Exact open quantum system dynamics using infinite tensor network influence functionals

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QOpen Many-body Open Quantum Systems

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Weak-coupling open quantum systems → stronger coupling, generalize "bath" → Interacting quantum many-body dynamics

Dynamical maps / Divisibility

Master equations

Quantum trajectories

…

Strongly correlated phenoma

Entanglement barrier

Thermalization / ETH

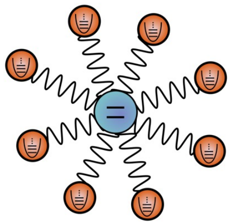
…

Quantum Impurity Models

Interacting system + strongly coupled non-interacting bath

Spin-boson model

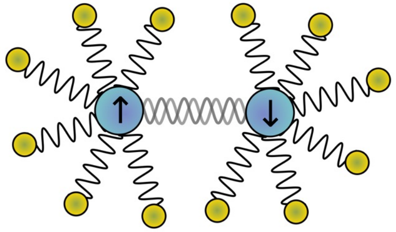
standard non-Markovian benchmark



$$H = \Omega\sigma_z + \sigma_x \sum_\lambda g_\lambda (b_\lambda + b_\lambda^\dagger) + \sum_\lambda \omega_\lambda b_\lambda^\dagger b_\lambda$$

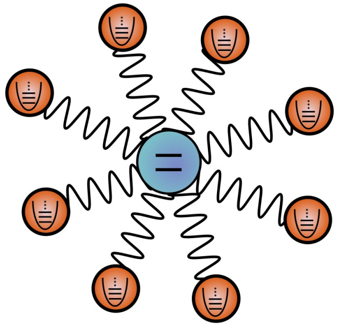
Single-impurity Anderson model

Kondo physics, DMFT



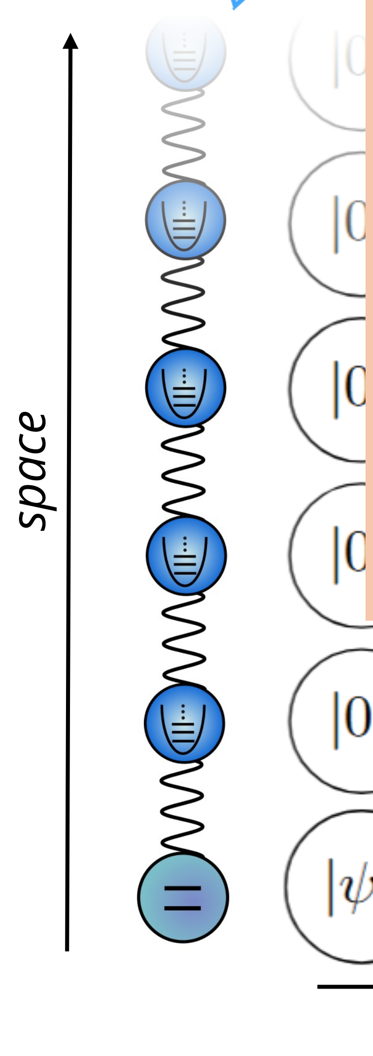
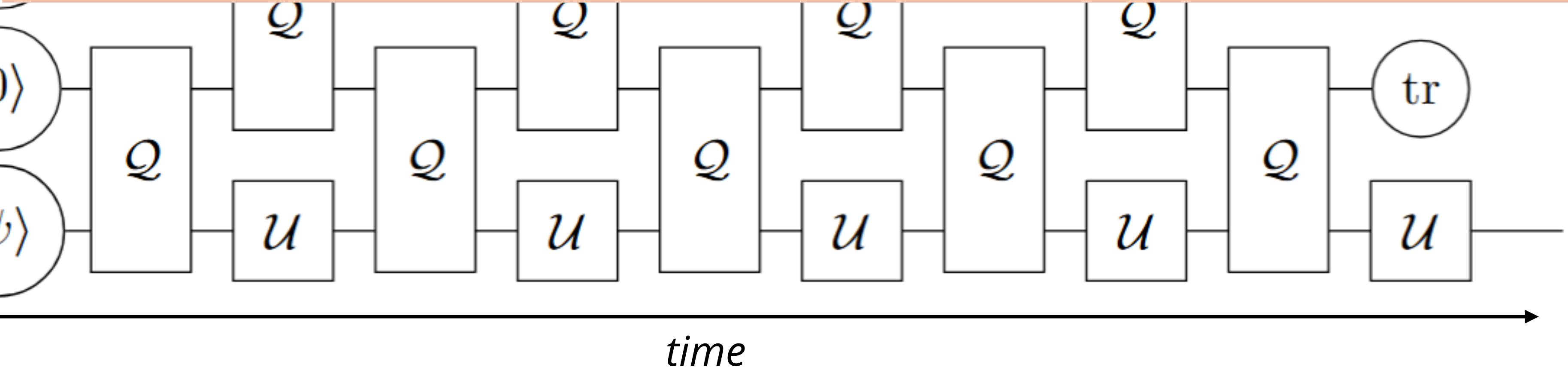
$$H = H_{\mathrm{imp}} + \sum_{\sigma=\uparrow,\downarrow} \sum_k g_k (d_\sigma f_{k\sigma}^\dagger + d_\sigma^\dagger f_{k\sigma}) + \sum_{k\sigma} \omega_k f_{k\sigma}^\dagger f_{k\sigma}$$

[Wilson (1975)]
[Chin et.al JMP 51,092109 (2010)]

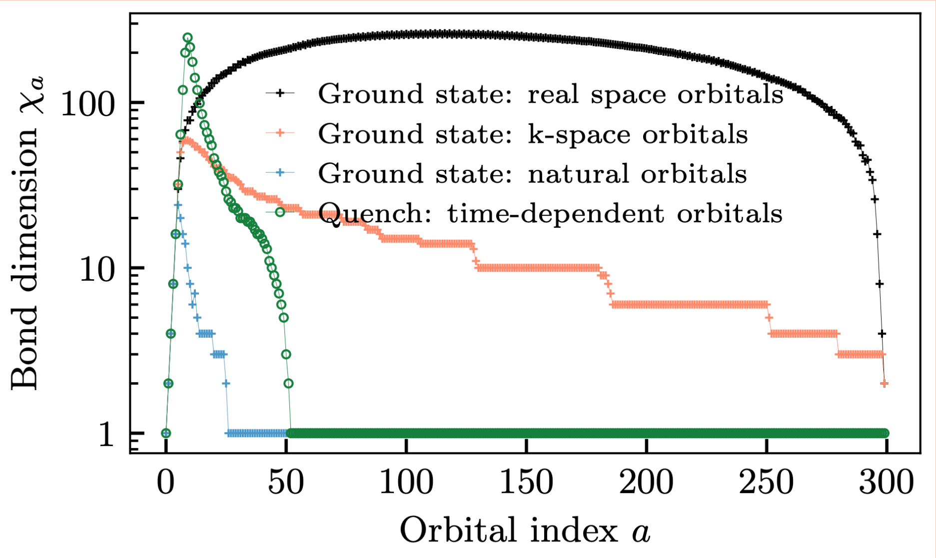


Spin-boson model

$$H = \Omega\sigma_z + \sigma_x \sum_\lambda g_\lambda (b_\lambda + b_\lambda^\dagger) + \sum_\lambda \omega_\lambda b_\lambda^\dagger b_\lambda$$

