optimal control of open quantum systems
— lessons to date —

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QUANTUM DYNAMICS & CONTROL

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WOMEN 4 QUANTUM

control of open qu systems

Koch, J Phys Condens Matter 28, 213001 (2016) & arXiv:2512.04990 (lecture notes)

control of open qu systems

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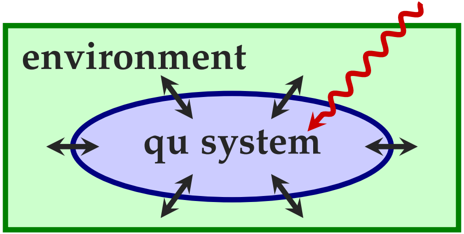

1. control "defines" the open quantum system
2. optimal control — set of tools to understand "what" and "how"

optimal control theory

steer a dynamical system
from an initial to a final state using an external knob

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steer a dynamical system from an initial to a final state using an external knob

navigate spacecraft:

- Newton's eq.
- initial state: $\vec{r}_0, \vec{v}_0$
  final state: $\vec{r}_f, \vec{v}_f$
- knob: change acceleration

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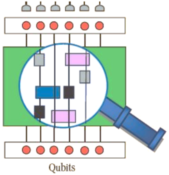
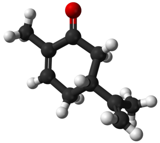
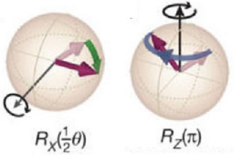
qubits



molecules





optimal control theory

steer dyn. system from initial to final state(s) using external knob

$$i\hbar \frac{\partial}{\partial t} |\Psi_{eln}(t)\rangle = \hat{H}[\{u_j(t)\}]\,|\Psi_{eln}(t)\rangle$$

$$\hat{H} = \hat{H}_0 + \sum_j u_j(t)\hat{H}_j$$

here: $\hat{H}_0 = \frac{\omega_0}{2}\hat{\sigma}_z$

$$\hat{H}_c = u_1(t)\,\hat{\sigma}_z + u_2(t)\,\hat{\sigma}_x$$

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initial states: $|\Psi_e(0)\rangle = \begin{cases} |0\rangle \\ |1\rangle \end{cases}$ $\quad |\Psi_n(0)\rangle = |\Phi_0\rangle$

final states: $|\Psi_e(T)\rangle = |\Psi_e(0)\rangle$ $\quad |\Psi_n(T)\rangle = \hat{O}|\Phi_0\rangle$

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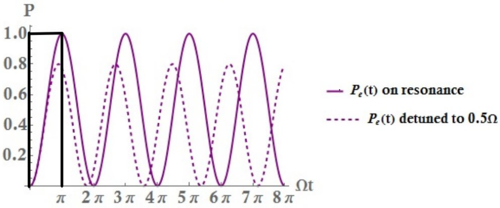
Solution to control problem:

resonant drive
with duration matching
desired Rabi angle



π- pulse

optimal control theory

steer dyn. system from initial to final state(s) using external knob

a more appropriate description:

$$\mathcal{D}_t : \hat{\rho}(0) \longrightarrow \hat{\rho}(t)$$

for open qu systems!

for example:

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right]_- + \sum_n \gamma_n \left(\hat{L}_n \hat{\rho} \hat{L}_n^+ - \frac{1}{2}\left[\hat{L}_n \hat{L}_n^+ , \hat{\rho}\right]_+\right)$$

$$\hat{H} = \hat{H}\left[\{u_j(t)\}\right]$$

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control problem:

given eqs. of motion and "target" find the $u_j(t)$

$$\hat{H} = \hat{H}\left[\{u_j(t)\}\right]$$

optimal control of quantum systems

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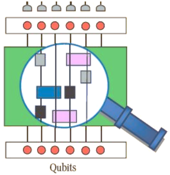
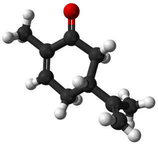
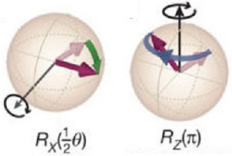
qubits *molecules*







optimal control of quantum systems

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Glaser et al., Eur. Phys. J. D 69, 279 (2015); Koch et al., Qu. Sci. Technol. 8, 045002 (2023)

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control problem :

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$\hat{H} = \hat{H}[\{u_j(t)\}]$

two choices

1. variational calculus (PMP)

   gradient-based optimization

   Python implementation of Krotov's method:
   https://github.com/qucontrol/krotov
   with detailed documentation & examples

   and many other variants (Tannor, Rabitz, Khaneja/Glaser,...)

2. parameter optimization

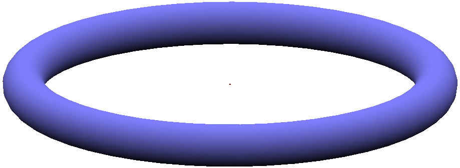
   gradient-free optimization

   many, many tools available (e.g. NLopt package from MIT)

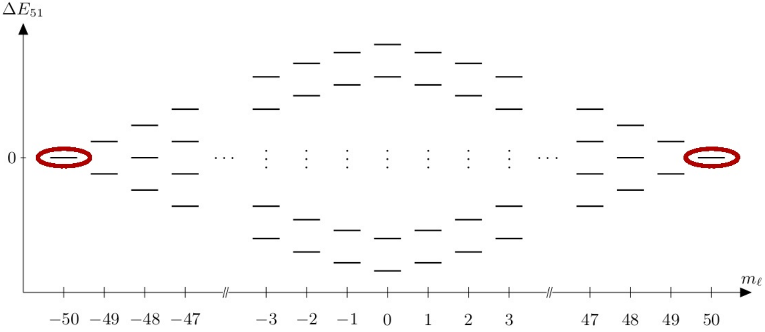
— an example —

preparation of (circular) Rydberg states

for quantum sensing or quantum computing



Rb valence electron in $|n,(l),m_l^{max}\rangle$

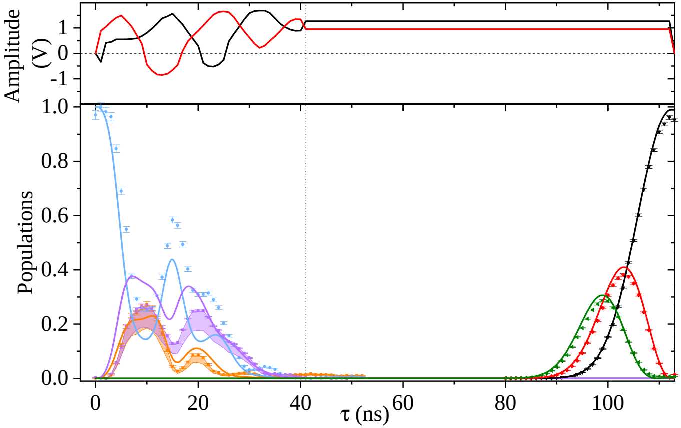


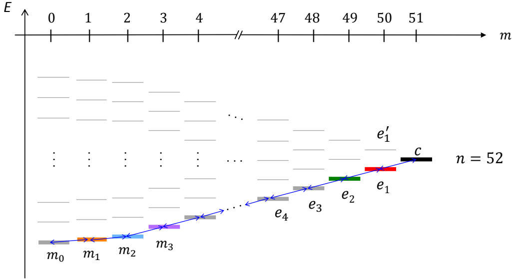
preparation of circular Rydberg state

theory & experiment

Larrouy, Patsch, . . . Koch, Gleyzes, Phys. Rev. X 10, 021058 (2020)

with optimized RF drives: fast & accurate

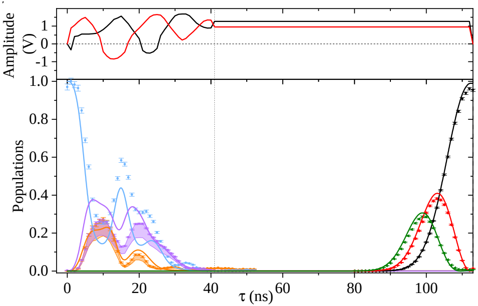


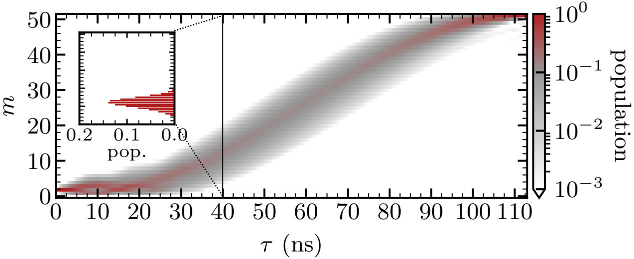


preparation of circular Rydberg state

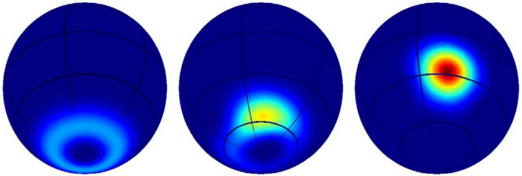
Larrouy, Patsch, . . . Koch, Gleyzes, Phys. Rev. X 10, 021058 (2020)

how do the optimized RF drives work? – via spin-coherent state





Husimi Q-function
(projection on spin-coh. state)
$Q_\Psi(\theta, \varphi) = \frac{2j+1}{4\pi} \, |\langle \theta, \varphi | \Psi \rangle|^2$



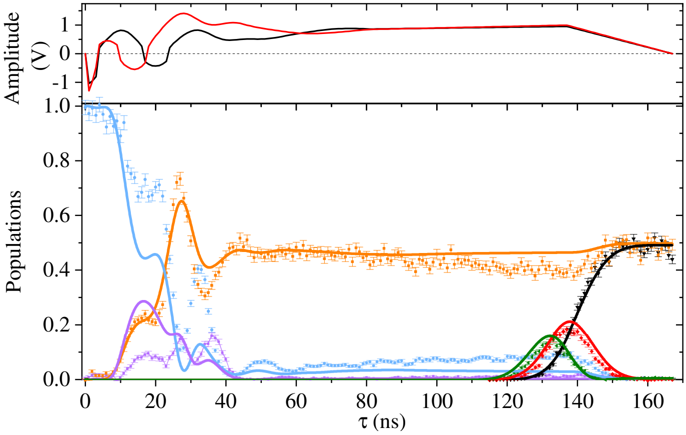
preparation of superposition state

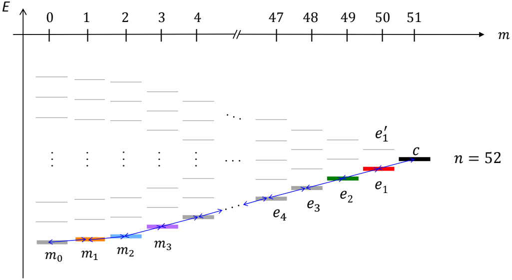
theory & experiment

Larrouy, Patsch, . . . Koch, Gleyzes, Phys. Rev. X 10, 021058 (2020)

non-classical superposition – no previously known protocol

similar accuracy : 97%





how to quantify success?

control target

does the evolved state match the target state?

