Numerically exact open quantum system work statistics with process tensors

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Q-Open Kick-off Workshop, Lisbon, 29 April 2026

Mike Shubrook, Moritz Cygorek, Erik Gauger, Jake Iles-Smith, AN, arXiv:2512.16823
Owen Diba, Harry Miller, Jake Iles-Smith, AN, PRL 132, 190401 (2024)

COST Action CA24109 – QOpen https://cost.eu https://qopen.eu/

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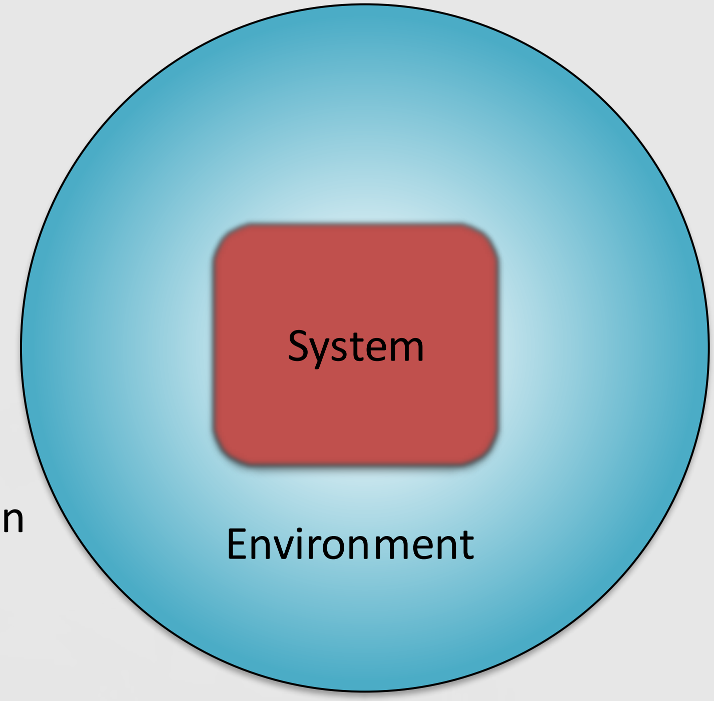
Outline

- **Quantum work statistics in driven open systems**
  - Two-point measurement protocol
  - Characteristic function and counting statistics
- **Strong reservoir coupling: polaron perspective**
  - Why weak-coupling work statistics fail
  - Landau–Zener work distributions
- **Numerically exact work statistics with process tensors**
  - Generalised-time axis and PT-MPO propagation
  - Landauer erasure beyond weak coupling and slow driving
- **Conclusions and outlook**



Motivation

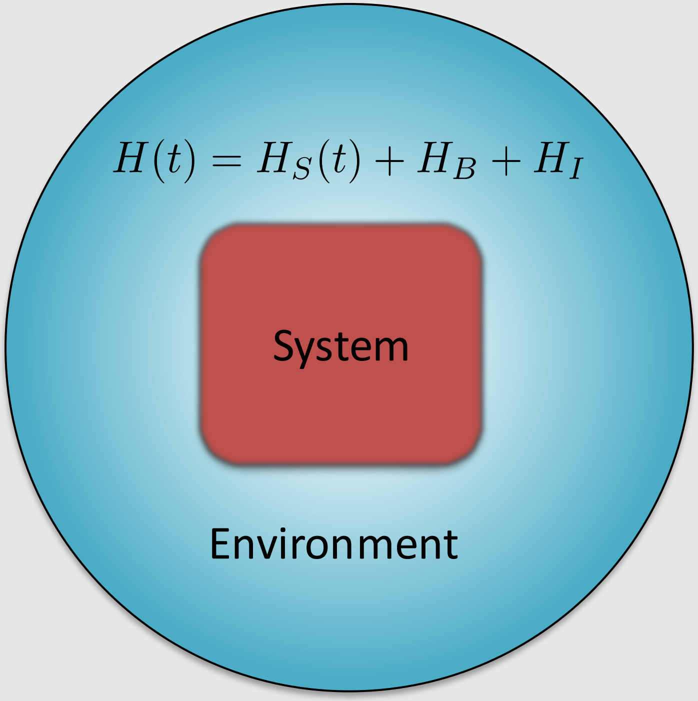
Strong-coupling thermodynamics

- Here we focus on calculating the full work distribution for a driven open system

- Standard approaches assume system-reservoir interactions play negligible role

- Determining statistics of work in strong coupling regimes is a **formidable task**
  - Requires calculation of full eigenspectrum of system and reservoir