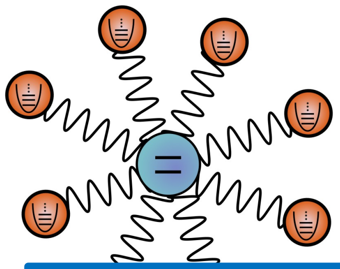



Chain (TEDOPA, Wilson) not a good basis for long evolution times?



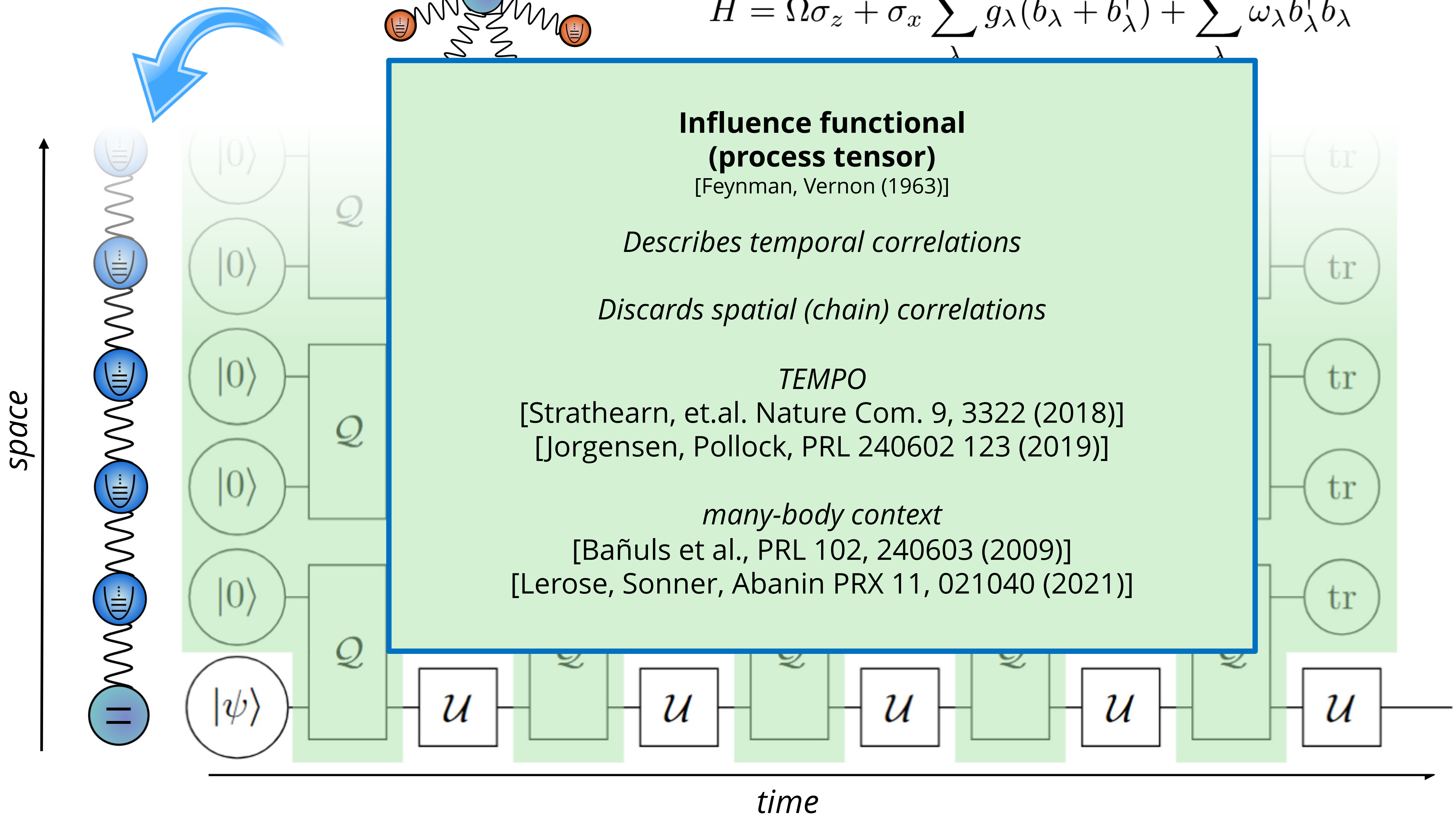
Resonant level model, figure from
[Fernández, Debertolis, Florens, arXiv.2503.13706]

[Wilson (1975)]
[Chin et.al JMP 51,092109 (2010)]



Spin-boson model

$$H = \Omega\sigma_z + \sigma_x \sum_\lambda g_\lambda (b_\lambda + b_\lambda^\dagger) + \sum_\lambda \omega_\lambda b_\lambda^\dagger b_\lambda$$



Influence functional
(process tensor)
[Feynman, Vernon (1963)]

Describes temporal correlations

Discards spatial (chain) correlations

TEMPO
[Strathearn, et.al. Nature Com. 9, 3322 (2018)]
[Jorgensen, Pollock, PRL 240602 123 (2019)]

many-body context
[Bañuls et al., PRL 102, 240603 (2009)]
[Lerose, Sonner, Abanin PRX 11, 021040 (2021)]

Outline

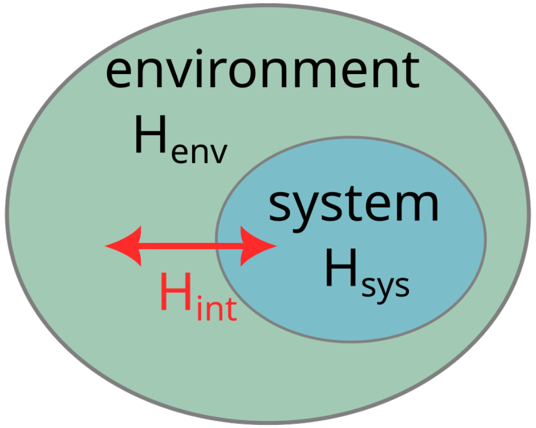
Open quantum systems + Influence functional

