-\gamma(t-t')}\,z(t')$$

Equation characteristics:

- Produces unconditional dynamics of the feedback system (equivalent of average dynamics from large number of Belavkin simulations)
- $\mathrm{Tr}\,[\hat{\rho}_t(D)] = P(D)$
- Equation consists of; feedback controlled system evolution; measurement back-action; and the detector time evolution
- Allows for non-linear feedback protocol

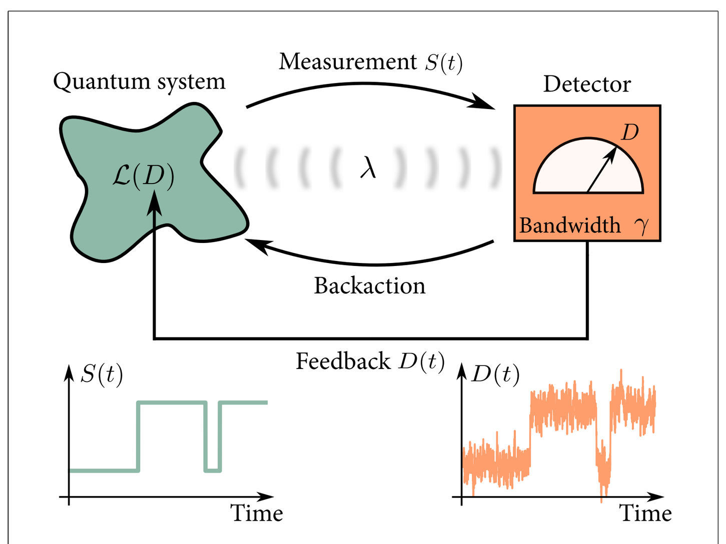

Figure 3: Illustration of a generic measurement and feedback setup with measurement strength λ, detector of bandwidth γ, feedback controlled system $\mathcal{L}(D)$ and feedback $D(t)$. Image: [2]

Measurement efficiency η:

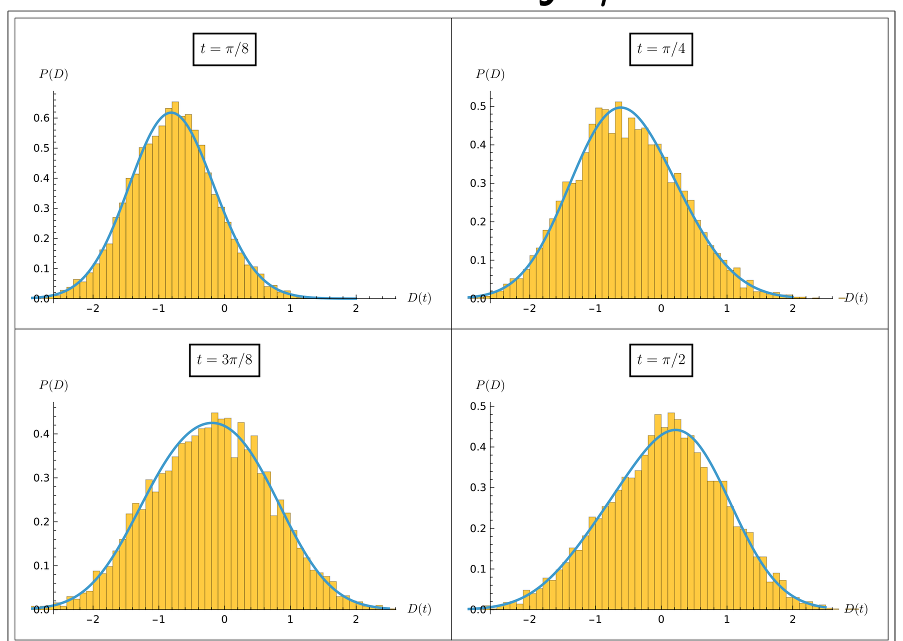

Figure 4: Measurement outcomes P(D) solving the QFP master equation (solid blue line) and collating 5000 stochastic trajectories (orange histograms) for isolated driven qubit system with $H = \sigma_x$, $A = \sigma_z$, $\lambda = 0.5$, $\gamma = 2$, $\eta = 0.5$.

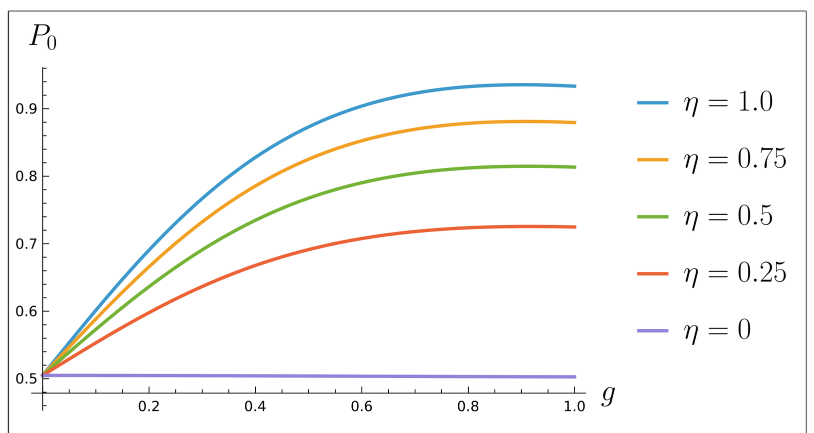

Figure 5: Ground state probability P_0 against feedback strenght g for different measurement efficiencies, for a qubit coupled to a thermal bath under the conditions $A = \sigma_x$, $\kappa = 0.01$, $n_B = 0.5$, $\gamma = 4$, $\lambda = 0.5$.