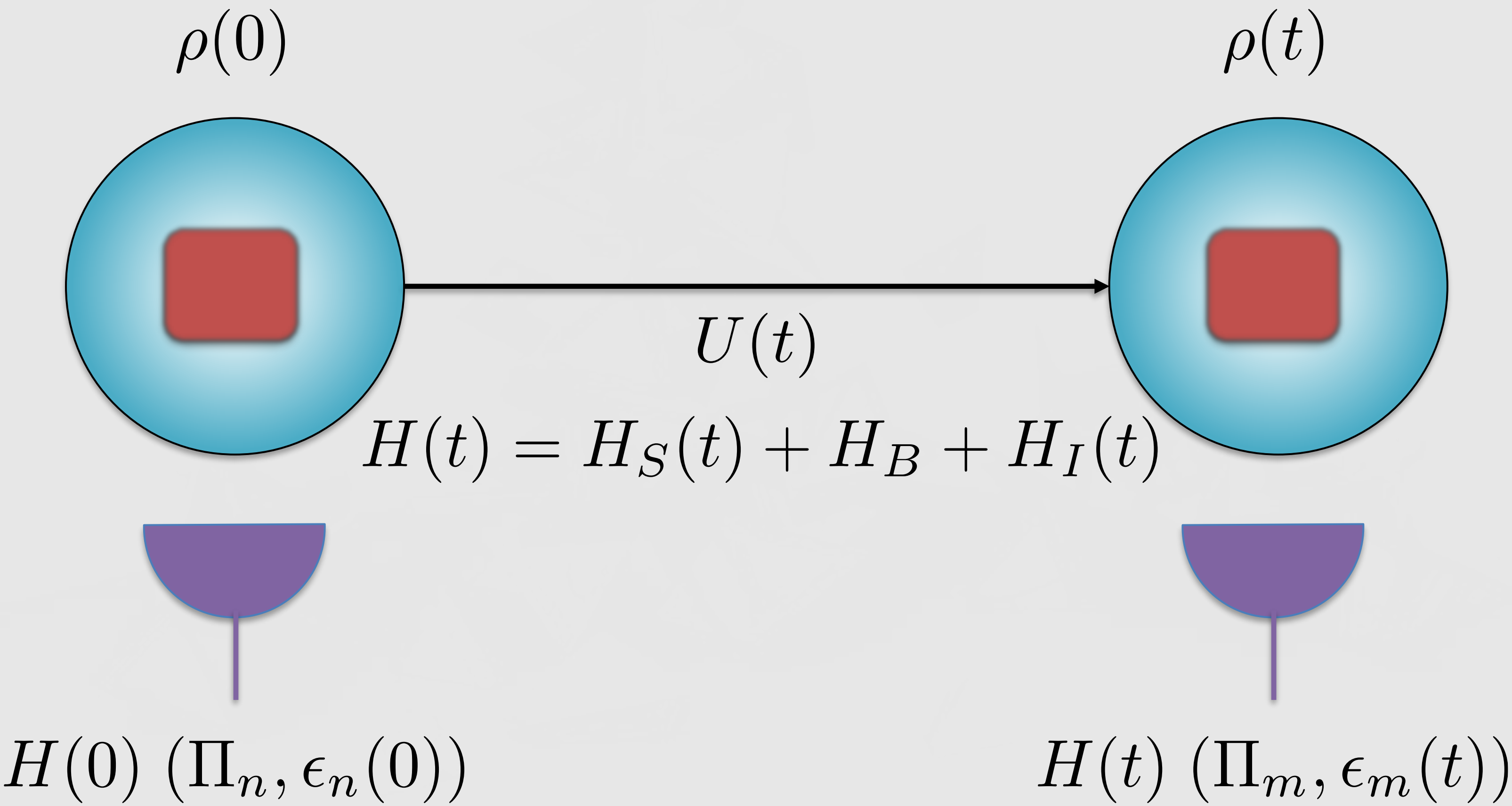
- **Question: how do we compute open-system work statistics beyond the regimes where conventional master equations are reliable?**
  - We will circumvent this issue using a unitary polaron transformation and process tensors



Work: two-point measurement

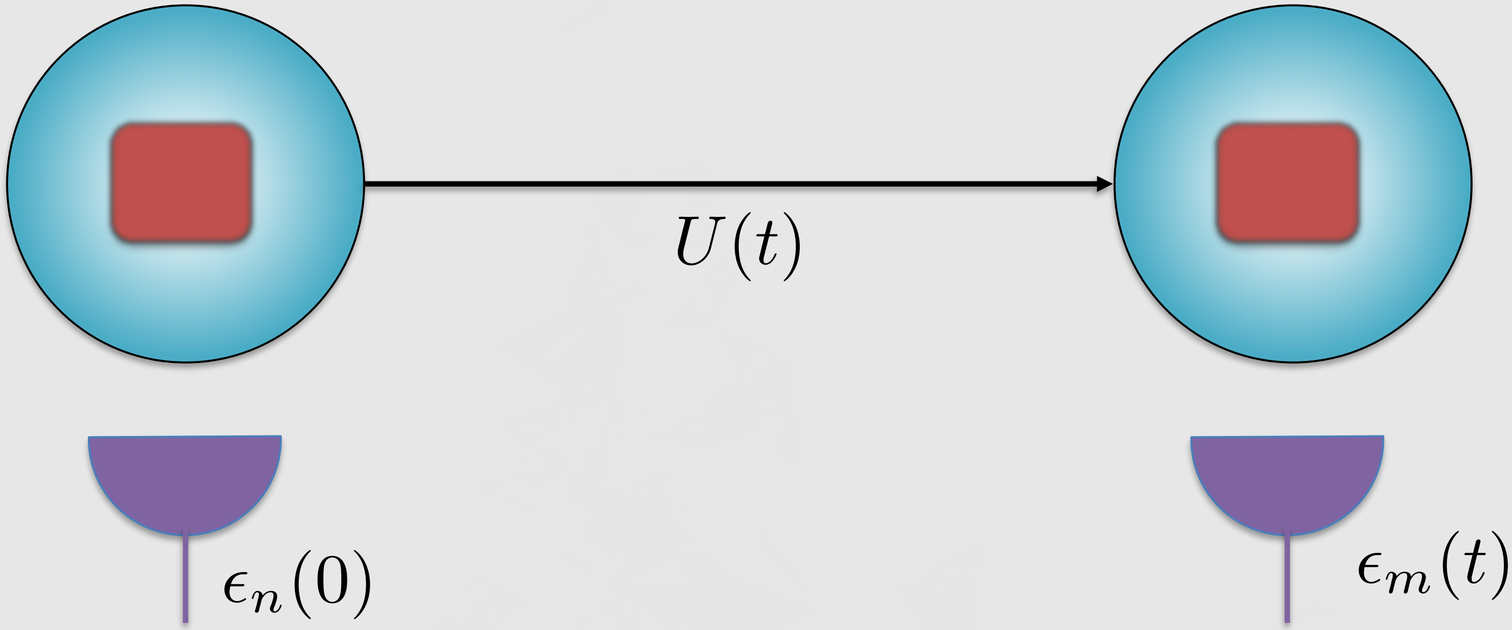
We characterise (stochastic) work via the two-point measurement protocol

- Consider projective energy measurements performed on **combined system and environment**, which is closed and evolves unitarily under the action of an external force



P. Talkner, E. Lutz, P. Hanggi, Phys. Rev. E 75, 050102(R) (2007)

Work probability density



$$p(n) = \mathrm{tr}(\Pi_n \rho(0)) \qquad p(m,t|n) = \frac{\mathrm{tr}(\Pi_m U(t) \Pi_n \rho(0) \Pi_n U^\dagger(t))}{p(n)}$$