Time-translation invariant influence functionals

(Driven) Spin-boson model

Fermionic impurity models

Open quantum systems



Real-time dynamics: unitary channels for (small) time step

$$\mathcal{U}\,\rho = \mathrm{e}^{-\mathrm{i}H_{\mathrm{sys}}\delta t}\,\rho\,\mathrm{e}^{\mathrm{i}H_{\mathrm{sys}}\delta t}$$

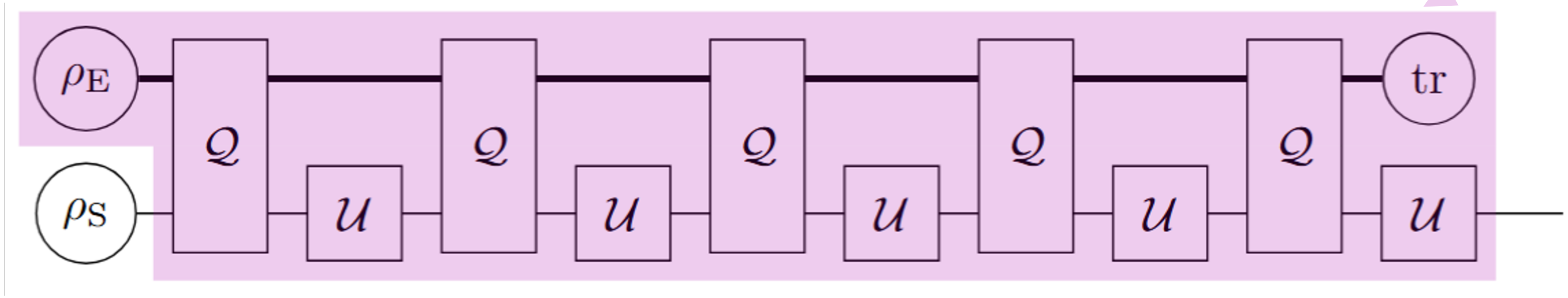
$$\mathcal{Q}\,\rho = \mathrm{e}^{-\mathrm{i}(H_{\mathrm{int}}+H_{\mathrm{env}})\delta t}\,\rho\,\mathrm{e}^{\mathrm{i}(H_{\mathrm{int}}+H_{\mathrm{env}})\delta t}$$

system state after N time steps

$$\rho_{\mathrm{S}}(N\delta t) = \mathrm{tr}_{\mathrm{E}}\left[(\mathcal{U}\mathcal{Q})^N \rho_{\mathrm{E}} \otimes \rho_{\mathrm{S}}\right]$$

Quantum circuit / tensor network notation

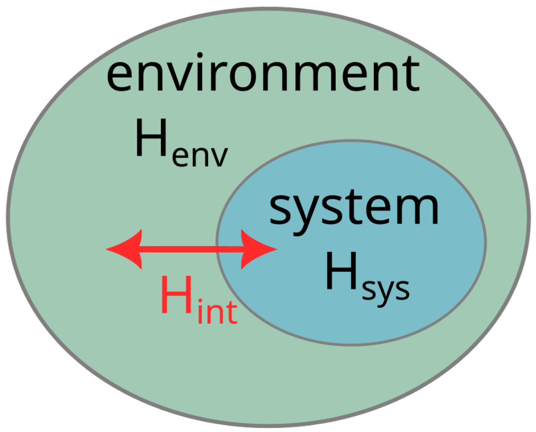
Dynamical map



$$\rho_{\mathrm{S}}(N\delta t) = \Phi_N \rho_{\mathrm{S}}$$

GKSL: Markovian semi-group propagator $\Phi_N = \mathrm{e}^{\mathcal{L}N\delta}$ $\Phi_M\Phi_N = \Phi_{N+M}$

Open quantum systems



Real-time dynamics: unitary channels for (small) time step

$$\mathcal{U}\,\rho = \mathrm{e}^{-\mathrm{i}H_{\mathrm{sys}}\delta t}\,\rho\,\mathrm{e}^{\mathrm{i}H_{\mathrm{sys}}\delta t}$$

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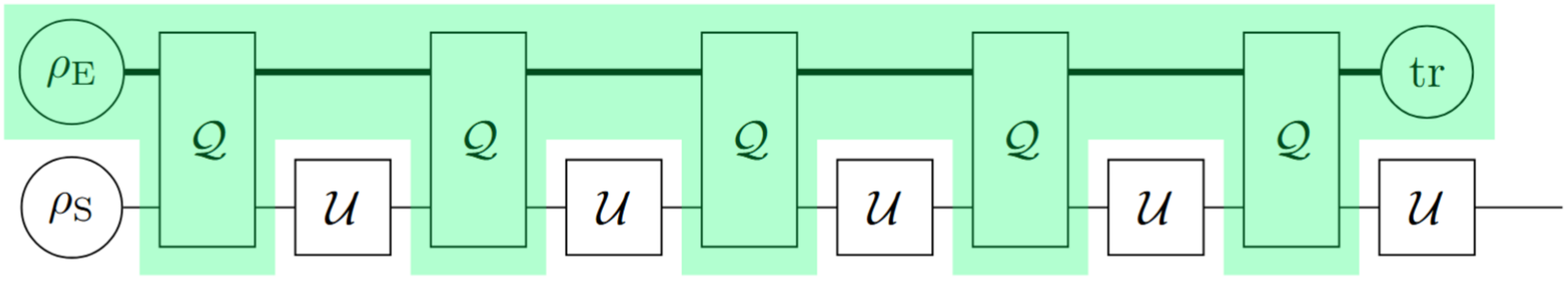
system state after N time steps

$$\rho_{\mathrm{S}}(N\delta t) = \mathrm{tr}_{\mathrm{E}}\left[(\mathcal{U}\mathcal{Q})^N \rho_{\mathrm{E}} \otimes \rho_{\mathrm{S}}\right]$$

Quantum circuit / tensor network notation

Influence functional/ process tensor



$$\mathcal{I}_N = \mathrm{tr}_{\mathrm{E}}\left[\mathcal{Q}^{\otimes_{\mathrm{S}} N} \rho_{\mathrm{E}}\right]$$

Influence functional as "temporal" MPS

Finite tensor networks

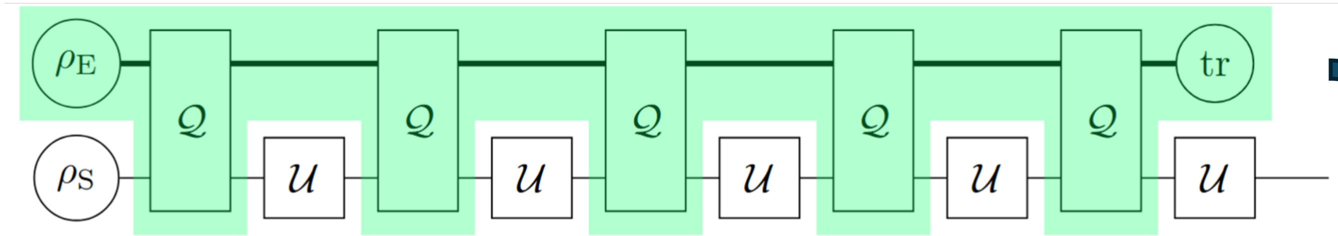
[Strathearn, et.al. Nature Com. 9, 3322 (2018)]