4. Non-driven qubit coupled to a thermal resevoir[3,4]

The following numerical results given in this section are derived from the following open feedback system: A qubit weakly coupled to a large thermal reservoir, with the goal of maintaining the qubit in its ground state through feedback control through the continuous measurement of σ_x. The Liouvillian superoperator describing the system is $\mathcal{L}_0\,\hat{\rho}_t(D) = \kappa(n_B)\mathcal{D}[\sigma_+]\hat{\rho}_t(D) + \kappa(n_B+1)\mathcal{D}[\sigma_-]\hat{\rho}_t(D)$, where the feedback Liouvillian is $\mathcal{L}_{fb}\,\hat{\rho}_t(D) = -i[g\sigma_y,\, \hat{\rho}_t(D)]$

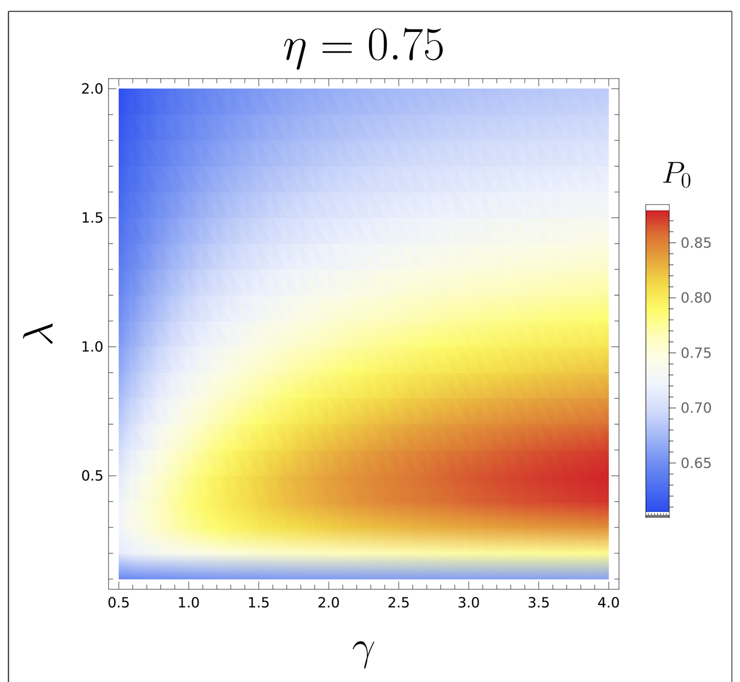

Figure 6: The relationship between λ and γ and their corresponding ground state probability for steady state solutions for $g = 1$, $\kappa = 0.01$, $n_B = 0.5$, $A = \sigma_x$.

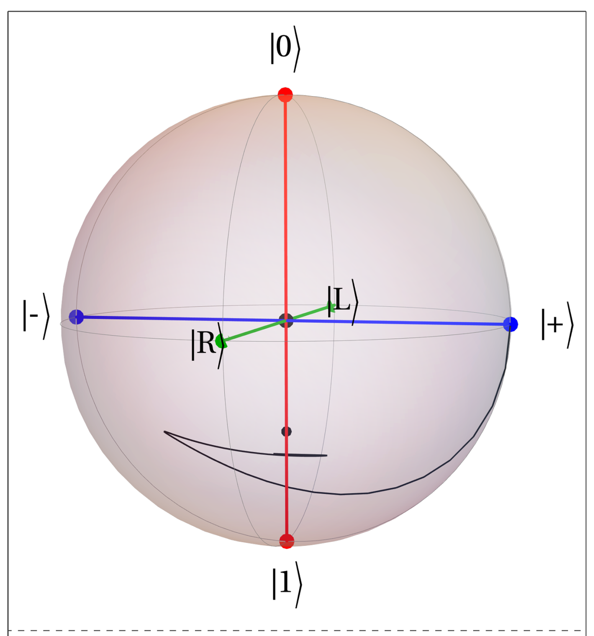

Figure 7: A quantum trajectory mapped to the bloch sphere for linear feedback control with $g = 1$, $\kappa = 0.01$, $n_B = 0.5$, $\lambda = 0.5$, $\gamma = 1$, $A = \sigma_x$.