Work done on system is measured energy difference, leading to probability density

$$\boxed{p(w) = \sum_{n,m} p(n) p(m,t|n) \delta(w - [\epsilon_m(t) - \epsilon_n(0)])}$$

P. Talkner, E. Lutz, P. Hanggi, Phys. Rev. E 75, 050102(R) (2007)

Work characteristic function

For practical purposes, it is easier to work with the characteristic function

$$\Phi(\chi) = \int dw e^{i\chi w} p(w)$$

Contains all statistical information about the work performed on our closed system+environment

$$\langle w^k \rangle = (-i)^k \frac{\partial^k \Phi(\chi)}{\partial \chi^k}\bigg|_{\chi=0}$$

Characteristic function can be written in terms of a counting-field dependent density operator

$$\Phi(\chi) = \mathrm{tr}(\rho(\chi,t)) = \mathrm{tr}_S(\rho_S(\chi,t))$$

With reduced density operator (which we look to approximate)

$$\rho_S(\chi,t) = \mathrm{tr}_B(\rho(\chi,t))$$

Work counting statistics

Counting-field dependent density operator obeys an equation of motion of the form

$$\dot{\rho}(\chi,t) = -i(H(\chi,t)\rho(\chi,t) - \rho(\chi,t)H(-\chi,t))$$

With a transformed Hamiltonian

$$H(\chi,t) = H(t) + i\partial_t(e^{i\chi H(t)/2})e^{-i\chi H(t)/2}$$

Reduced density operator obeys a generalised master equation

$$\boxed{\dot{\rho}_S(\chi,t) = \mathcal{L}(\chi,t)\rho_S(\chi,t)}$$

Form of the Liouvillian depends on the approximations made

Weak coupling

Born–Markov, small interaction energy

→

Polaron

strong coupling captured by dressing transformation

→

Process tensor

non-perturbative memory via tensor network influence functional

M. Esposito, U. Harbola, and S. Mukamel, Rev. Mod. Phys. 81, 1665 (2009), Rev. Mod. Phys. 86, 1125 (2014)

Moving beyond weak coupling: solid-state quantum optics

How do we modify standard quantum optics to model solid-state devices?

- e.g. what effect do lattice vibrations (phonons) have on system properties?

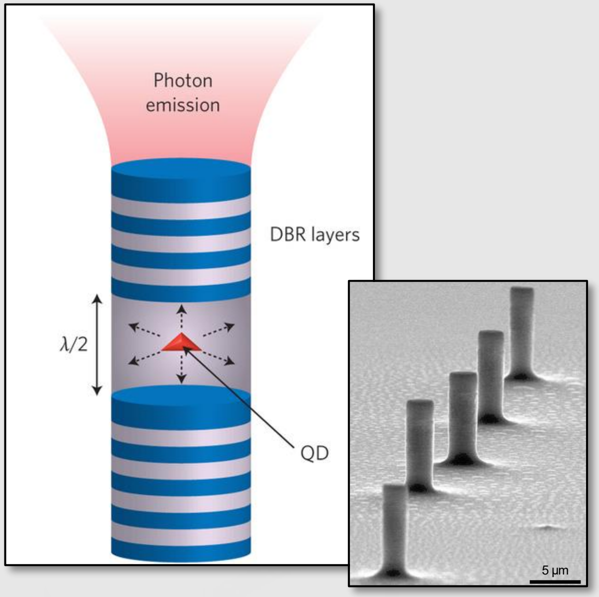
Model

- two-level system
- coherent manipulations through external laser addressing (adiabatic)
- oscillations damped by phonon interactions

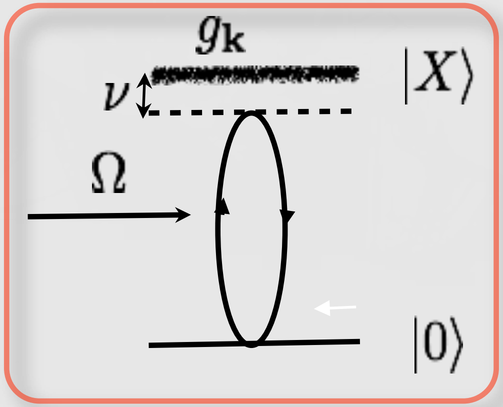
$$H_S(t) = \frac{\omega_0}{2}\sigma_z + \Omega(e^{-i\omega_l t}\sigma_+ + e^{i\omega_l t}\sigma_-)$$

$$H_B = \sum_k \omega_k b_k^\dagger b_k$$

$$H_I = \sigma_z \sum_k g_k (b_k^\dagger + b_k)$$



R. Oulton, Nature Nano. 9, 169 (2014)



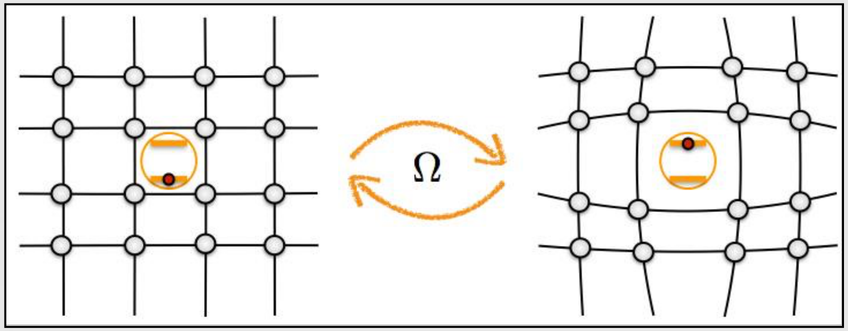
D. P. S. McCutcheon and AN, PRL 110, 217401 (2013)

Phonon interactions

Lattice displaces in response to changes in charge configuration

- polaron formation – new boundary
- we incorporate this physics into our master equation via a unitary transformation
- allows strong electron-phonon interactions
- direct expt-theory comparisons

$$H_P = e^S H e^{-S}$$



Topical Review: AN and D. P. S. McCutcheon, JPCM 28, 103002 (2016)