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control target

does the evolved state match the target state?

evolved state $|\Psi(t)\rangle = \hat{U}(t,0;\{u_j\})|\Psi(0)\rangle$ target state $|\varphi_{targ}\rangle$

matching $\langle\Psi(T)|\varphi_{targ}\rangle = \langle\Psi(0)|\hat{U}^{\dagger}(T,0;\{u_j\})|\varphi_{targ}\rangle$

figure of merit $\in \mathbb{R}$ $J_T = |\langle\Psi(T)|\varphi_{targ}\rangle|^2$ or $J_T = \mathrm{Re}\{\langle\Psi(T)|\varphi_{targ}\rangle\}$ $J_T = J_T[\{u_j\}]$

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for open quantum systems: Hilbert-Schmidt product

$$J_T \sim \mathrm{tr}\left(\mathcal{D}_T(\hat{\rho}_{ini})\,\hat{\rho}_{targ}\right)$$

for the experts: while not a distance measure, the HS product is sufficient except if the target state is mixed

Basilewitsch, Koch, Reich, Adv. Quantum Technol. 2, 1800110 (2019)

how to quantify success?

control target

does the evolution of *all* states match the target operation?

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target op. $\hat{O}$ actual evolution $\hat{U}(T,0;\{u_j\})$

$$J_T = \frac{1}{N} \mathrm{Re}\left\{ \mathrm{tr}\left(\hat{O}^{+} \hat{P}_N \hat{U}(T,0,\{u_j\}) \hat{P}_N \right) \right\}$$

$\hat{P}_N$ projector onto logical subspace

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desired op. for a single initial state $\hat{\rho}_{ini}$: $\hat{O}\,\hat{\rho}_{ini}\,\hat{O}^+$

actual evolution $\hat{\rho}(T) = \mathcal{D}_T(\hat{\rho}_{ini})$

matching via HS product $\mathrm{tr}\left(\hat{O}\,\hat{\rho}_{ini}\,\hat{O}^+\,\hat{\rho}(T)\right)$

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matching via HS product $\mathrm{tr}\left(\hat{O}\,\hat{\rho}_{ini}\,\hat{O}^+\,\hat{\rho}(T)\right)$ how many $\hat{\rho}_{ini}$?

open quantum system characterization

how many initial states $\hat{\rho}_j$ need to be checked in $\sum_j \mathrm{tr}\left(\hat{\mathbf{O}}\hat{\rho}_j\hat{\mathbf{O}}^+\hat{\rho}_j(T)\right)$?

1. span all of Liouville space : N^2

Schulte-Herbrüggen et al, J Phys B (2011)

$$J_T = \frac{1}{N^2}\sum_{j=1}^{N^2} \mathrm{tr}\left(\hat{\mathbf{O}}\hat{\rho}_j\hat{\mathbf{O}}^+\hat{\rho}_j(T)\right)$$

where $N = 2^n$ for n qubits

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2. exploit that $\hat{\mathbf{O}}$ is unitary : 3 (or $N+1$)

Reich, Gualdi, Koch, Phys Rev A 88, 042309 (2013)
Reich, Gualdi, Koch, Phys Rev Lett 111, 200401 (2013)
Goerz, Reich, Koch, New J Phys 16, 055012 (2014)

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for coherent evolution, two states are sufficient to distinguish any two unitaries

$$\hat{\rho}_1 = \sum_i \lambda_i \hat{\mathbf{P}}_i \text{ (with } \lambda_i \neq \lambda_j) \qquad \hat{\rho}_2 = \hat{\mathbf{P}}_{rot} \text{ (with } \hat{\mathbf{P}}_{rot}\hat{\mathbf{P}}_i \neq 0 \;\; \forall i)$$

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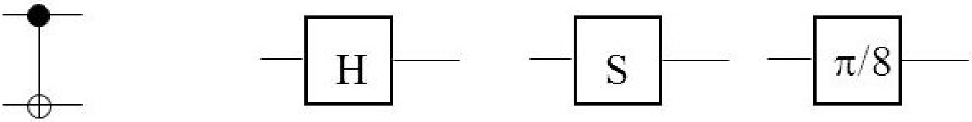
for open qu system, third state checks unitality on logical subspace $\hat{\rho}_3 = \mathbb{1}/N$

targeting quantum gates

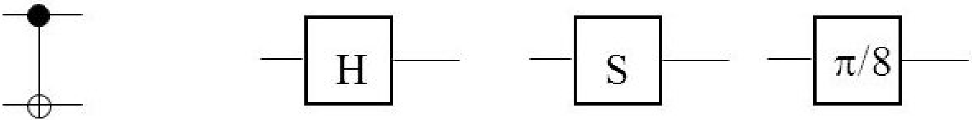
quantum gates

quantum gates

which gates? a universal set!



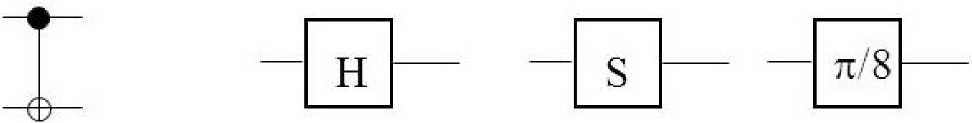
quantum gates
which gates? a universal set!



which entangling gate?

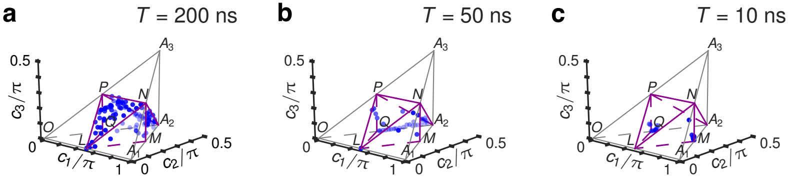
quantum gates

which gates? a universal set!



which entangling gate? optimal control can tell us!

Goerz, Motzoi, Whaley, Koch, npj Quantum Inf. 3, 37 (2017)



almost all PE

diagonal, $\sqrt{\text{SWAP}}$, $\sqrt{i\,\text{SWAP}}$

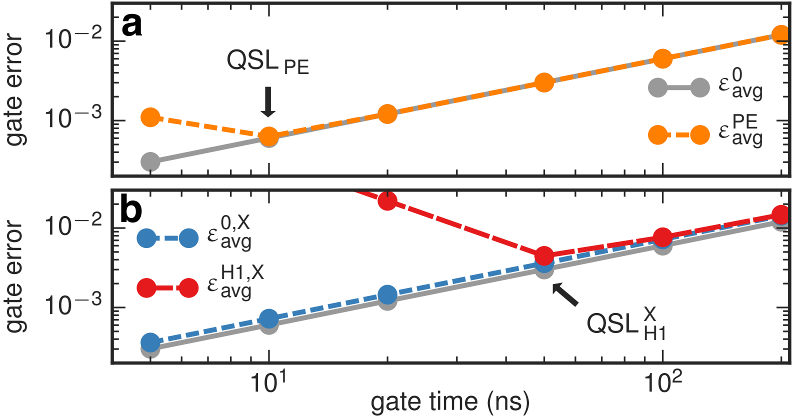
$\sqrt{i\,\text{SWAP}}$

fastest universal set of gates for transmons

— quantum speed limit —

Goerz, Motzoi, Whaley, Koch, npj Quantum Inf. 3, 37 (2017)

entangling: 10 ns **single-qubit: 50 ns**



lesson No. 1 :
avoid decoherence by operating fast

optimal control for avoiding decoherence

operate fast

utilize protection

optimal control for avoiding decoherence

operate fast

utilize protection

qu sensor near decoherence-free subspace

Basilewitsch, Yuan, Koch, Phys Rev Research 2, 033396 (2020)

STIRAP-like solutions for Rydberg gate

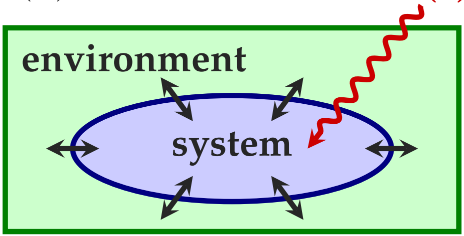
Goerz, Halperin, Aytac, Koch, Whaley, Phys Rev A 90, 032329 (2014)

control of open quantum systems

Koch, J Phys Condens Matter 28, 213001 (2016) & arXiv:2512.04990

$$\hat{\mathsf{H}} = \hat{\mathsf{H}}_S(\epsilon) + \hat{\mathsf{H}}_{SE} + \hat{\mathsf{H}}_E$$

$\epsilon(t)$ **control ?**



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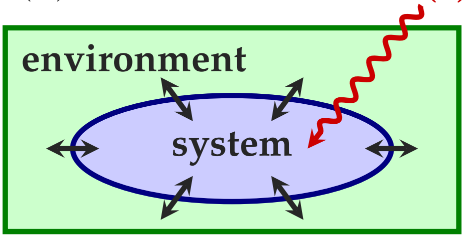

- what are viable quantum control strategies ?
- dependence on properties of environment / coupling ?
- fundamental limits to quantum control ?

control of open quantum systems

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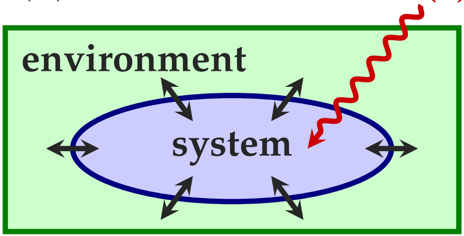

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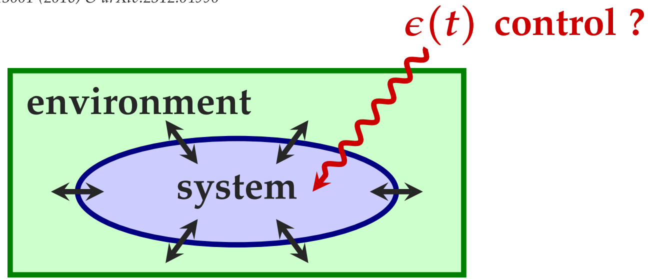
but also dissipation-enabled control: cooling, qubit reset

control of open quantum systems

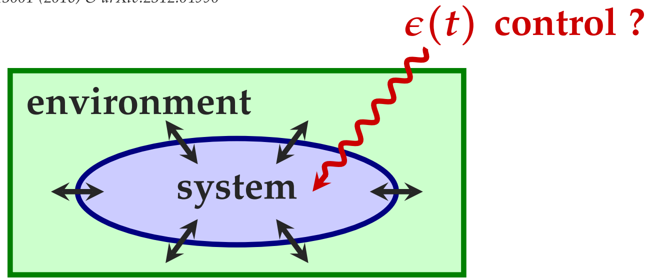
Koch, J Phys Condens Matter 28, 213001 (2016) & arXiv:2512.04990



1. beat decoherence ... using optimal control

control of open quantum systems

Koch, J Phys Condens Matter 28, 213001 (2016) & arXiv:2512.04990



1. beat decoherence … using optimal control
2. cooperate with environment … quite hard & non-generic