[Jorgensen, Pollock, PRL 240602 123 (2019)]

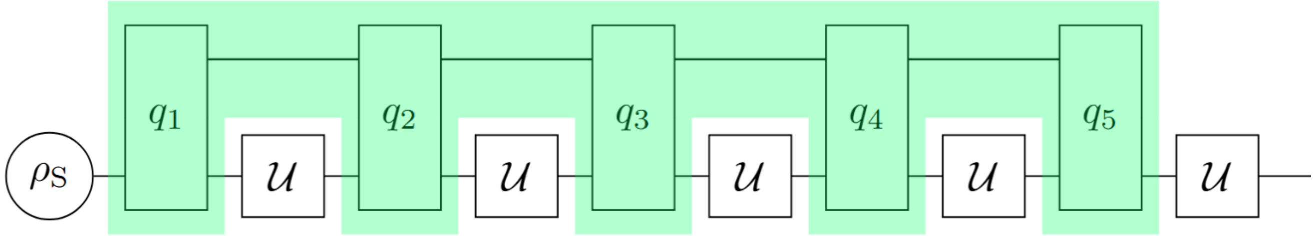
[Fux, et.al., PRL 126 200401 (2021)]

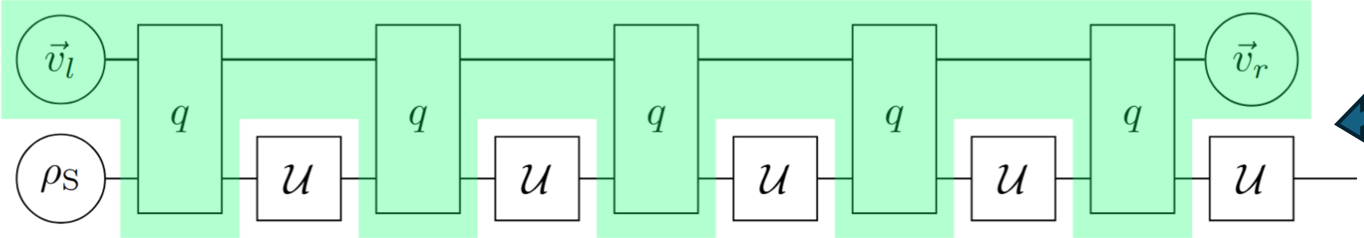
[Lerose, Sonner, Abanin PRX 11, 021040 (2021)]

...



Already has MPS form! But "bond dimension" is huge.





The MPS is *homogeneous* due to underlying *dynamical semi-group.*
(time-translation invariance)

iMPS at Berlin wall

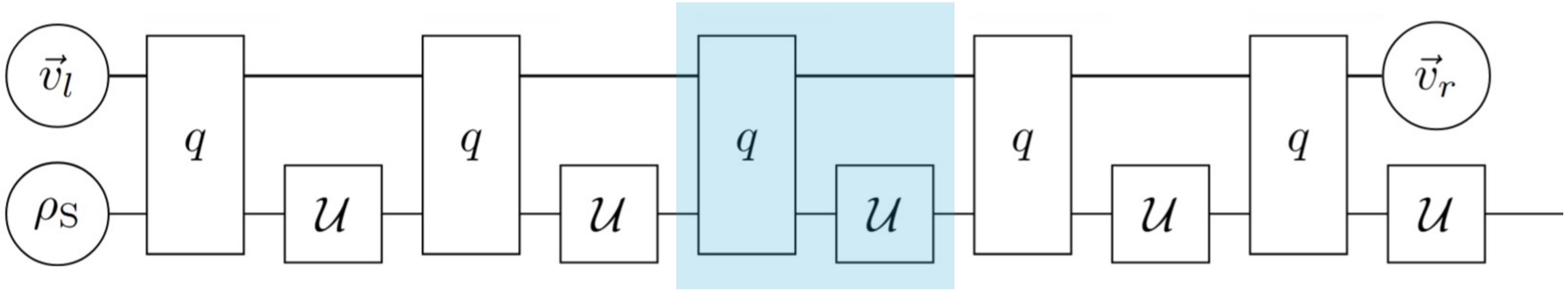
Infinite tensor networks

[VL, Tu, Strunz, PRL 132 200403 (2024)]

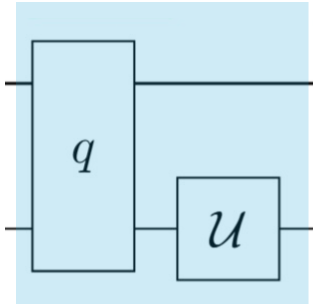
[Chu, Chen, PRB 110, 045106 (2024)]

[Sonner, VL, Abanin, PRL 135 170402 (2025)]

(HEOM, pseudomodes)



Semi-group propagator



$= \mathrm{e}^{\mathcal{G}\delta t}$

"auxiliary degrees of freedom"

$\partial_t \Gamma = \mathcal{G}\Gamma \qquad \rho = \Gamma \cdot \vec{v_r}$

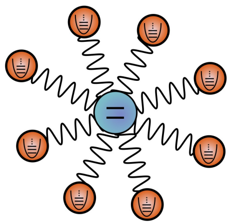
Steadystate = fixed point

cf Lindblad $\Phi_N = \mathrm{e}^{\mathcal{L}N\delta} \qquad \Phi_M \Phi_N = \Phi_{N+M}$

→ **Semi-group Markovian embedding**

But unphysical "bond space"

Non-interacting (Gaussian) environments

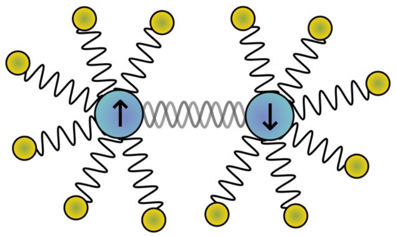


Bosonic

[VL, Tu, Strunz, PRL 132 200403 (2024)]

$$H = H_{\mathrm{sys}} + S\sum_{\lambda} g_\lambda (b_\lambda + b_\lambda^\dagger) + \sum_\lambda \omega_\lambda b_\lambda^\dagger b_\lambda$$

Fermionic



[Sonner, VL, Abanin, PRL 135 170402 (2025)]