Spectral density –
system-bath coupling strength
weighted by bath density of states

$$J(\omega) = \alpha \frac{\omega^3}{\omega_c^2} e^{-\omega/\omega_c}$$

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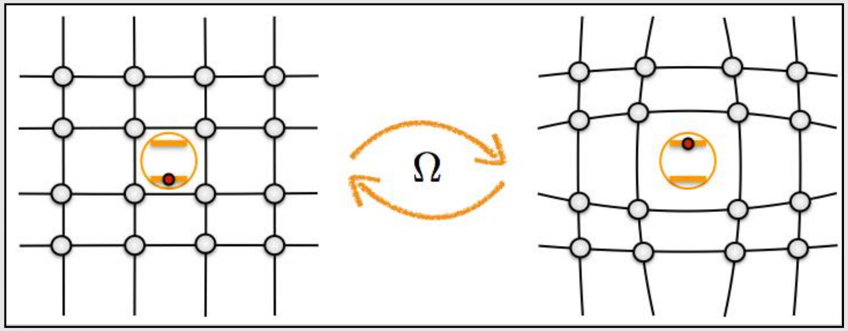
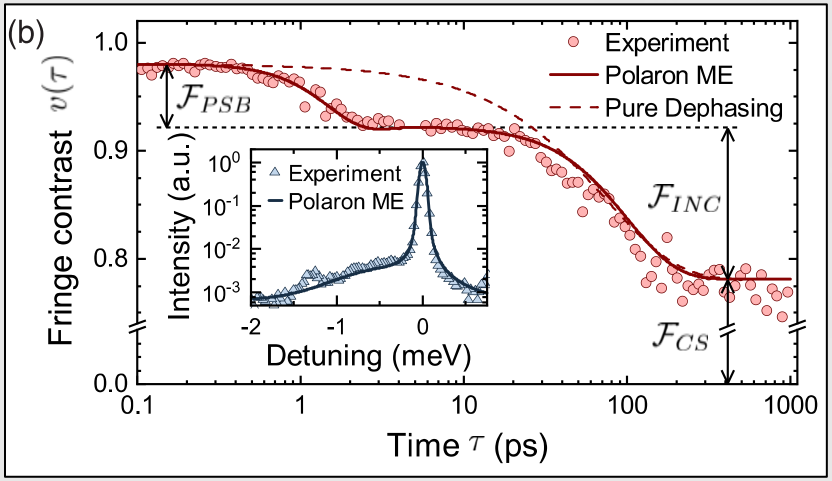


Topical Review: AN and D. P. S. McCutcheon, JPCM 28, 103002 (2016)

Damping of coherence



A. Brash et al., PRL 123, 167403 (2019)

Spectral density –
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weighted by bath density of states

$$J(\omega) = \alpha \frac{\omega^3}{\omega_c^2} e^{-\omega/\omega_c}$$

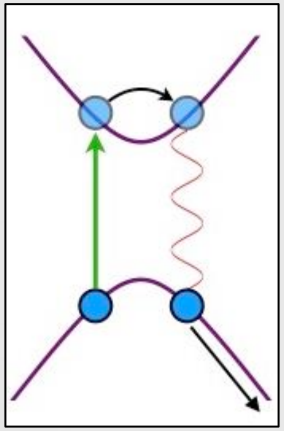
Polaron transformation

We perform a unitary polaron transformation to our Hamiltonian to identify a new perturbation term

$$H_P(t) = e^S H(t) e^{-S},\ S = \sigma_z \sum_k (g_k/\omega_k)(b_k^\dagger - b_k)$$

This leads to a non-perturbative temperature and coupling strength dependent renormalisation of system driving

$$H_{PS}(t) = \frac{1}{2}\nu t \sigma_z + \frac{1}{2}\Delta\kappa\sigma_x,\ 0 \leq \kappa \leq 1$$

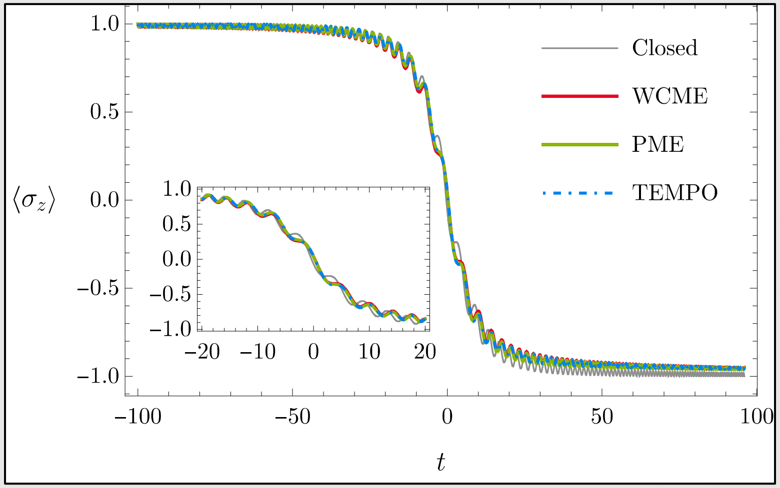


System now couples through raising and lowering operators to environmental **displacements**.

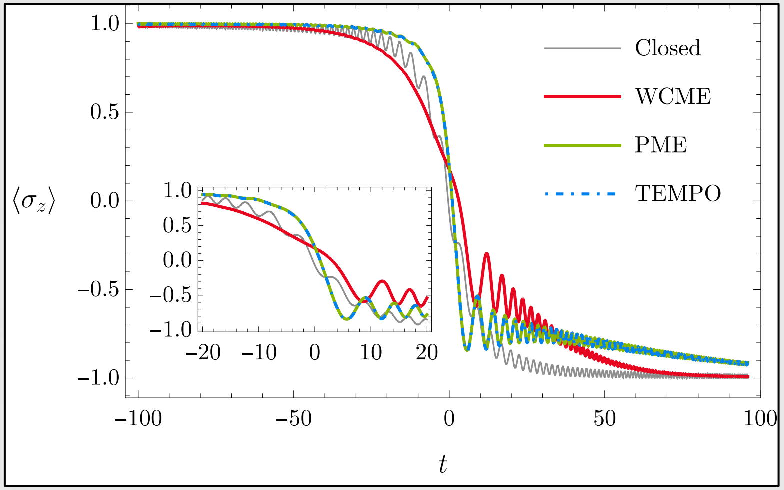
Following an adiabatic, Markovian procedure in the **polaron frame** we obtain a generalised polaron mater equation

$$\dot{\rho}_{P_S}(\chi, t) = \mathcal{L}_P^{\mathrm{ad}}(\chi, t)\rho_{P_S}(\chi, t)$$