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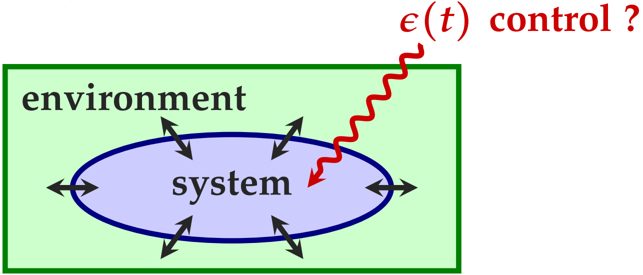

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3. exploit environment ... laser cooling, qubit reset
   ... quantum reservoir engineering

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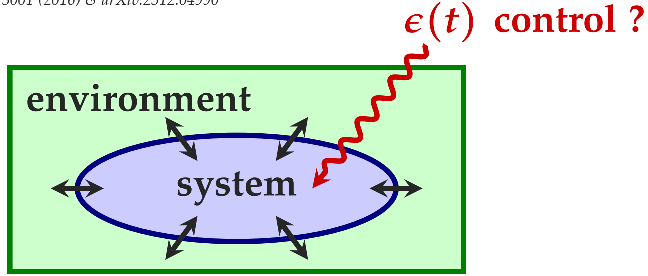

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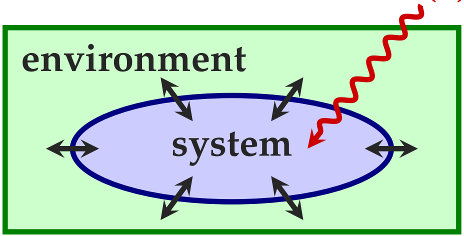
$\epsilon(t)$ **control ?**



1. beat decoherence … using optimal control
2. cooperate with environment … quite hard & non-generic
   role of non-Markovian effects?
3. exploit environment … laser cooling, qubit reset
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Markovian vs non-Markovian evolution

non-Markovian evolution: loss of energy and phase not monotonic

quantifiers of non-Markovianity

- distinguishability of two optimal states
  (recovery of previously lost information) *Breuer, Laine, Piilo, Phys Rev Lett 103, 210401 (2009)*
- divisability of the dynamical map
  (increase of correlations in a multi-partite system)
  Rivas, Huelga, Plenio, Phys Rev Lett 105, 050403 (2010)
- ...
- accessible volume in Liouville (state) space
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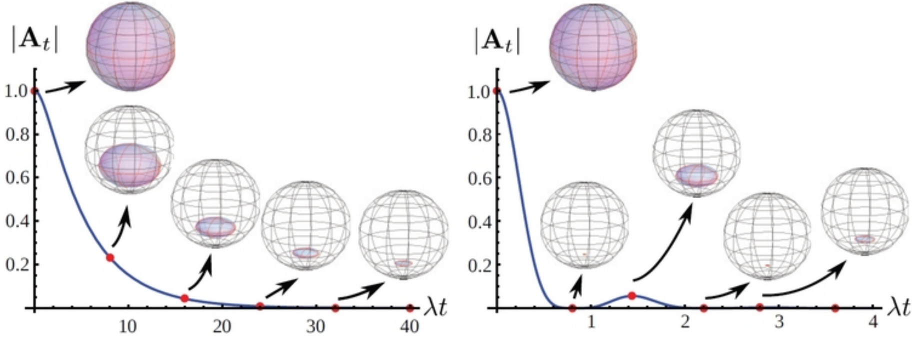
$\curvearrowright$ **exploit these features for control**

Markovian vs non-Markovian evolution

does non-Markovian evolution allow for more control ?

example: set of reachable states and Liouville space determinant

Lorenzo, Plastina, Paternostro, Phys Rev A 88, 020102 (2013)



non-Markovianity enhanced control

flux-biased phase qudit with limited control

$$\hat{\mathbf{H}} = \hbar\omega_0\hat{\mathbf{n}} + \frac{\beta}{2}\hat{\mathbf{n}}\left(\hat{\mathbf{n}} - 1\right) + \underbrace{\kappa_1 I_c(t)\hat{\mathbf{n}}}_{\substack{\text{low}\\ \text{frequency}}} + \underbrace{\kappa_2 I_c(t)\left(\hat{\mathbf{a}} + \hat{\mathbf{a}}^+\right)}_{\substack{\text{high frequency:}\\ \text{all SO(4) ops}}}$$

Svetitsky et al. Nature Commun. 5, 5617 (2014)

missing for full SU(4) controllability ?

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Cartan decomposition of $\mathfrak{su}(N)$

$$\forall\, U \in SU(N): \qquad U = k_1\, A\, k_2 \qquad k_1, k_2 \in SO(N)$$

A diagonal, unitary matrix

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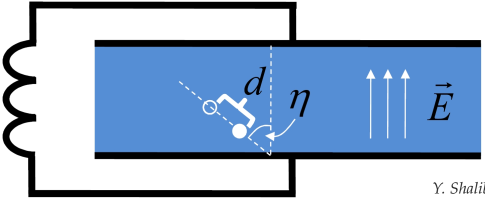
$\curvearrowright$ **diagonal unitaries (with 'non-local' phases)**

non-Markovianity enhanced control

main source of decoherence in Josephson junctions: charge defects (TLSs)



Y. Shalibo, PhD thesis (Katz group, Hebrew U Jerusalem)

$$\hat{\mathsf{H}}^{(i)} = \Delta^{(i)}\sigma_z^{(i)} \qquad \hat{\mathsf{H}}^{(i)}_{int} = \frac{S^{(i)}}{2}\left(\hat{\mathsf{a}}\hat{\sigma}_i^{+} + \hat{\mathsf{a}}^{+}\hat{\sigma}_i^{-}\right)$$

parameters can be measured in exp. *Shalibo et al. PRL 105, 177001 (2010)*

1. $\Delta^{(i)} \gg \omega_0 \Longrightarrow$ no effect on qudit
2. $\Delta^{(i)} \ll \omega_0 \Longrightarrow T_2$: pure dephasing of qudit
3. $\Delta \simeq \omega_0$, S weak $\Longrightarrow T_1$: relaxation of qudit
4. $\Delta \simeq \omega_0$, S strong $\Longrightarrow$ non-Markovian dynamics

non-Markovianity enhanced control

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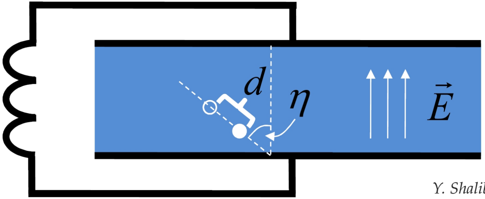

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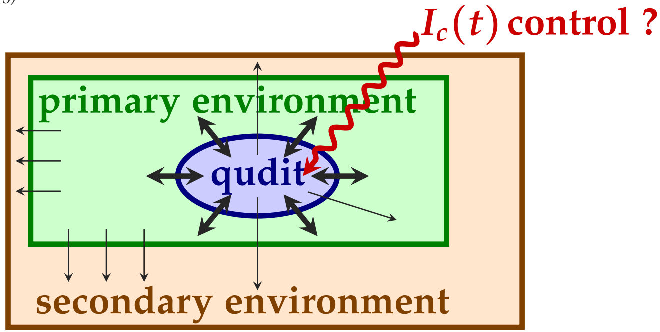
non-Markovianity enhanced control

Reich, Katz, Koch, Sci Rep 5, 12430 (2015)



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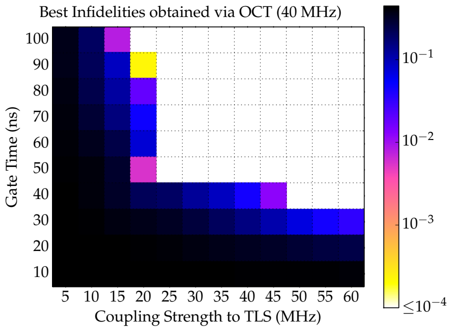
} secondary environment master eq. (items 1–3)

primary env. (item 4)

non-Markovianity enhanced control

Reich, Katz, Koch, Sci Rep 5, 12430 (2015)

error after optimization for $\mathrm{diag}(1,-1,1,1)$ w/ single primary bath TLS



$T_1 = \infty$ (no loss)

→ **for weak coupling no control**

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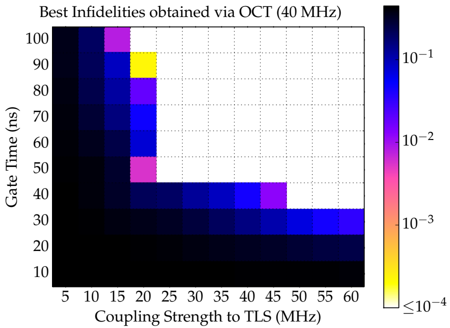

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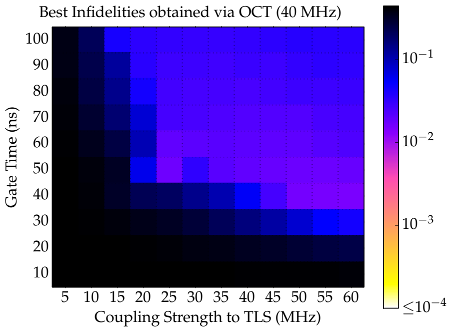


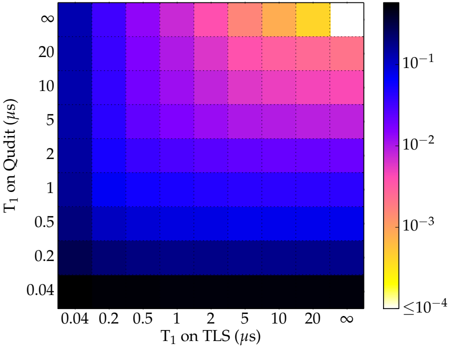
$T_1 = 5\mu$s, $T_1^{TLS} = 1\mu$s

→ **for strong enough coupling error small even for lossy TLS**

→ **for weak coupling no control**

non-Markovianity enhanced control

Reich, Katz, Koch, Sci Rep 5, 12430 (2015)



control lost if Markovian decay too fast: $T_1 < 1/S$

non-Markovianity enhanced control

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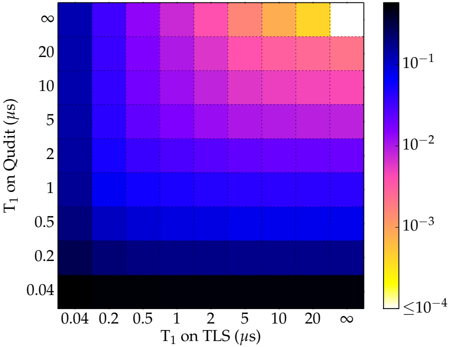


larger primary bath ?

up to 4 TLS

↷ similar errors except for strongly coupled resonant very lossy TLS

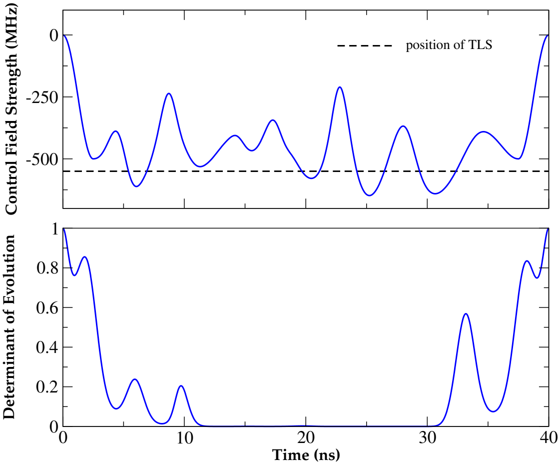
control lost if Markovian decay too fast: $T_1 < 1/S$

non-Markovianity enhanced control

how does the control work?