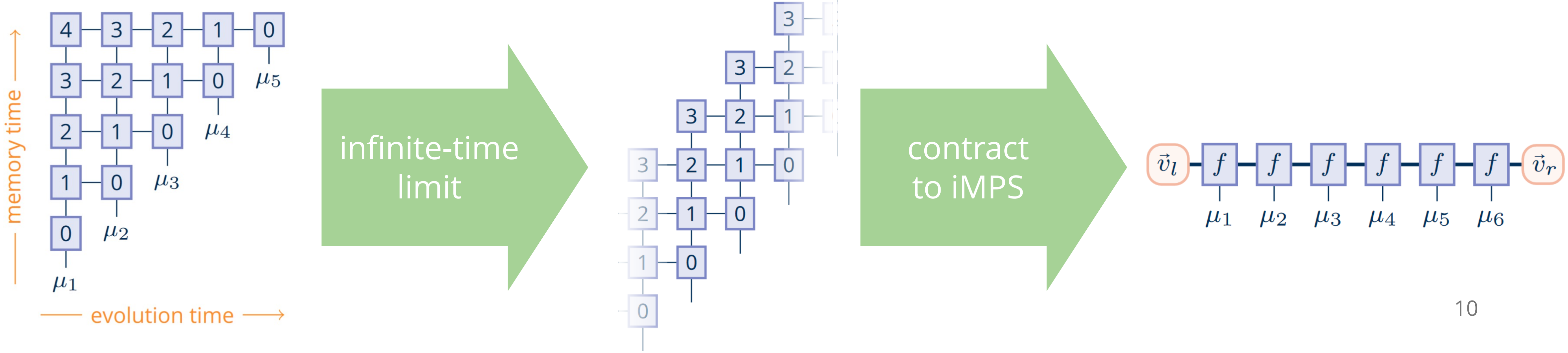
$$H = H_{\mathrm{imp}} + \sum_{\sigma=\uparrow,\downarrow}\sum_k g_k (d_\sigma f_{k\sigma}^\dagger + d_\sigma^\dagger f_{k\sigma}) + \sum_{k\sigma} \omega_k f_{k\sigma}^\dagger f_{k\sigma}$$

Exact expressions for influence functional. MPS form non-trivial

$$\mathcal{F}_N^{\mu_1\ldots\mu_N} = \exp\left(-\delta t^2 \sum_{t=1}^{N}\sum_{s=1}^{n} S_-^{\mu_t}\left(\mathrm{Re}[\alpha(t-s)]S_-^{\mu_s} - \mathrm{i}\,\mathrm{Im}[\alpha(t-s)]S_+^{\mu_s}\right)\right)$$

$$|I_N^\sigma\rangle = \prod_{n=1}^{N}\prod_{m=1}^{n} \exp\left(\vec{c}_n^{\,\dagger} \cdot G_{n,m}^\sigma \vec{c}_m^{\,\dagger}\right)|0\rangle$$

[Strathearn, et.al. Nature Com. 9, 3322 (2018)]
[Jorgensen, Pollock, PRL 240602 123 (2019)]



Spin boson - precision benchmark

Can the MPS influence functional capture algebraic decay?

$$H = \Omega\sigma_z + \sigma_x \sum_\lambda g_\lambda (b_\lambda + b_\lambda^\dagger) + \sum_\lambda \omega_\lambda b_\lambda^\dagger b_\lambda$$

Sub-Ohmic bath (s = 0.5, T = 0) $\quad J(\omega) = \eta\sqrt{\omega}\mathrm{e}^{-\omega/\omega_c}$

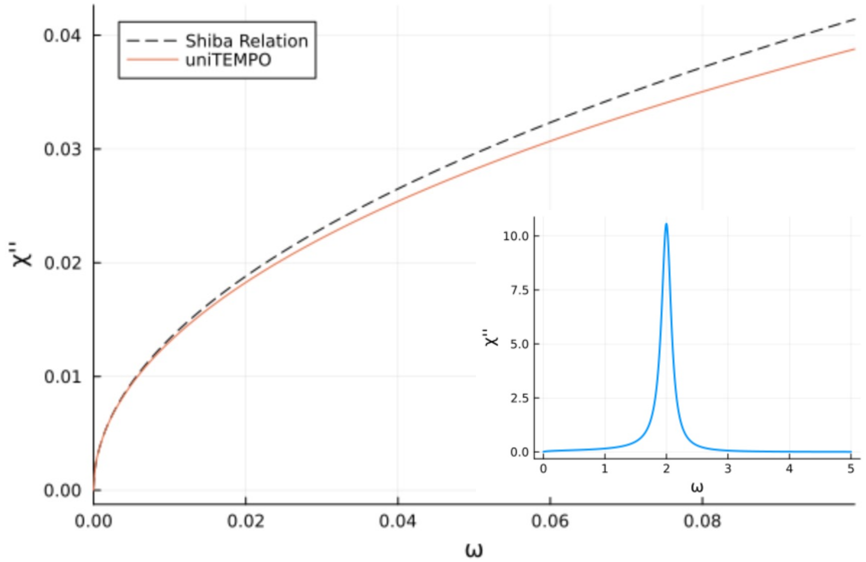
$$\chi_z(\omega) = -\mathrm{i}\int_0^\infty \mathrm{d}t \langle[\sigma_z(t), \sigma_z(0)]\rangle \mathrm{e}^{-\mathrm{i}\omega t}$$

Analytical result at low frequencies (deloc. phase)

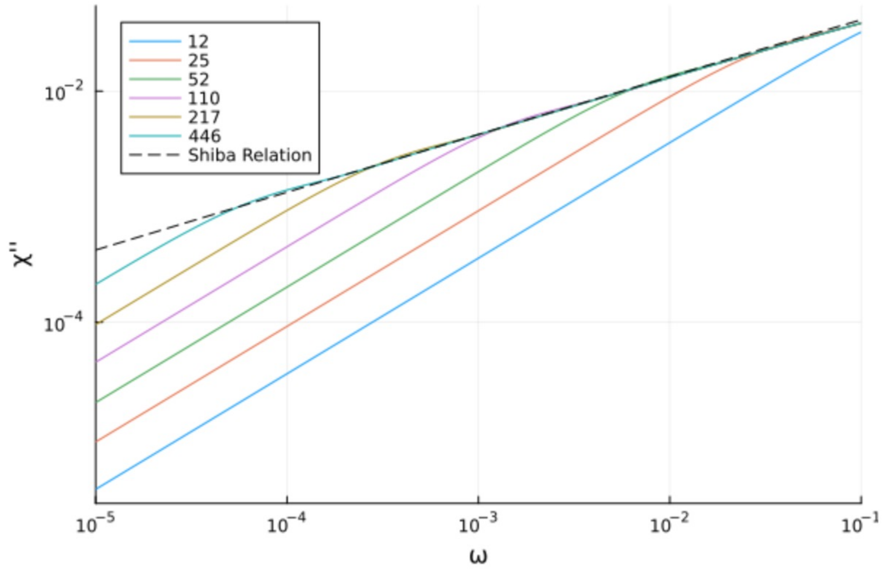
Shiba relation
[Shiba, Prog. Theor. Phys. 54, 967 (1975)]
[Xu et.al. Phys. Rev. Lett. 129, 230601 (2022)]

$$\mathrm{Im}(\chi_z(\omega)) = \frac{\pi}{4} J(|\omega|)\,[\mathrm{Re}(\chi_z(0))]^2$$