Benchmarking – Landau-Zener

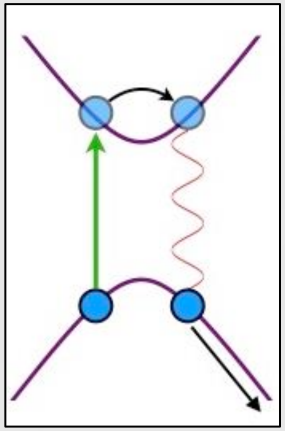


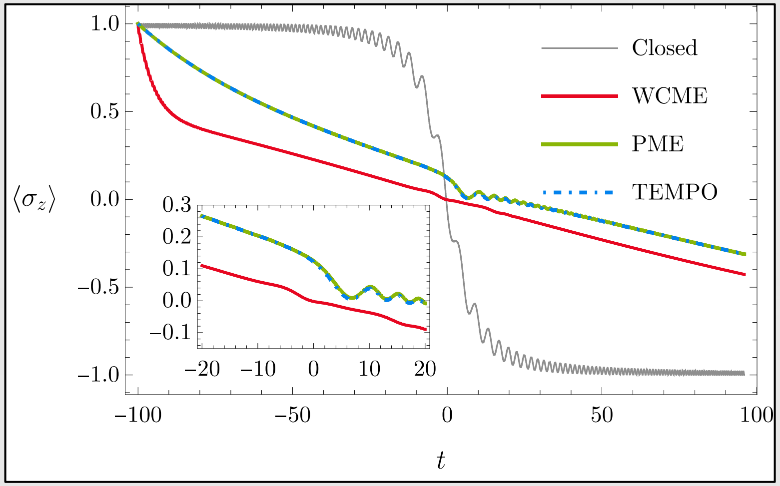
$\alpha = 0.02, \Delta\beta = 1$



$\alpha = 0.4, \Delta\beta = 1$

Benchmark Landau-Zener dynamics against numerically exact TEMPO method

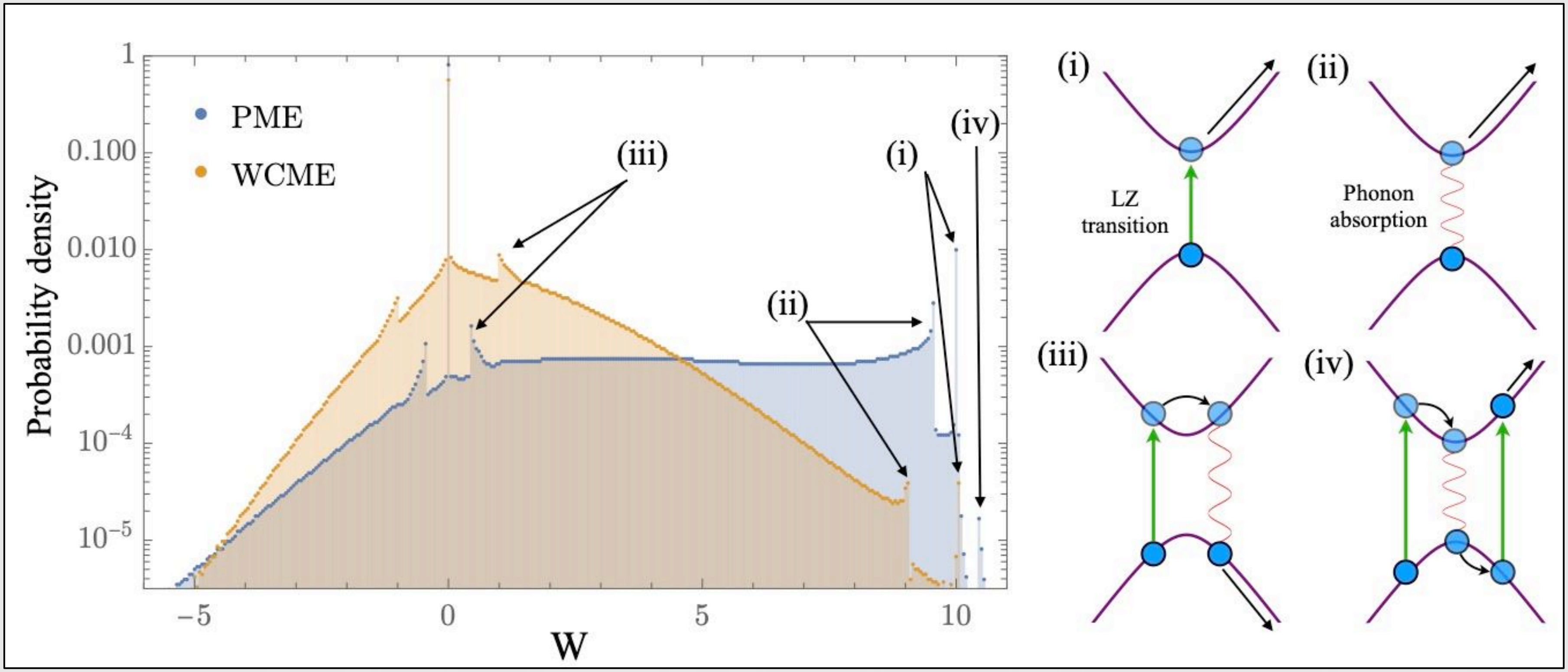




$\alpha = 0.4, \Delta\beta = 0.1$

J. Iles-Smith, O. Diba, AN, PRL 132, 190401 (2024)

Work distributions



$$\alpha = 0.4, \Delta\beta = 1$$

Jarzynski fluctuation relation also reproduced, consistent with laws of stochastic thermodynamics

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta F_{PS}}$$

PRL 132, 190401 (2024)

From strong-coupling intuition to numerically exact statistics

Strong coupling matters — but can we remove the remaining approximations?