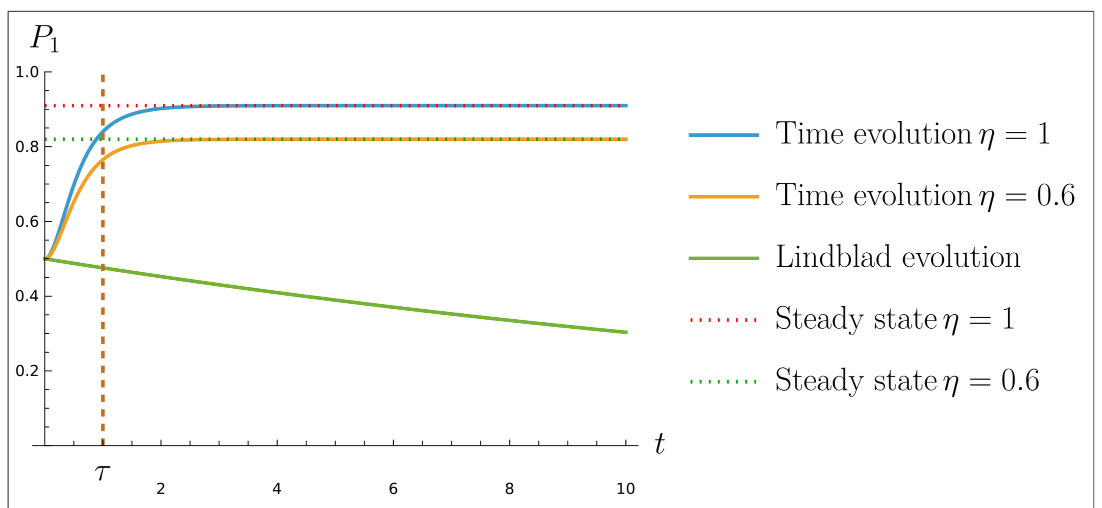

Figure 8: A comparison between the feedback controlled evolution [time-evolution (blue,orange) and steady steady (dotted red, dotted green) simulated separately] for different measurement efficiency values, the Lindblad evolution (green), and the Liouvillian-gap timescale (τ) for parameters $g = 1$, $\kappa = 0.01$, $n_B = 0$, $A = \sigma_x$, $\lambda = 0.5$, $\gamma = 4$, .

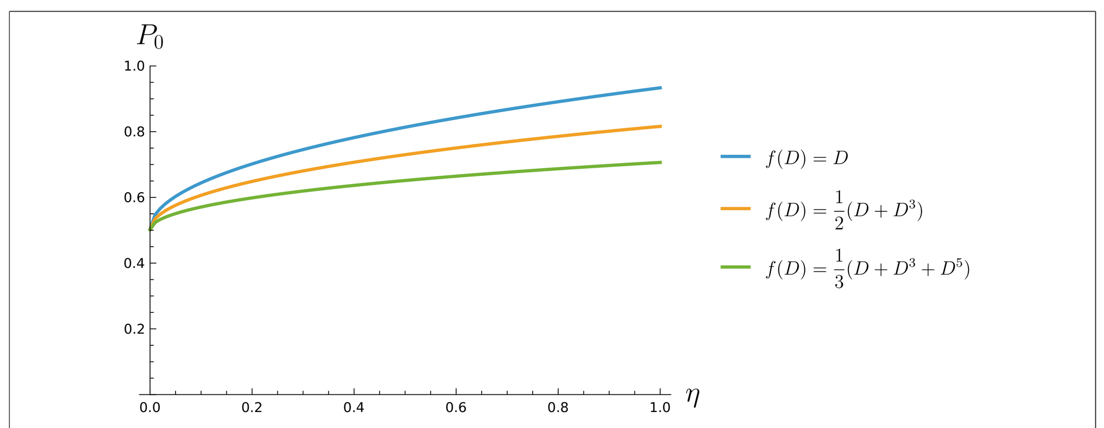

Figure 8: P_0 plotted against measurement efficiency for different order polynomial feedback protocols for $g = 1$, $\kappa = 0.01$, $n_B = 0.5$, $A = \sigma_x$, $\lambda = 0.5$, $\gamma = 4$,

Goal for optimisation

→ Implement non-linear feedback protocol
→ Implement optimisation techniques such as gradient descent: $\vec{a}_{n+1} = \vec{a}_n - \alpha\,\nabla_{\vec{a}}\big(1 - F(\vec{a})\big)$ to achieve higher fidelity

References

[1] Wiseman, H. M., & Milburn, G. J. (2010). *Quantum Measurement and Control.* Cambridge University Press.
[2] Annby-Andersson et al,. (2022). *Quantum Fokker-Planck master equation for continuous feedback control.* Physical Review Letters, 129(5), 050401.
[3] Gabriel T. Landi et al,. (2024). *Current fluctuations in open quantum systems.* PRX Quantum 5, 020201.
[4] Kiely, Anthony., & Landi, Gabriel T. (2026). (*Extracting filtered signal statistics of continuously measured quantum systems.* Phys. Rev. A 113, 032442