Spectra via ED: [Garbellini et.al. arXiv:2603.04970]




```
Ω = 1;
s = 0.5;
ω_c = 5;
η = 0.1;
```



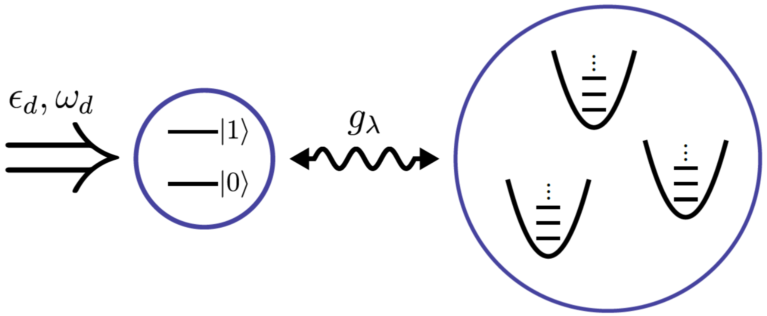
Driven-dissipative dynamics

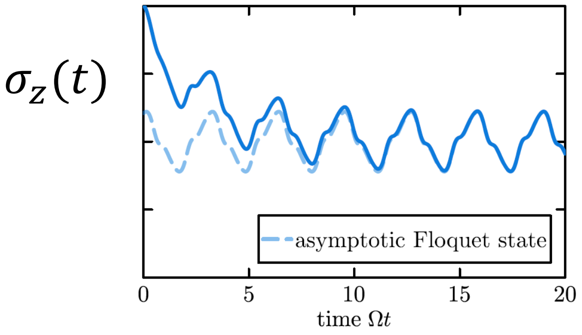
Konrad
Mickiewicz
(TU Dresden)

Driven spin boson model
[Mickiewicz, VL, Strunz, arXiv:2511.08754]

$$H = \Omega\sigma_z + \sigma_x \sum_\lambda g_\lambda (b_\lambda + b_\lambda^\dagger) + \sum_\lambda \omega_\lambda b_\lambda^\dagger b_\lambda$$
$$+\, \epsilon_d \cos(\omega_d t)\sigma_{x/z}$$

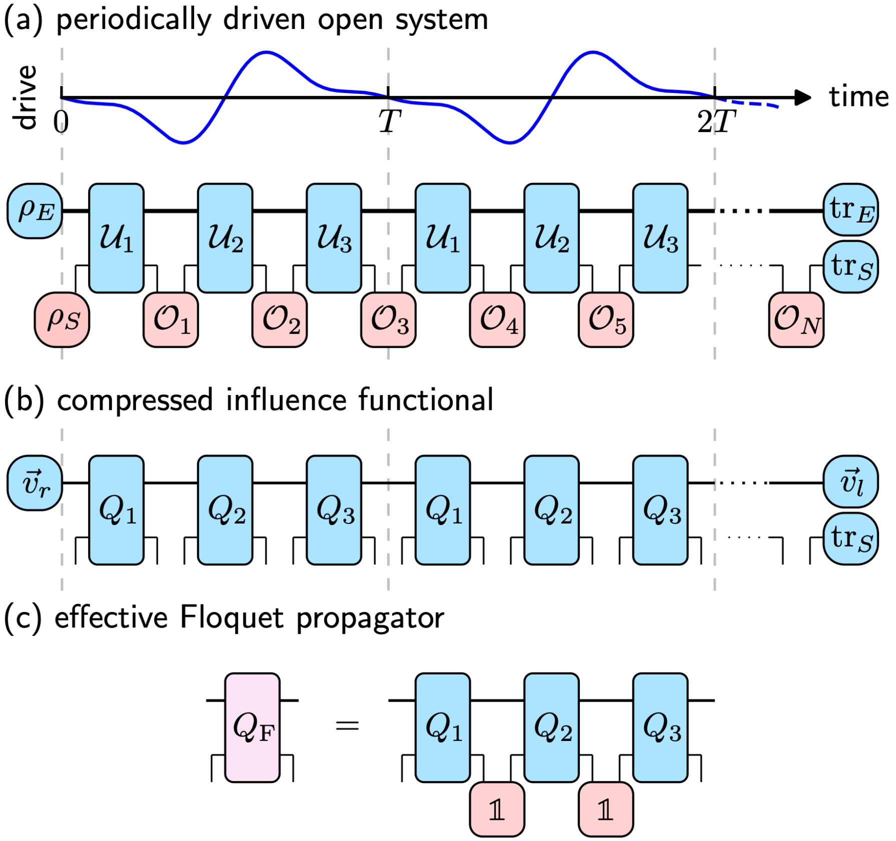
transversal longitudinal





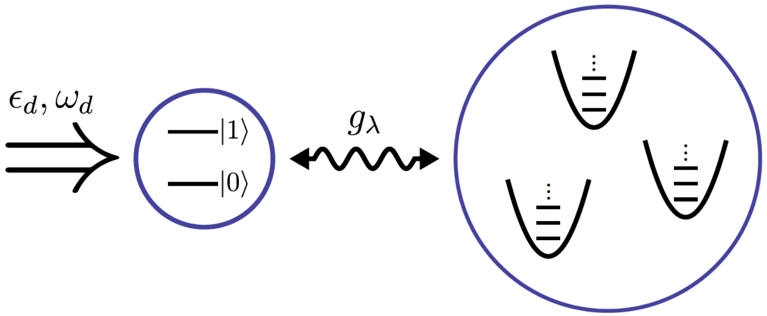
Semi-Group Influence Functional
+
Local periodic driving

→ Periodic Floquet Influence Functional



Driven-dissipative dynamics

Driven spin boson model
[Mickiewicz, VL, Strunz, arXiv:2511.08754]



$$H = \Omega\sigma_z + \sigma_x \sum_\lambda g_\lambda (b_\lambda + b_\lambda^\dagger) + \sum_\lambda \omega_\lambda b_\lambda^\dagger b_\lambda$$
$$+ \epsilon_d \cos(\omega_d t)\sigma_{x/z}$$