Reich, Katz, Koch, Sci Rep 5, 12430 (2015)

ramp into resonance + acquire non-local phase from interaction



non-Markovianity enhanced control

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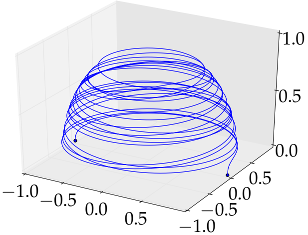
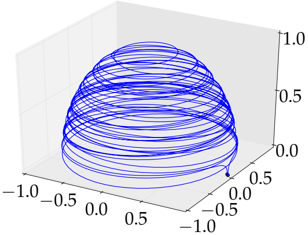
ramp into resonance + acquire non-local phase from interaction

evolution of $|1\rangle$



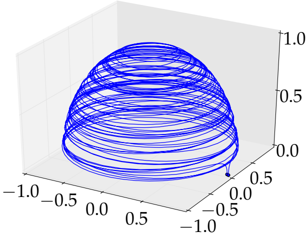
non-local phase acquired

evolution of $|2\rangle$



original phase restored

evolution of $|3\rangle$



original phase restored

proper phase alignment possible thanks to optimal control

lesson No. 2 :

more control with structured environments (non-Markovian dynamics)

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more control with structured environments (non-Markovian dynamics)

but no general picture yet

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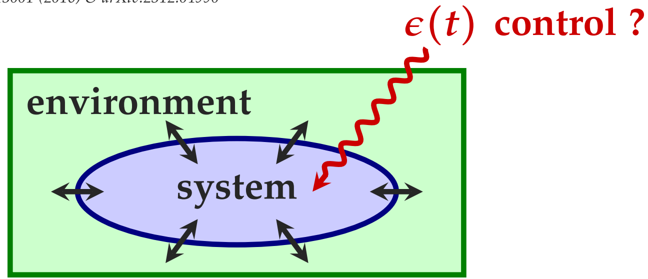

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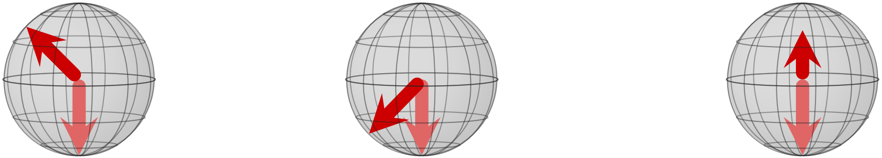
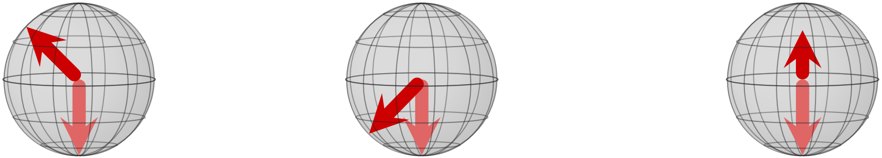
exploiting the environment for control

fast qubit reset



well-protected qubit $\longleftrightarrow$ reset as fast as possible

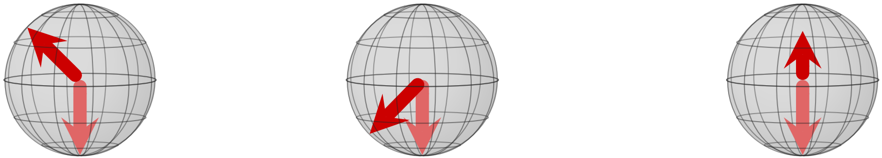
fast qubit reset



well-protected qubit $\longleftrightarrow$ reset as fast as possible

a structured environment is crucial

fast qubit reset



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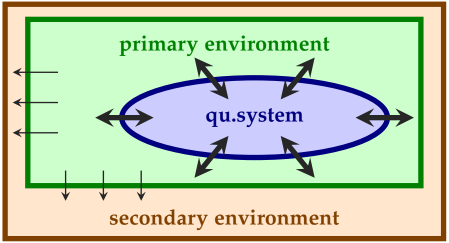
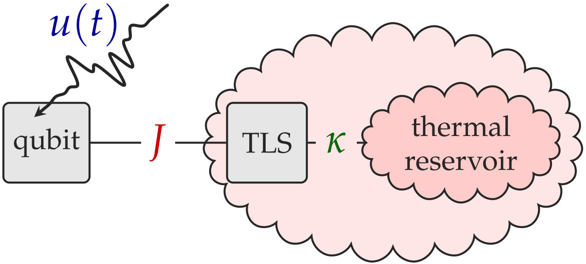


paradigm

- auxiliary degrees of freedom ("primary" environment) for fast reset
- secondary environment for thermalization

fast qubit reset

simplest setup without violation of 2nd law



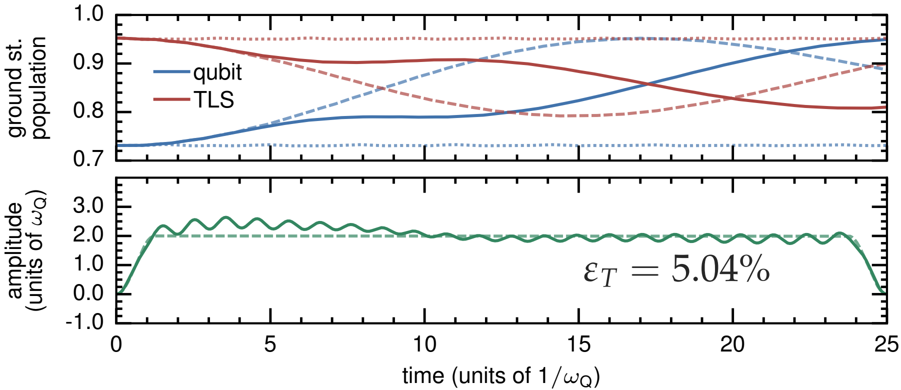
timescale separation: $\kappa < J < \omega_Q$

primary bath = TLS:
strong coupling J for fast reset

reservoir:
weak coupling κ for thermalization

optimal control for fast qubit reset

Basilewitsch, Schmidt, Sugny, Maniscalco, Koch, New J Phys 19, 113042 (2017)



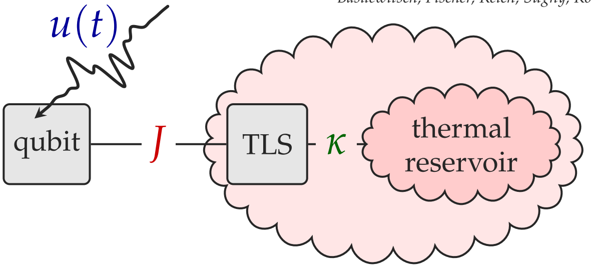
solution: ramp into resonance & swap populations

$\curvearrowright$ final error determined by temperature

related exp. work: Magnard et al., Phys Rev Lett 121, 060502 (2018)

time-optimal qubit reset

Basilewitsch, Fischer, Reich, Sugny, Koch, Phys Rev Research 3, 013110 (2021)



$$\hat{\mathsf{H}}(t) = \frac{\omega_Q}{2}\hat{\sigma}_z \otimes \mathbb{1} + \frac{u(t)}{2}\hat{\sigma}_i \otimes \mathbb{1} - \mathbb{1} \otimes \frac{\omega_{TLS}}{2}\hat{\sigma}_z + J\left(\hat{\sigma}_n \otimes \hat{\sigma}_m\right)$$

- if combination of control $\hat{\sigma}_i$ and coupling $\hat{\sigma}_n \otimes \hat{\sigma}_m$ optimal
- drawback: slow thermalization $\longleftrightarrow \kappa$

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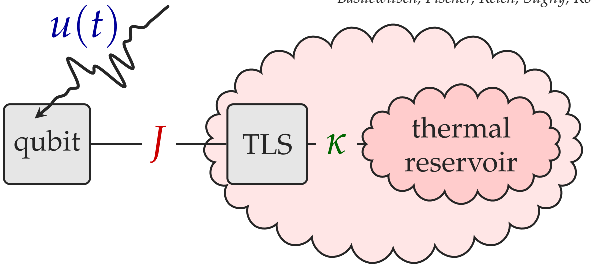

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can we *engineer* effective κ?

what sets decay rates & channels ?

standard GKLS master equation

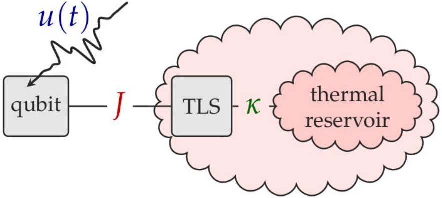
$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right]_- + \sum_n \gamma_n\left(\hat{L}_n\hat{\rho}\hat{L}_n^+ - \frac{1}{2}\left[\hat{L}_n\hat{L}_n^+,\hat{\rho}\right]_+\right)$$

$$\hat{H} = \hat{H}\left[\{u_j(t)\}\right]$$

with "natural" rates γ_n

$\hat{L}_n \sim |\varphi_i\rangle\langle\varphi_j|$ eigenstates of $\hat{H}_S$

structured environments



rates set by J (and κ)

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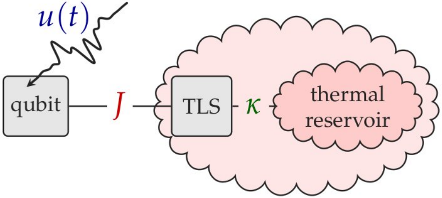


rates set by J (and κ)

(non)adiabatic master equations

for strong (and fast) driving:

derive ME in “dressed” (instantaneous) eigenbasis of $\hat{H}_S$

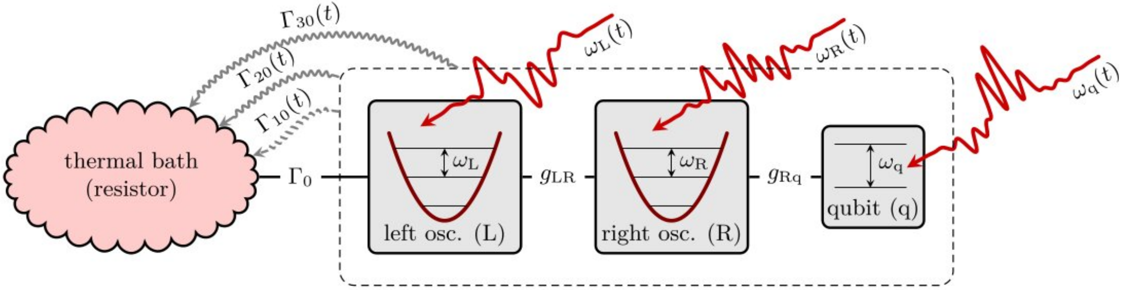
Albash, Boixo, Lidar, Zanardi, New J Phys 14, 123016 (2012); Dann, Levy, Kosloff, Phys. Rev. A 98, 052129 (2018)