Polaron approach

strong-coupling physics with physical transparency, but still uses adiabatic/Markovian assumptions in the transformed frame

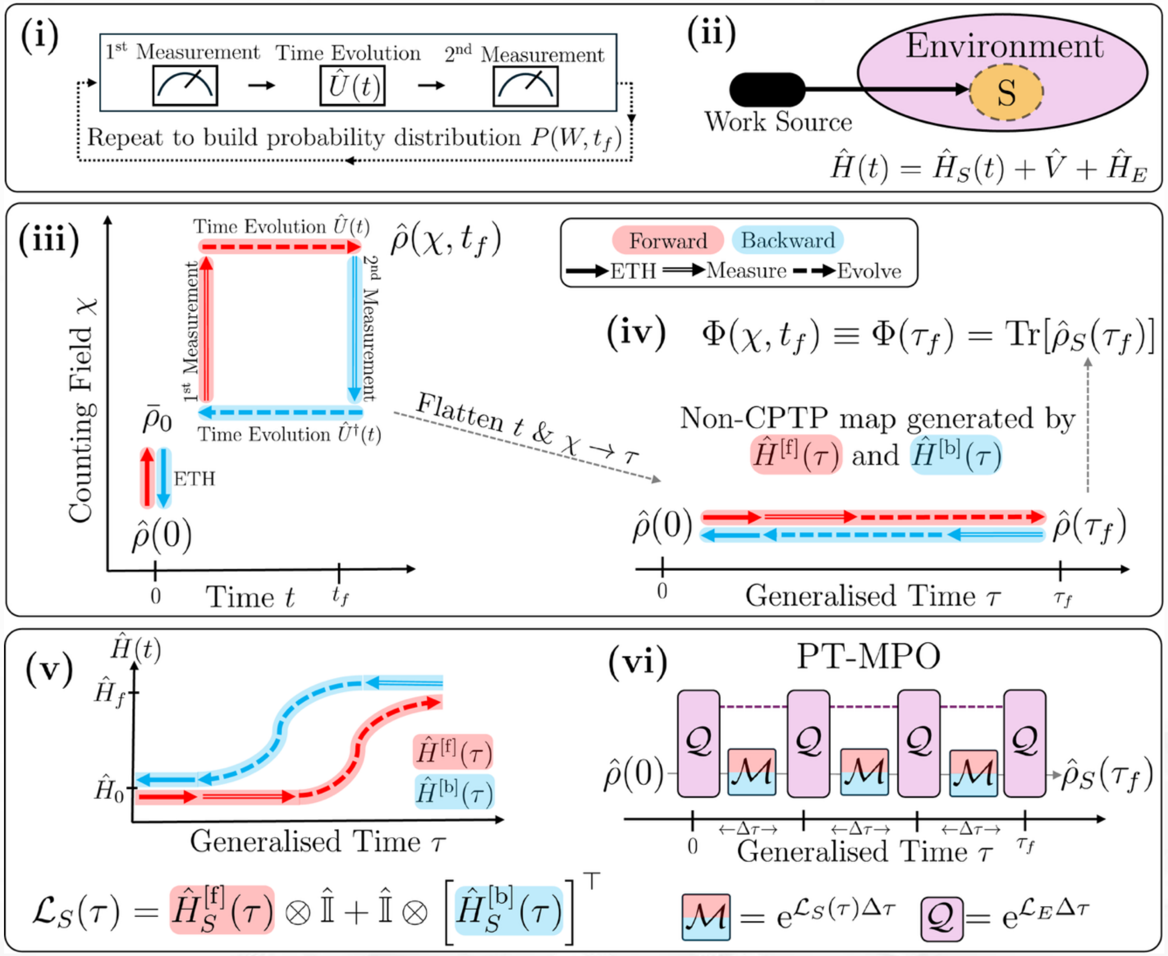
→

Process tensor approach

full WPD from a non-perturbative influence functional; no weak-coupling, no Markovian or slow-driving assumption

Next: encode the work characteristic operator as an open-system propagation problem.

Process-tensor framework for work statistics



Start from the TPMP work characteristic function

Treat the counting field χ as a time-like coordinate

Combine physical time and χ into a single generalised-time axis τ

All environmental memory is encoded in a PT-MPO influence functional

Turn work counting into a tensor-network propagation problem

M. Shubrook, M. Cygorek, E. Gauger, J. Iles-Smith, AN, arXiv:2512.16823

Mapping to a generalised-time propagation

The work characteristic operator evolves under different forward and backward Hamiltonians.

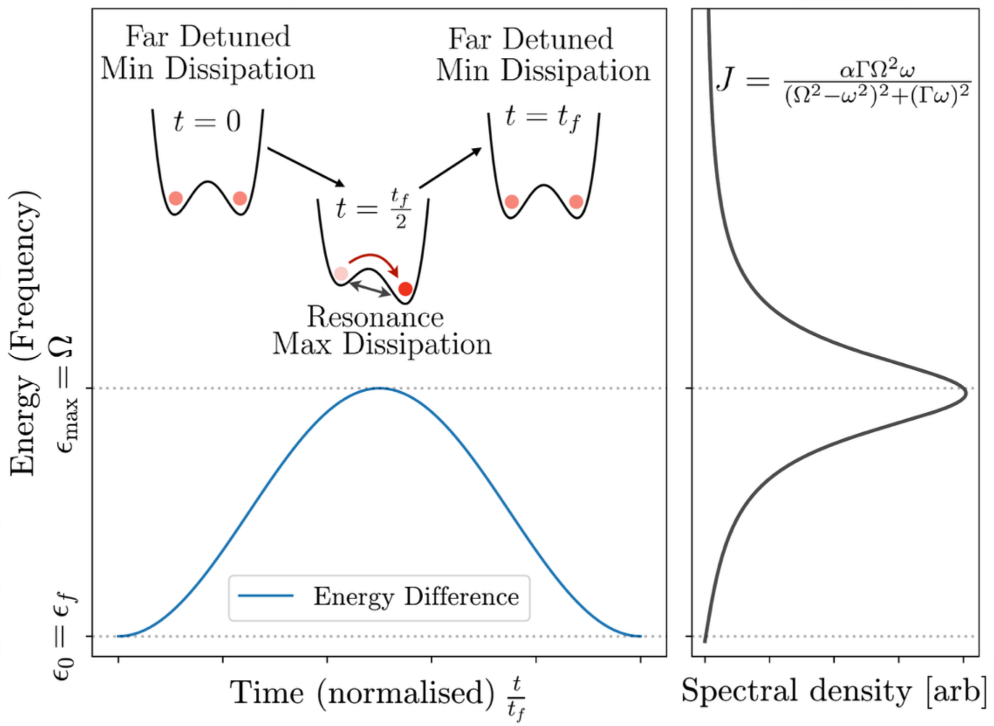
$$\rho(\chi, t_f) \longrightarrow \rho(\tau_f), \qquad \tau = t + \chi$$

