Ergotropy transport in a one-dimensional spin chain

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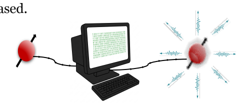
Motivation

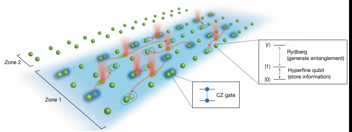
- Landauer's principle encodes a fundamental link between information and energy. When information is erased there is a lower bound placed on how much energy is released.



$$\Delta E \geq k_B T \ln 2$$

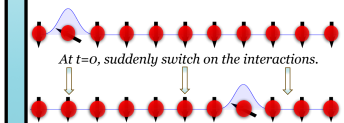
- Despite high expectations from governments and private companies, the energetic footprint of quantum technologies has largely been overlooked in current deployment strategies [1].

- Networks of spins are a promising experimental realisation for the hardware of computers and have been realised in many settings, including quantum dots, trapped ions, and even photonic systems [2].



Thermodynamics of state transfer

Our goal is to examine protocols to transport energy and extractable work in quantum networks.



$$H_{init.} = -h\sum_{i=1}^{N}\sigma_z^i \qquad \text{Eq. (a)}$$

$$H_f = \sum_{i=1}^{N-1} J_\alpha\left(\sigma_x^i\sigma_x^{i+1} + \sigma_y^i\sigma_y^{i+1}\right) + H_{init.}$$

Key Thermodynamic Figures of Merit

We ask two questions: Firstly, what is the **work cost of switching on the interactions?** Secondly, how much useful **work can we extract** from one end of the chain?

Work Distribution

$$P(W) = \sum_{n,m}^{N} p_n p_{m|n}\,\delta\left(W - W_{m,n}\right) \qquad \text{Eq. (b)}$$

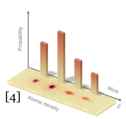
$p_n = |\langle E_n^i|\Psi_i\rangle|^2$, $\quad p_{m|n} = \left|\langle E_m^f|E_n^i\rangle\right|^2$, $\quad W_{m,n} = E_m^f - E_n^i$

In quantum mechanics, work is not an observable but is defined through a two-point measurement scheme [3].

Extractable Work – Ergotropy

$$\mathcal{E} = Tr\left\{\left(\rho_N - U_\tau\rho_N U_\tau^\dagger\right)H\right\} \qquad \text{Eq. (c)}$$
$$= Tr\{\rho_N\sigma_z\} - Tr\{\rho_p\sigma_z\}\ [5]$$

Ergotropy, ε, is the maximum amount of work that can be extracted from a quantum system ("quantum battery") through unitary, cyclic operations. To extract work, the system must be in a non-passive state. We demonstrate the difference in transporting energy and *useful* energy.



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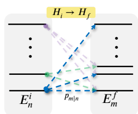

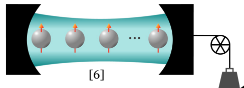

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Work Statistics of State Transfer

We consider two different interaction regimes: uniform coupling and perfect state transfer (PST) coupling, the latter yielding PST for all system sizes. We initialise the first qubit in the chain in three different states, one pure excitation (S1), and two states, with the same initial ergotropy, which is embedded in populations (S2) and the other in coherences (S3). Work distributions calculated from Eq. (a) and Eq. (b) for N=50.

$$J_\alpha = \frac{1}{2}\left[(\alpha-1)J + \alpha J_{i,i+1}\right]$$

Uniform Coupling

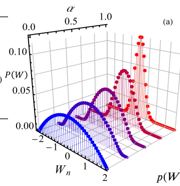
$$J_\alpha = J$$

Finite N

$$P(W) = \frac{2}{N+1}\sum_k^N \sin^2\frac{k\pi}{N+1}\,\delta(W - W_k)$$

$N \to \infty$

$$p(W) = \frac{1}{2\pi J^2}\sqrt{4J^2 - W^2}$$



PST Coupling

$$J_\alpha = J_{i,i+1} = G_N\frac{2J}{N}\sqrt{i(N-i)}$$

Finite N

$$P(W) = \left(\frac{1}{2}\right)^{N-1}\sum_k^N \binom{N-1}{k-1}\delta(W - W_k)$$

$N \to \infty$

$$p(W) = \frac{1}{\sqrt{2\pi\mathrm{Var}(W)}}\exp\left[-\frac{W^2}{2\mathrm{Var}(W)}\right]$$

State 1 (S1)

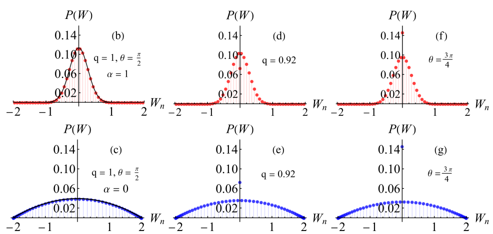
$$|\Psi_i\rangle = |10\cdots00\rangle$$

State 2 (S2)

$$\rho_i = q|1\rangle\langle 1| + (1-q)\,|0\rangle\langle 0|$$

State 3 (S3)

$$|\Psi_i\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$



Both figures demonstrate that, given two states, one in a quantum coherent superposition (S2) and the other a classical mixture (S3), both with the same initial ergotropy, there is a higher probability of doing zero work for the coherent state. While they both have zero average work, the PST distribution has smaller variance, and is therefore thermodynamically cheaper.

Dynamics of extractable work

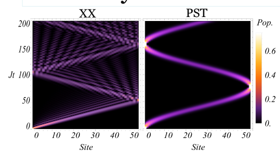
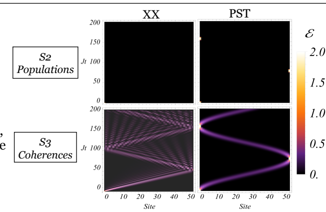


Fig. 3: Population of each site in the chain over time, for uniform and PST coupling. This plot shows us that given, there is no difference in how energy/fidelity is transported in the chain.

Fig. 4: Ergotropy of each site in the chain over time, for uniform and PST coupling. Although the population dynamics are the same, this is not the case for extractable work. Coherences enable more efficient and robust work extraction, surpassing that of a classical mixture of states, serving as a resource for ergotropy transport.



S2 $\quad \varepsilon_{mix}(t) = 2\left(2q|f_n(t)|^2 - 1\right)$

S3 $\quad \varepsilon_{coh}(t) = 2|f_n(t)|^2\sin^2\left(\frac{\theta}{2}\right) - 1 + \sqrt{1 + 4\sin^4\left(\frac{\theta}{2}\right)|f_n(t)|^2(|f_n(t)|^2 - 1)}$

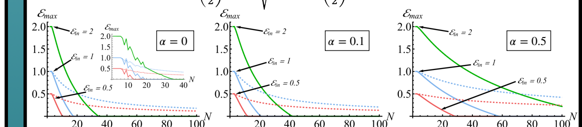


Fig. 5: The maximum ergotropy to reach the end of the chain at the first reflection point as a function of N (the left inset is for a larger time window Jt=1000). The dashed lines are for pure coherent states. The solid lines are mixed states. The mixed and pure states are initialised with the same ergotropy.

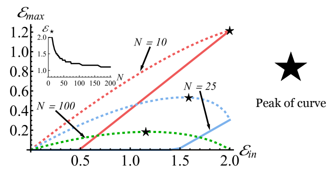


Fig. 6: The maximum ergotropy to reach the end of the chain at the first reflection point as a function of the initial ergotropy. The dashed lines are for pure coherent states. The solid lines are mixed states. Inescapable conclusion: coherences are better than populations.

References

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