$\curvearrowright$ $\hat{L}_n$ (and thus also γ_n) become dependent on $u_j(t)$!

adiabatic ME — tunable decay rates

optimal control of qubit reset

Basilewitsch, Cosco, LoGullo, Möttönen, Ala-Nissilä, Koch, Maniscalco, New J Phys 21, 093054 (2019)

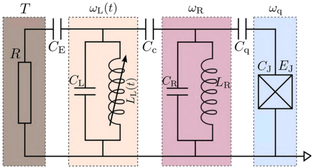


$$\Gamma_{i0}(t) = \Gamma_0 \left| \langle \Psi_0(t) | \hat{a}_L^\dagger + \hat{a}_L | \Psi_i(t) \rangle \right|^2 \frac{\omega_L(t)\omega_i(t)}{\omega_R(t)^2} \frac{1}{1 - e^{-\omega_i(t)/k_B T_{env}}}$$

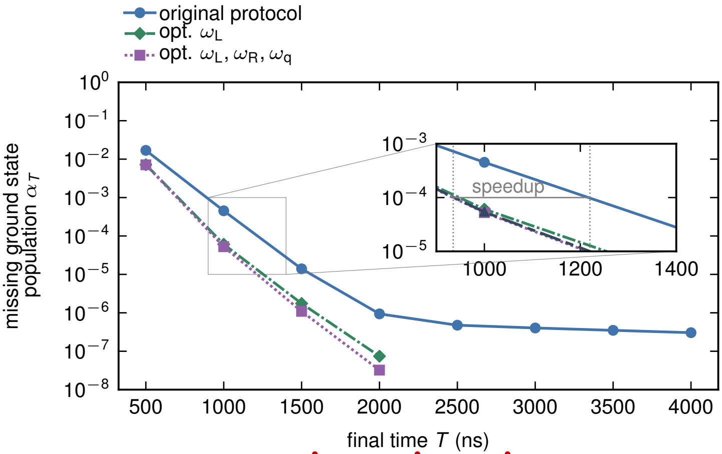
use all possible external drives & optimize

adiabatic ME — tunable decay rates

optimal control of qubit reset



Basilewitsch, Cosco, LoGullo, Möttönen, Ala-Nissilä, Koch, Maniscalco, New J Phys 21, 093054 (2019)

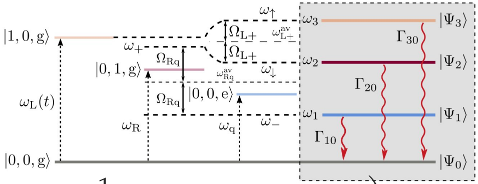


—faster quantum reservoir engineering—

control of open quantum systems

control-dependent decay rates & channels

1. strong drives: adiabatic master equations



- speed-up of decay
- limited by assumptions on model

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}\left[\hat{H}\left[\{u_j\}\right],\hat{\rho}\right]_- + \sum_j \gamma_n\left[\{u_j\}\right]\left(\hat{L}_n\left[\{u_j\}\right]\hat{\rho}\hat{L}_n\left[\{u_j\}\right]^+ - \frac{1}{2}\hat{L}_n\left[\{u_j\}\right]\hat{L}_n\left[\{u_j\}\right]^+\hat{\rho}\right)$$

2. strong drives: non-adiabatic master equations

Dann & Kosloff; Aroch, Kallush & Kosloff (Hebrew U Jerusalem)

3. strong drives: non-Markovian dynamics

Ankerhold group (U Ulm)

full potential of strong drives remains to be explored

lesson No. 3 :

exploitation of environment as entropy sink can be optimized

lesson No. 3 :

exploitation of environment as entropy sink
can be optimized

best strategies ?
strong couplings? strong drives? both?

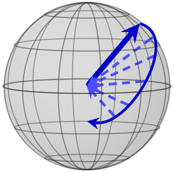
faster quantum reservoir engineering

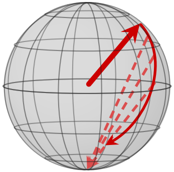
what about *better*
quantum reservoir engineering ?

quantum reservoir engineering

$$\partial_t \hat{\boldsymbol{\rho}} = \mathcal{L}(\hat{\boldsymbol{\rho}}) = -\frac{i}{\hbar}[\hat{\mathbf{H}}, \hat{\boldsymbol{\rho}}]_- + \mathcal{L}_D(\hat{\boldsymbol{\rho}})$$

$$\text{steady state} \quad \longleftrightarrow \quad \partial_t \hat{\boldsymbol{\rho}} = 0$$

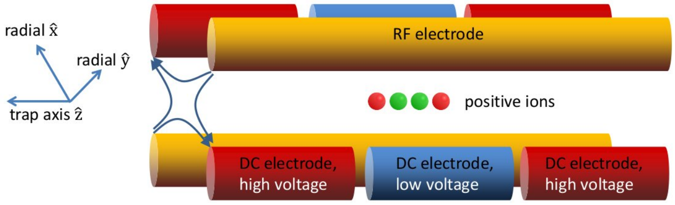




use external fields + unidirectional evolution due to $\mathcal{L}_D$
to create non-trivial steady state

an experimental implementation (almost)

Lin, Gaebler, Reiter, Tan, Bowler, Sorensen, Leibfried, Wineland, Nature 504, 415 (2013)



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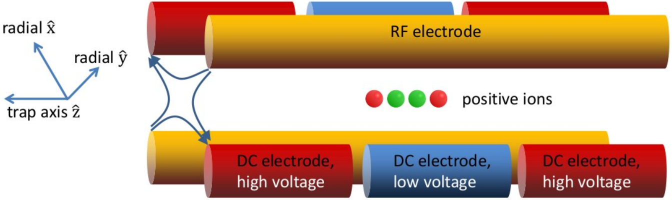
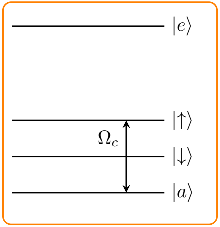
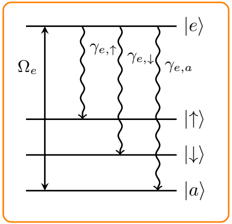


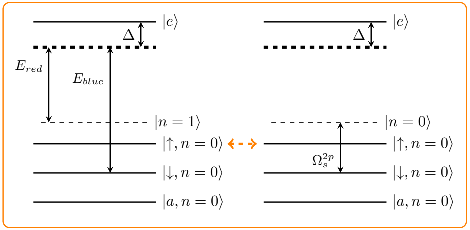
microwave



repump



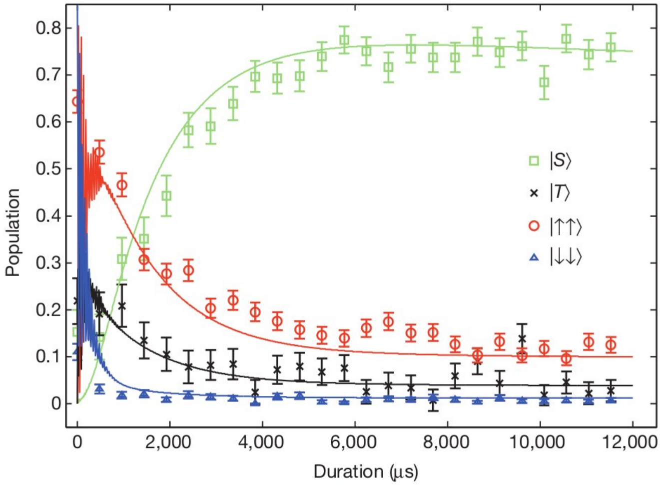
sideband



plus laser cooling on Mg^+

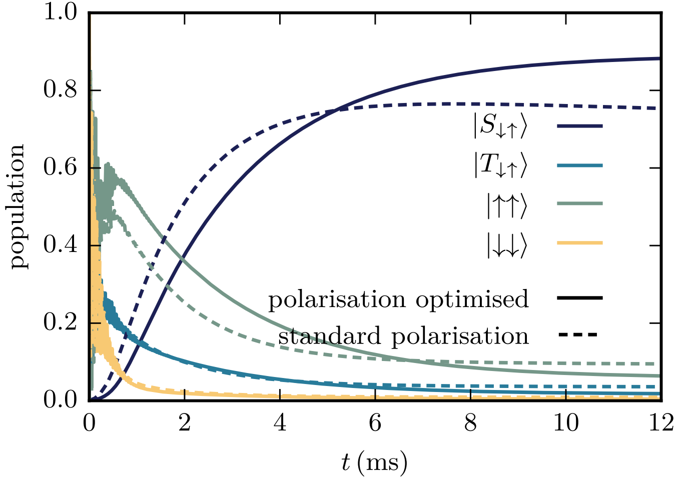
an experimental implementation (almost)

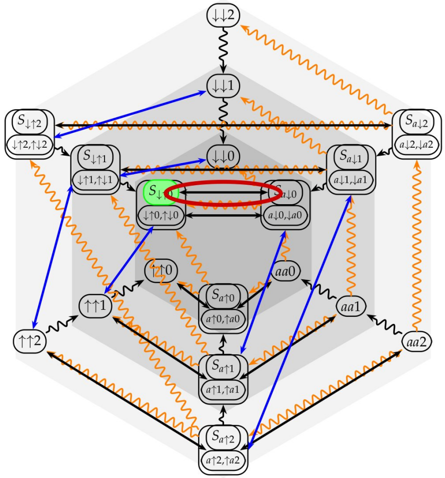
Lin, Gaebler, Reiter, Tan, Bowler, Sorensen, Leibfried, Wineland, Nature 504, 415 (2013)



sub-optimal choice of drives

leakage out of target state





optimal control of qu reservoir engineering

ensure the desired steady state

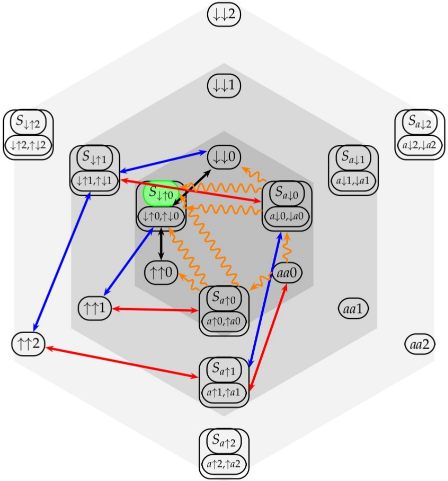
Horn, Reiter, Lin, Leibfried, Koch, New J Phys 20, 123010 (2018)

result of optimization

- keep repump & blue sideband
- different microwave transition
- replace cooling on Mg^+ by red sideband