physical time — counting field — generalised time τ

$$\hat{\rho}(\tau_f) \approx \prod_{j=1}^{N} e^{\mathcal{L}_E \Delta\tau} e^{\mathcal{L}_S(\tau_j)\Delta\tau} [\hat{\rho}_S(0) \otimes \hat{\rho}_E(0)]$$

- System-local propagators contain the work-counting operations
- PT-MPO formalism contains full non-Markovian environmental correlations
- Sweep along τ to obtain Φ, then Fourier transform to P(W)

Application: Landauer erasure of a qubit

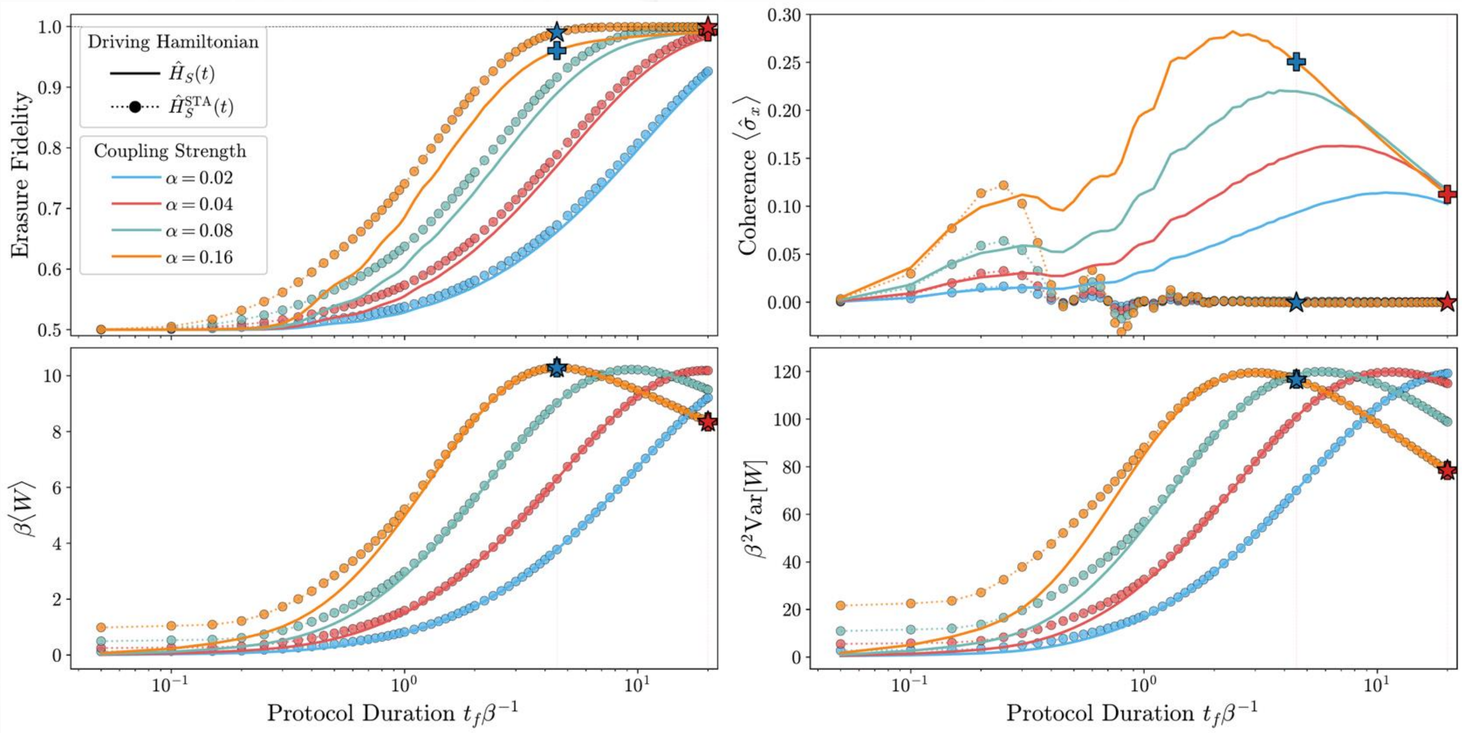


- Initial two-level system is maximally mixed
- Drive the splitting towards resonance with a structured bath
- Return to the far-detuned point to keep the protocol cyclic
- Compare reference driving with a closed-system shortcut to adiabaticity

Goal: compute the full work distribution beyond weak coupling, Markovianity and slow driving.