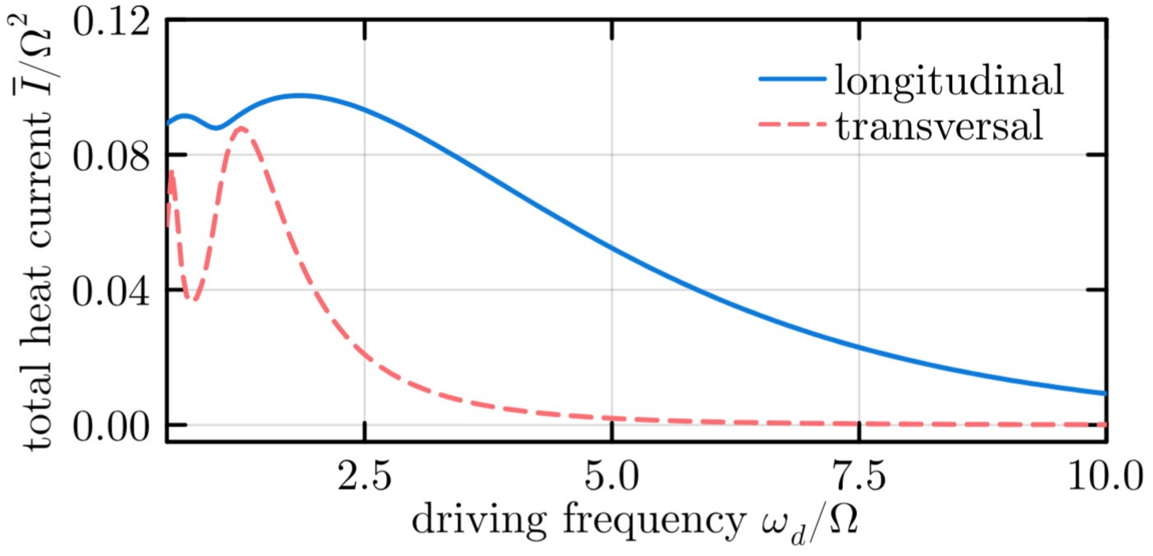
non-equilibrium signature: bath heating

$$j(t,\omega) = \sum_\lambda \frac{\mathrm{d}}{\mathrm{d}t} \langle \omega_\lambda b_\lambda^\dagger(t) b_\lambda(t)\rangle \delta(\omega - \omega_\lambda)$$

Period-averaged current from
period-averaged correlation function

$$\overline{C}(\tau) = \frac{1}{T}\int_t^{t+T} dt' \langle S(t'+\tau)S(t')\rangle$$

heating of the reservoir in steady state



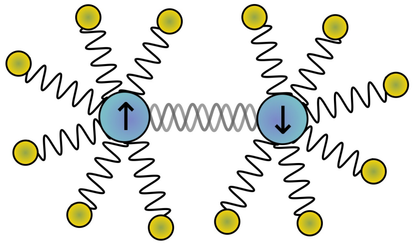
$$J(\omega) = \sum_\lambda |g_\lambda|^2 \delta(\omega - \omega_\lambda) = \alpha\omega \mathrm{e}^{-\omega/\omega_c} \quad \alpha = 0.05,\ \omega_c = 2.5\Omega \quad \epsilon_d = 1\Omega$$

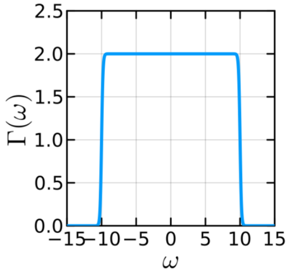
Fermions - Dissipative Kondo effect

[Sonner, VL, Abanin. PRL 135 170402 (2025)]

Michael
Sonner
(Berkeley)

$$H = \cancel{H_{\mathrm{imp}}} + \sum_{\sigma=\uparrow,\downarrow} \sum_k g_k (d_\sigma f^\dagger_{k\sigma} + d^\dagger_\sigma f_{k\sigma}) + \sum_{k\sigma} \omega_k f^\dagger_{k\sigma} f_{k\sigma}$$

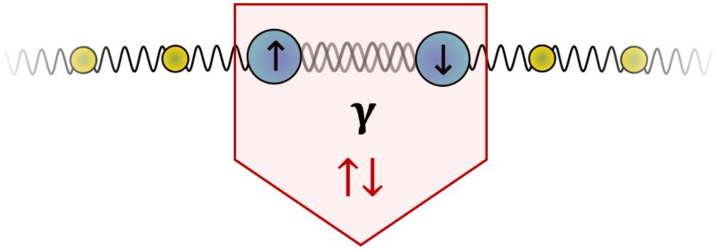




Nonequilibrium realization of Kondo physics

Local double losses [Stefanini et.al, Commun. Phys. 8, 212 (2025)]

$$\mathcal{L}\rho = -\mathrm{i}\varepsilon_d [n_\uparrow + n_\downarrow, \rho] + \gamma d_\uparrow d_\downarrow \rho d^\dagger_\downarrow d^\dagger_\uparrow - \frac{\gamma}{2}\{d^\dagger_\downarrow d^\dagger_\uparrow d_\uparrow d_\downarrow, \rho\}$$