and (of course) opt parameter values

↷ no transitions out of target state

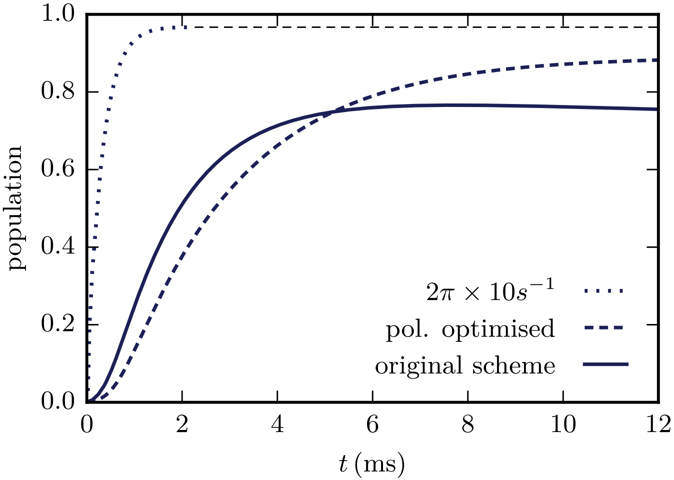


only remaining source of error: anomalous heating

optimal control of qu reservoir engineering

ensure the desired steady state

Horn, Reiter, Lin, Leibfried, Koch, New J Phys 20, 123010 (2018)



optimal control of qu reservoir engineering

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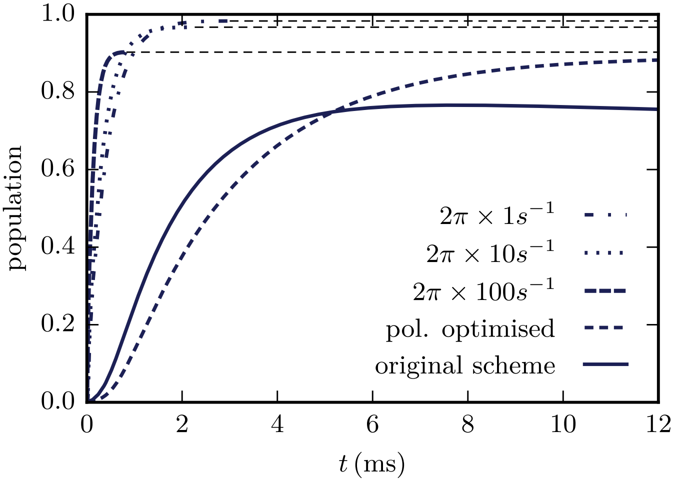
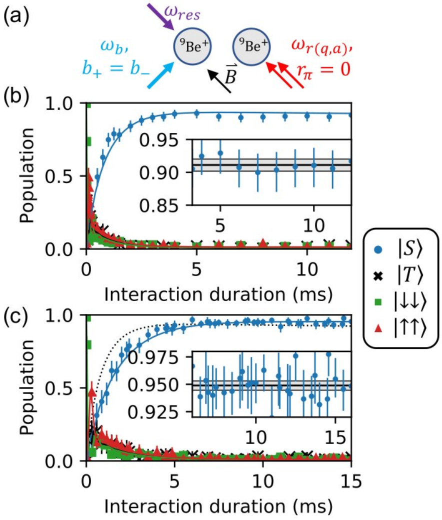
Horn, Reiter, Lin, Leibfried, Koch, New J Phys 20, 123010 (2018)



an experimental implementation ~~(almost)~~

steady state = target state



Cole, . . . Koch, Leibfried, Phys Rev Lett 128, 080502 (2022)
[experiment at NIST Boulder]

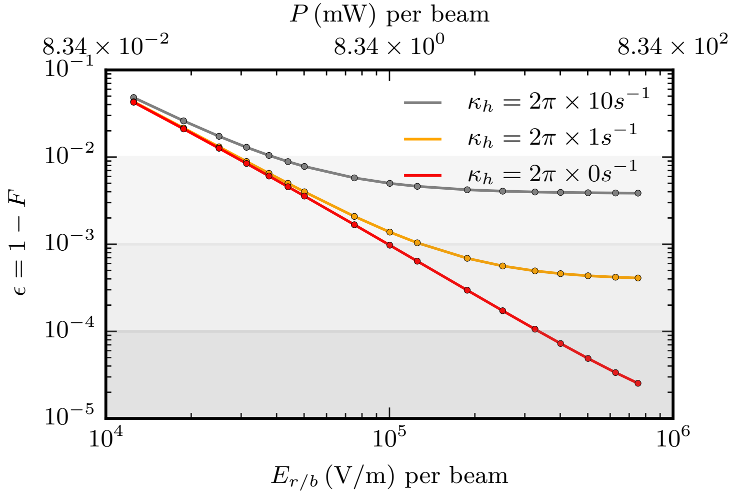
gain in exp. performance due to optimal control !

remaining error sources in experiment well understood

optimal control of qu reservoir engineering

ultimately attainable error

Horn, Reiter, Lin, Leibfried, Koch, New J Phys 20, 123010 (2018)



optimal control of qu reservoir engineering

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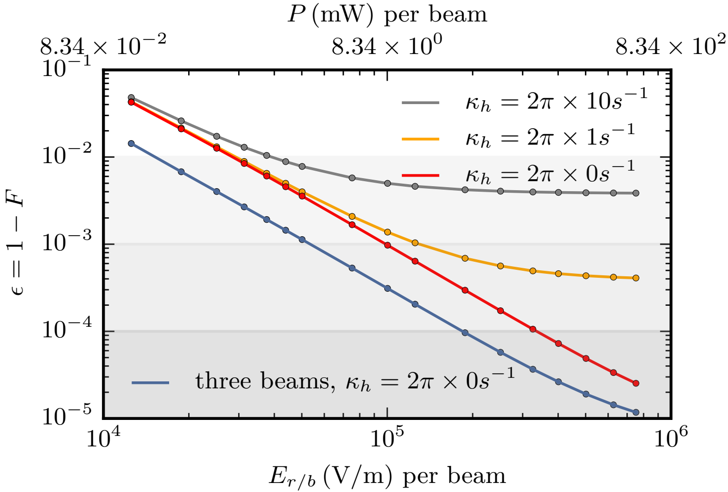
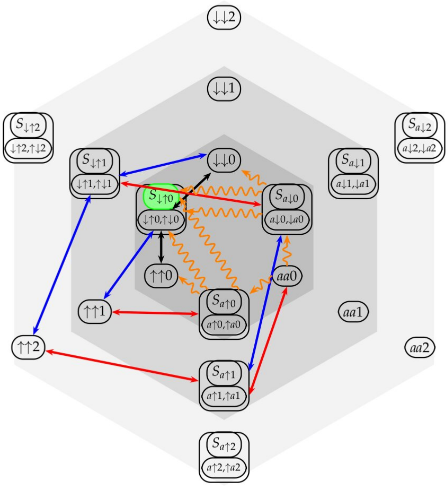


lesson No. 4 :

in desired Markovian dynamics

target state must be / can be made fixed state of $\mathcal{L}$

which fields are needed to reach a control target?



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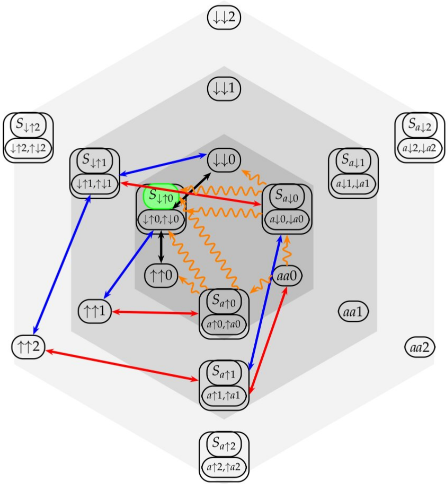


is there a better way to find out?

control theory

controllability analysis vs optimal control

Glaser et al., Eur. Phys. J. D 69, 279 (2015)
Koch et al., EPJ Quantum Technol. 9, 19 (2022)

control problem:

given eqs. of motion and "target" find the $u_j(t)$

$\hat{H} = \hat{H}[\{u_j(t)\}]$

1. is the problem solvable?

- controllability analysis

2. how to solve it?

optimal control theory

- variational calculus (PMP)
  gradient-based optimization
- parameter optimization
  gradient-free optimization

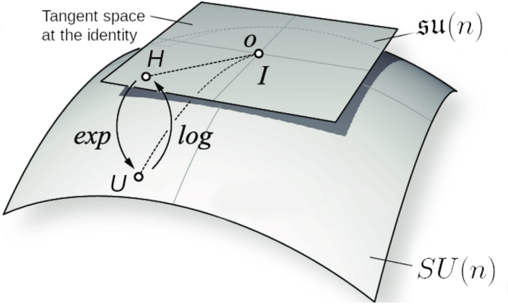
controllability analysis

rank of dynamical Lie algebra

D'Alessandro: Introduction to Qu Control & Dynamics (2007, rev. 2021), ch. 3

$$\hat{H}(t) = \hat{H}_0 + \sum_{i=1}^{k} v_i(t)\hat{H}_i$$

$$dim(\mathcal{H}) = n$$

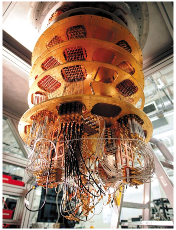


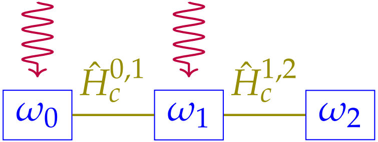
Dynamical Lie algebra $(\mathcal{L})$

$$\mathcal{L} = Lie\left(\{i\hat{H}_0, i\hat{H}_1, ..., i\hat{H}_k\}\right)$$

$$\boxed{dim(\mathcal{L}) = dim(\mathfrak{su}(n)) = n^2 - 1} \iff \boxed{\text{controllable}}$$

controllability of qubit arrays

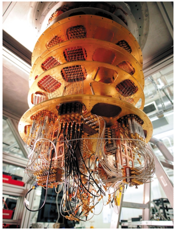




Gago-Encinas, Leibscher, Koch, Quantum Sci. Technol. 8, 045002 (2023)

how many controls necessary for universal quantum computing?

controllability of qubit arrays



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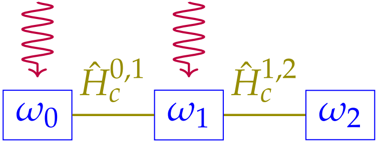

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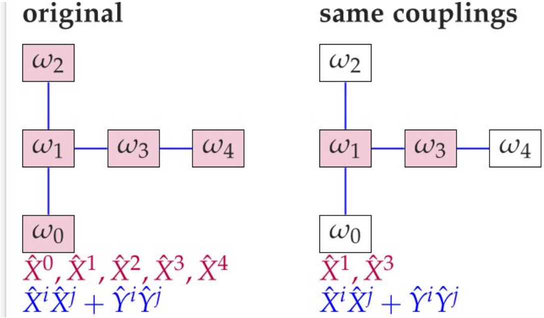

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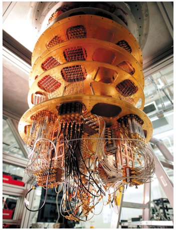