M. Shubrook et al., arXiv:2512.16823

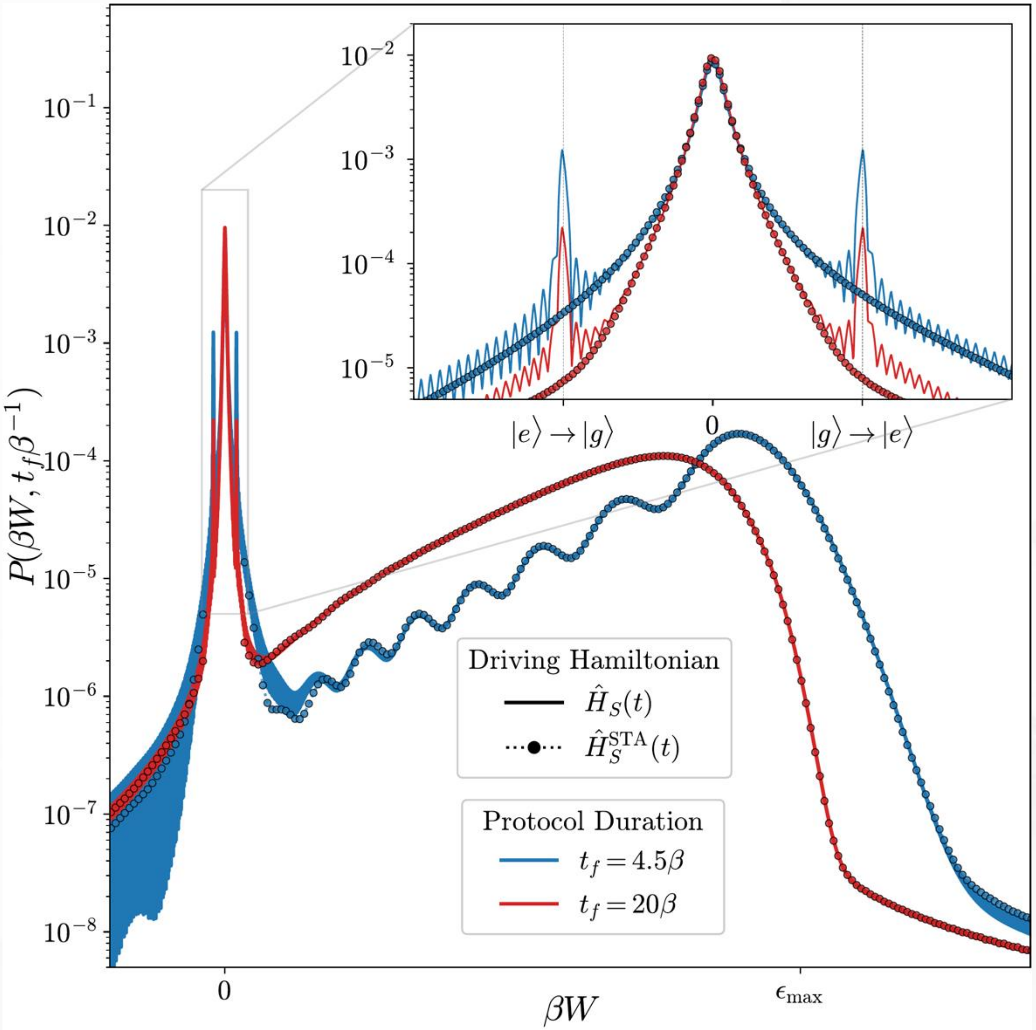
Erasure fidelity and low-order work statistics



- Fidelity improves with stronger coupling and slower driving
- STA improves intermediate-time erasure by suppressing non-adiabaticity
- Mean and variance do not always clearly distinguish the protocols
- The full WPD is needed to see the underlying pathways

M. Shubrook et al., arXiv:2512.16823

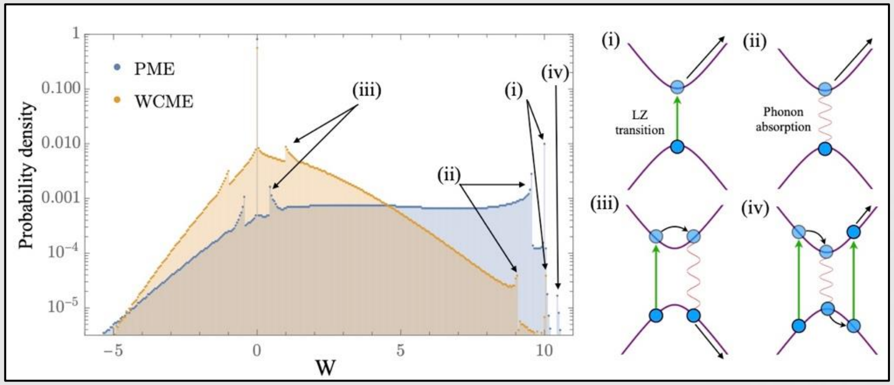
The full work distribution reveals hidden structure

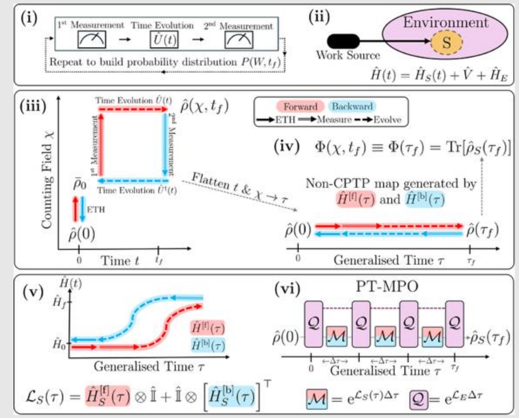


- Broad positive-work feature: energetic cost of erasure via dissipation
- Sharp side peaks near W=0: non-adiabatic closed-system transitions
- STA suppresses the unwanted side peaks while leaving most of the WPD unchanged
- Explains why fidelity can change even when mean/variance look similar

M. Shubrook et al., arXiv:2512.16823

Summary and future work





Polaron theory shows that strong coupling qualitatively reshapes work statistics
– renormalised gaps and phonon-assisted pathways are visible in P(W)

Process tensors make open-system work statistics numerically exact
– no weak-coupling, Markovian, or slow-driving assumption is required
– Landauer erasure shows why the full WPD matters

Ongoing

- How do we probe these work distributions experimentally?
- Optimal control in non-Markovian environments
- Many-body driven open systems

PRL 132, 190401 (2024)

arXiv:2512.16823

Acknowledgments

- Owen Diba
- Harry Miller
- Jake Iles-Smith
- Mike Shubrook
- Moritz Cygorek
- Erik Gauger

Engineering and Physical Sciences Research Council

The University of Manchester Photon Science Institute

Thank You

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