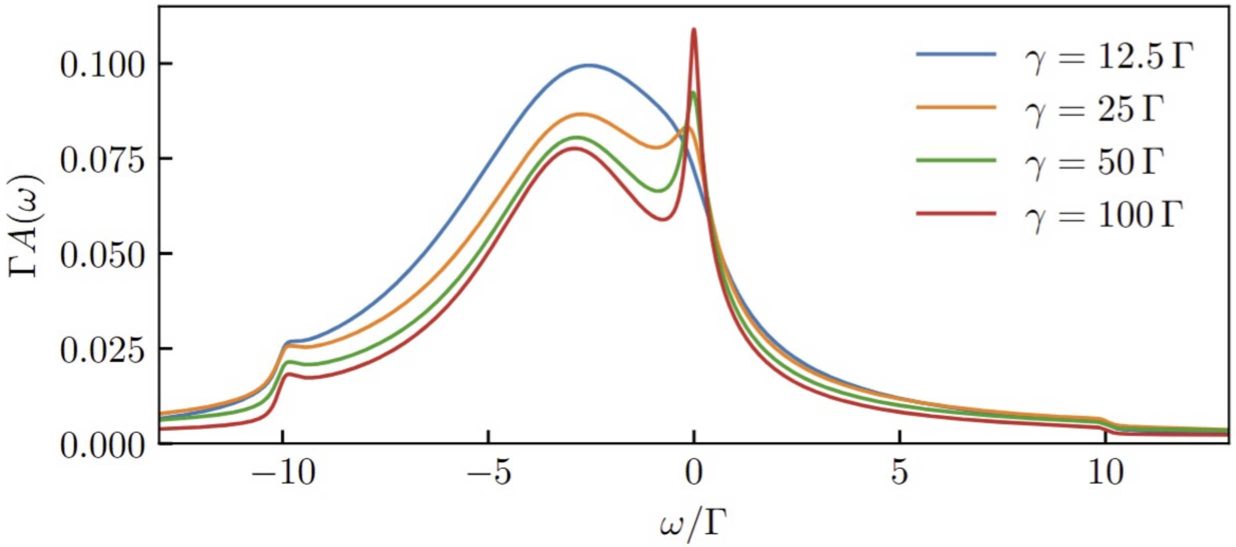


spectral function

$$G^R(t) = -\mathrm{i}\Theta(t)\langle\{d(t), d^\dagger(0)\}\rangle$$

$$A(\omega) = -\frac{1}{\pi}\mathrm{Im}G^R(\omega)$$

bond dimension 239/bath

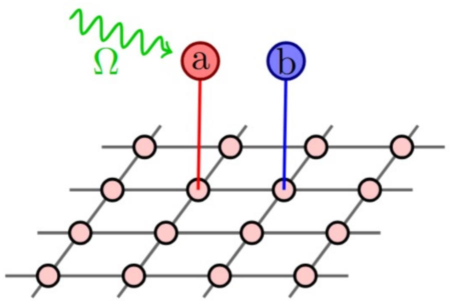


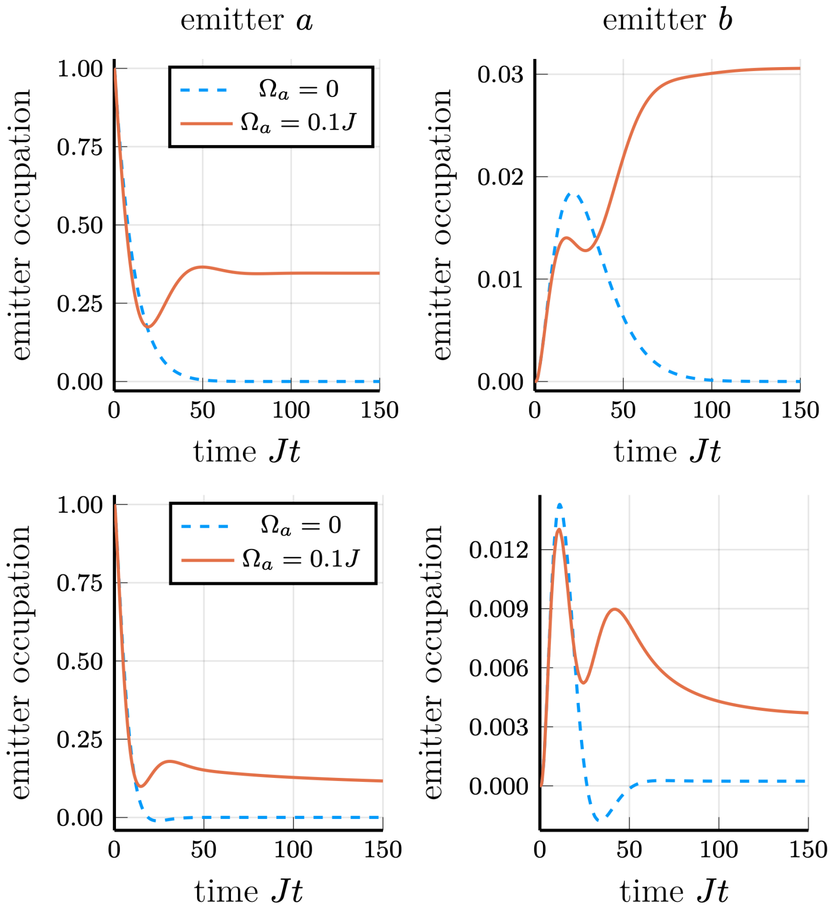
Multiple subsystems - Limitations

Highly coherent, non-additive reservoir [VL, arXiv:2603.23432 (2026)]

Emitters coupled to d-dimensional cubic lattice [Papaefstathiou et.al. PRA 112, 013721 (2025)]

$$H_{\mathrm{sys}} = \frac{\Delta}{2}(\sigma_z^a + \sigma_z^b) + \frac{\Omega}{2}\sigma_x^a \qquad H_{\mathrm{env}} = -J\sum_{\langle \vec{x},\vec{y}\rangle}(b_{\vec{x}}^\dagger b_{\vec{y}} + b_{\vec{y}}^\dagger b_{\vec{x}}) \qquad H_{\mathrm{int}} = g\sum_{l=a,b}(\sigma_-^l b_{\vec{x}^l}^\dagger + \sigma_+^l b_{\vec{x}^l})$$





$g^2 = 0.1J^2,\ \Delta = 0$

3D

(bond dim 117)

2D

not converged

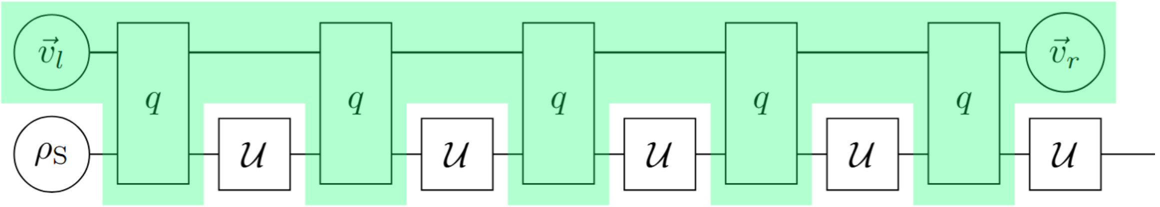
(bond dim 357)

Influence functional physical leg dimension

$2^8 = 256$

(filtering + low-rank SVD)

zero-detuning: low-frequency resonance (2D)



Summary

General “black-box“ method for strongly coupled open quantum systems / impurity models

Efficient contraction of influence matrix to infinite MPS (semi-group)

Competitive for both non-equilibrium and equilibrium dynamics [Nguyen et.al, *J. Chem Phys.* 161 104111 (2024)]

Challenges

Size of coupled system Hilbert space

Machine precision / Finite environments / Sharp resonances

Collaborators:
Michael Sonner, Konrad Mickiewicz, Walter Strunz, Hong-Hao Tu, Dima Abanin

QOpen Many-body Open Quantum Systems proposal

Define common benchmark set for non-Markovian methods

(TEMPO, ACE, HEOM, HOPS, TEDOPA, Pseudomodes, ML-MCTDH, SMat-PI, ...)

Thanks for your attention!