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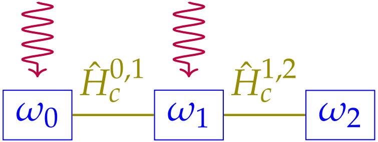
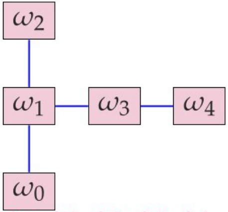


how many controls necessary for universal quantum computing?

a single one ! at the expense of gate durations

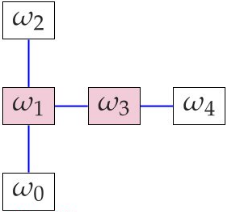
original



$\hat{X}^0, \hat{X}^1, \hat{X}^2, \hat{X}^3, \hat{X}^4$

$\hat{X}^i\hat{X}^j + \hat{Y}^i\hat{Y}^j$

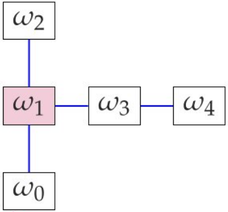
same couplings



$\hat{X}^1, \hat{X}^3$

$\hat{X}^i\hat{X}^j + \hat{Y}^i\hat{Y}^j$

minimal no. controls



$\hat{X}^1$

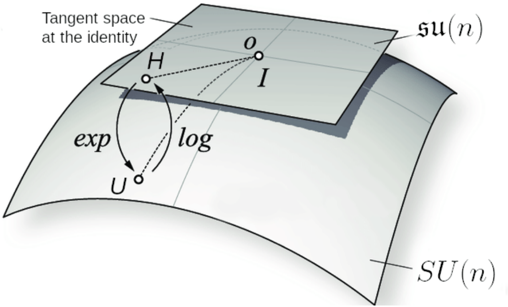
$\hat{X}^i\hat{X}^j + \hat{Y}^i\hat{Y}^j + \hat{Z}^i\hat{Z}^j$

controllability analysis

rank of dynamical Lie algebra

$$\hat{H}(t) = \hat{H}_0 + \sum_{i=1}^{k} v_i(t)\hat{H}_i$$

$$dim(\mathcal{H}) = n$$



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The curse of dimensionality prevents scalability!

controllability analysis

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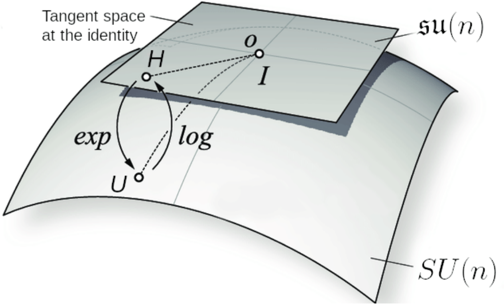

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The curse of dimensionality prevents scalability!

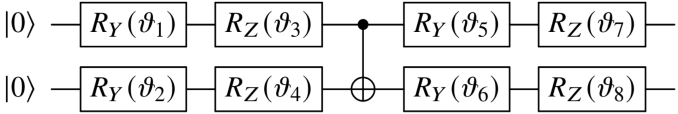
let a quantum device do the job...

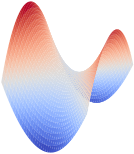
$\curvearrowright$ **parametric quantum circuits!**

controllability via dimensional expressivity

dimensional expressivity analysis — find redundant parameters

Funcke, Hartung, Jansen, Kühn, Stornati, Quantum 5, 422 (2021)





$$\vartheta_k \text{ redundant} \iff \left\{ \frac{\partial C}{\partial \vartheta_0}, \ldots, \frac{\partial C}{\partial \vartheta_k} \right\} \text{ linearly dependent}$$

dimensional expressivity: number of non-redundant parameters

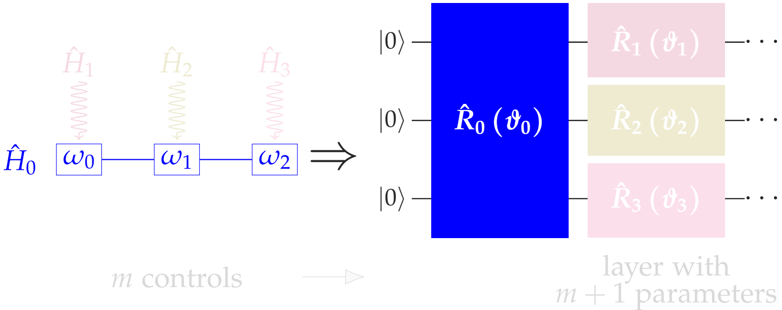
implementation via hybrid qu-class algorithm with only one auxiliary qubit

controllability via dimensional expressivity

quantum circuit for pure state controllability

Gago-Encinas, Hartung, Reich, Jansen, Koch, Quantum 7, 1214 (2023)

$$\hat{R}_j(\alpha) := \exp\left(i\,\alpha \hat{H}_j\right) \qquad (l = 0, ..., m)$$



add a total of n_l layers with different parameters

controllability via dimensional expressivity

quantum circuit for pure state controllability

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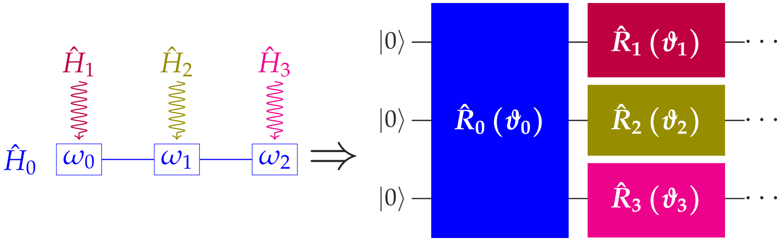

m controls $\longrightarrow$ layer with $m+1$ parameters

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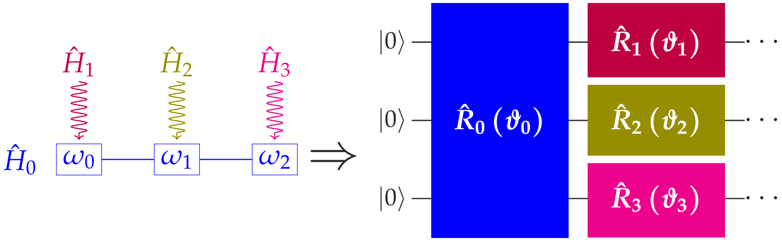

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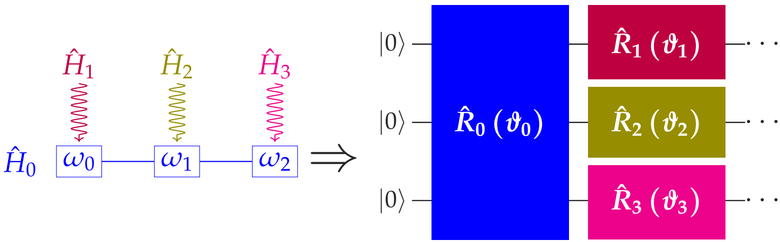


m controls $\longrightarrow$

layer with
$m+1$ parameters

add a total of n_l layers with different parameters

why does it work?

cyclic parameters

$$\hat{R}_j(\alpha) = \hat{R}(\alpha + T_j)$$

$$T_j \in \mathbb{R}$$

$\curvearrowright$ $\mathcal{P}$ compact

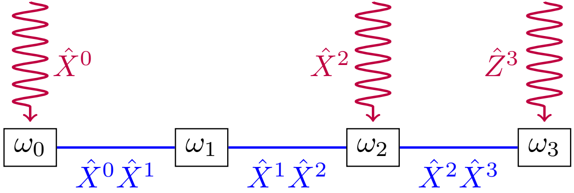
$\curvearrowright$ image = compact manifold w/ max. dimension

extension to operator controllability via Choi–Jamiołkowski isomorphism

controllability via dimensional expressivity

— examples —

Gago-Encinas, Hartung, Reich, Jansen, Koch, Quantum 7, 1214 (2023)



$$dim(\mathcal{H}) = 16$$
$$dim_E = 29 < 2 * 16 - 1$$

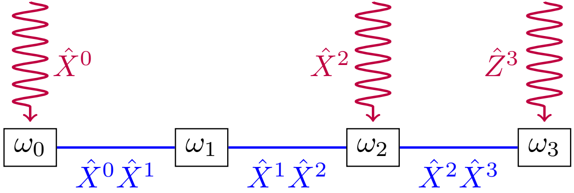
$\Longrightarrow$ **not controllable**

tested with different $|\psi_0\rangle$ and random arays $\vec{\vartheta}$

controllability via dimensional expressivity

— examples —

Gago-Encinas, Hartung, Reich, Jansen, Koch, Quantum 7, 1214 (2023)



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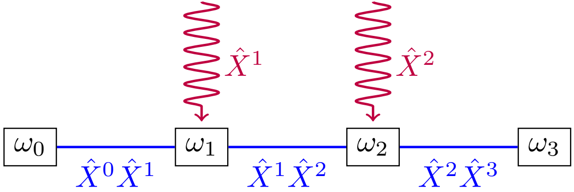


$$dim(\mathcal{H}) = 16$$
$$dim_E = 31$$

$\Longrightarrow$ **controllable**

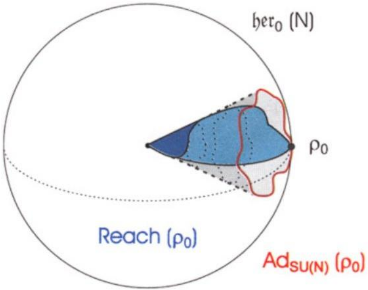
controllability of open quantum systems ?

controllability of open quantum systems

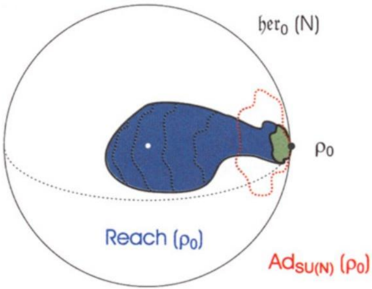
- for Markovian dynamics & assuming $\varepsilon(t)$ does not change $\mathcal{L}_D$
  - time evolution: Lie group $\Longrightarrow$ Lie semi-group

Dirr, Helmke, Kurniawan, Schulte-Herbrüggen, Rep Math Phys 64, 93 (2009)

$\mathcal{L}_D \sim \mathbb{1}$



generic



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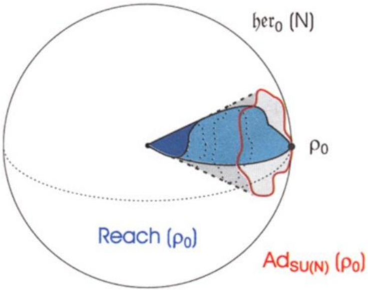

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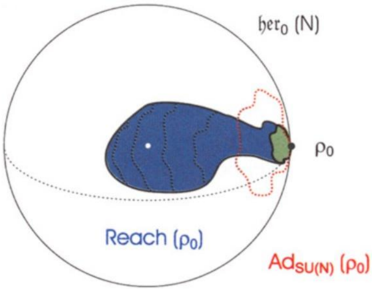


- **no rigorous results for non-Markovian dynamics**

Markovian vs non-Markovian evolution

non-Markovian evolution: loss of energy and phase not monotonic

quantifiers of non-Markovianity

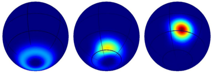
- distinguishability of two optimal states
  (recovery of previously lost information) *Breuer, Laine, Piilo, Phys Rev Lett 103, 210401 (2009)*
- divisability of the dynamical map
  (increase of correlations in a bi/multi-partite system)
  Rivas, Huelga, Plenio, Phys Rev Lett 105, 050403 (2010)
- ...
- accessible volume in Liouville (state) space
  Lorenzo, Plastina, Paternostro, Phys Rev A 88, 020102 (2013)

non-Markovian evolution should allow for more control than Markovian evolution but under which conditions is non-Markovianity a resource for quantum control?

control of open quantum systems

— lessons to date —

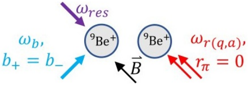
1. detrimental decoherence $\longrightarrow$ avoid decoherence



operate fast / utilize protection

qu speed limit / DFS / dyn decoupling

2. beneficial decoherence 1 $\longrightarrow$ exploit dissipation



make the steady state of $\mathcal{L}$ equal $\hat{\rho}_{\text{targ}}$

quantum reservoir engineering

control of open quantum systems

— lessons to date —

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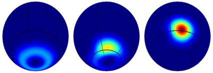

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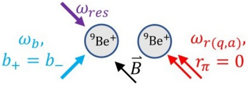


make the steady state of $\mathcal{L}$ equal $\hat{\rho}_{\mathrm{targ}}$

quantum reservoir engineering

3. beneficial decoherence 2 $\longrightarrow$ create desired dissipation

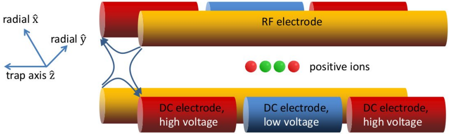
by coupling to auxiliary qu systems or measurement

quantum reservoir engineering 2.0

driven dissipative evolutions

promises & limitations

trapped ion example



’natural’ dissipation:
fast spontaneous emission

limitations
- may lead to leakage
- not always available
- local

PHYSICAL REVIEW A **78**, 042307 (2008)

Preparation of entangled states by quantum Markov processes

B. Kraus,[1] H. P. Büchler,[2] S. Diehl,[1,3] A. Kantian,[1,3] A. Micheli,[1,3] and P. Zoller[1,3]
[1]*Institute for Theoretical Physics, University of Innsbruck, Innsbruck, Austria*
[2]*Institute for Theoretical Physics III, University of Stuttgart, Pfaffenwaldring 57, 70550 Stuttgart, Germany*
[3]*Institute of Quantum Optics and Quantum Information of the Austrian Academy of Sciences, Innsbruck, Austria*
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Quantum states and phases in driven open quantum systems with cold atoms

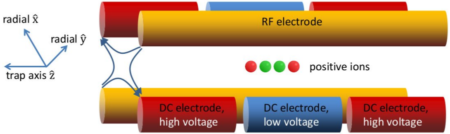
S. Diehl, A. Micheli, A. Kantian, B. Kraus, H. P. Büchler & P. Zoller

Nature Physics **4**, 878–883 (2008) | Cite this article

driven dissipative evolutions

promises & limitations

trapped ion example



'natural' dissipation:
fast spontaneous emission

limitations
- may lead to leakage
- not always available
- local

PHYSICAL REVIEW A **78**, 042307 (2008)

Preparation of entangled states by quantum Markov processes

B. Kraus,[1] H. P. Büchler,[2] S. Diehl,[1,3] A. Kantian,[1,3] A. Micheli,[1,3] and P. Zoller[1,3]
[1]*Institute for Theoretical Physics, University of Innsbruck, Innsbruck, Austria*
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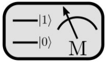
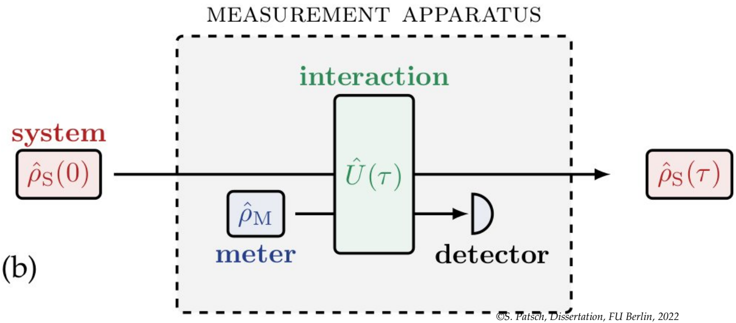
↷ engineer dissipation via measurements



theory of quantum measurements



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state of system after measurement $$\frac{\hat{\mathbf{M}}_a|\psi_S\rangle}{\sqrt{\langle\psi_S|\hat{\mathbf{M}}_a^+\hat{\mathbf{M}}_a|\psi_S\rangle}}$$

measurement operator $\hat{\mathbf{M}}_a = \langle a_M|\hat{\mathbf{U}}(\tau)|\psi_M\rangle$

interaction with meter defines effective dissipators

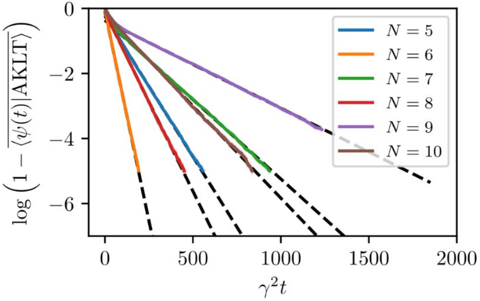
Haroche & Raimond: Exploring the quantum; Wiseman, Milburn: Quantum measurement & control

quantum control as quantum engineering

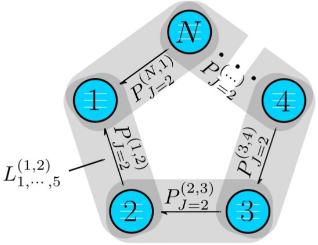
— creating dissipation via measurements in many-body systems —

Langbehn, Snizhko, Morigi, Gefen, Koch, PRX Quantum 5, 030301 (2024)

dilute "blind" cooling into ground state of AKLT model



$$
\begin{aligned}
L_1 &= |1,1\rangle\langle 2,2| \\
L_2 &= |1,0\rangle\langle 2,1| \\
L_3 &= |0,0\rangle\langle 2,0| \\
L_4 &= |1,-1\rangle\langle 2,-1| \\
L_5 &= |1,-1\rangle\langle 2,-2|
\end{aligned}
$$



attaching meter qubits to single link is sufficient !

quantum control as quantum engineering

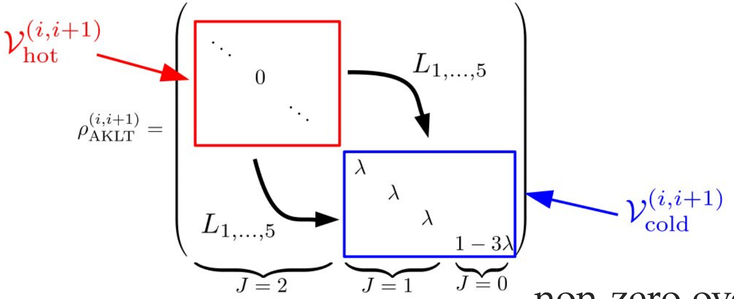
— creating dissipation via measurements —

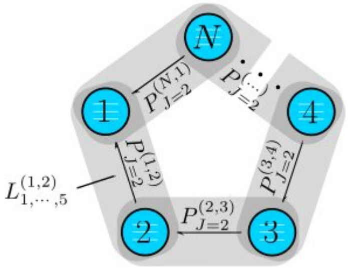
Langbehn, Snizhko, Morigi, Gefen, Koch, PRX Quantum 5, 030301 (2024)

dilute "blind" cooling into ground state of AKLT model

why does it work?

target state projected onto cooled link





all excited states have
non-zero overlap with hot subspace

quantum control as quantum engineering

— creating dissipation via measurements —

Langbehn, Snizhko, Morigi, Gefen, Koch, PRX Quantum 5, 030301 (2024)

dilute "blind" cooling into many-body states: general recipe from control theory

Kernelizer

$$K_{|\psi_\oplus\rangle} \equiv \{A \in \text{span}\,\mathcal{O} : A\,|\psi_\oplus\rangle = 0\}$$

$$\mathcal{O} = \text{span} \cup_{i=1}^{N} \mathcal{O}^{(i,i+1)}$$

$$\mathcal{O}^{(i,i+1)} = \text{span}\left\{\sigma_j^{(i)} \otimes \sigma_k^{(i+1)} : j,k \leq d^2\right\}$$

quasi-local op.s respecting $\rho_\oplus$

Sufficient condition for cooling:
Unitary controllability on the complement space of $|\psi_\oplus\rangle$

$$\dim \mathfrak{L}\left(K_{|\psi_\oplus\rangle}\right) = \left(d^N - 1\right)^2 - 1$$

Lie-Algebra generated by $K_{|\psi_\oplus\rangle}$

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quasi-local op.s respecting $\rho_\oplus$

Target state $\rho_\oplus = |\psi_\oplus\rangle\langle\psi_\oplus|$

1. Check necessary condition $\mathcal{V}_{\text{hot}}^{(i,i+1)} \neq \emptyset$
2. Check sufficient condition $\dim \mathfrak{L}\left(K_{|\psi_\oplus\rangle}\right) = \left(d^N - 1\right)^2 - 1$
3. Choose $L_j = |\phi_{\text{c},j}\rangle\langle\phi_{\text{h},j}|$
4. Choose some $H \in K_{|\psi_\oplus\rangle}$

works for frustration-free quasi-local H

Sufficient condition for cooling:
Unitary controllability on the complement space of $|\psi_\oplus\rangle$

$$\dim \mathfrak{L}\left(K_{|\psi_\oplus\rangle}\right) = \left(d^N - 1\right)^2 - 1$$

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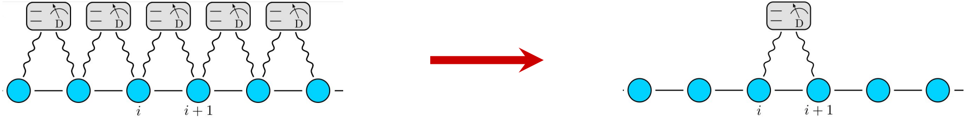
quantum control as quantum engineering

— creating dissipation via measurements —

dilute “blind” cooling into many-body states



Langbehn, Snizhko, Morigi, Gefen, Koch, PRX Quantum 5, 030301 (2024)

- finite cooling time for finite systems
- works for systems with ground state s.t. $\mathcal{V}_{hot}^{(i,i+1)} \neq \emptyset$
  e.g. frustration-free quasi-local H but also PEPS
- requires engineering of $L_{1,\dots,5}$ (three-body interactions)

can we simplify this & make it more generic ?

quantum control as quantum engineering

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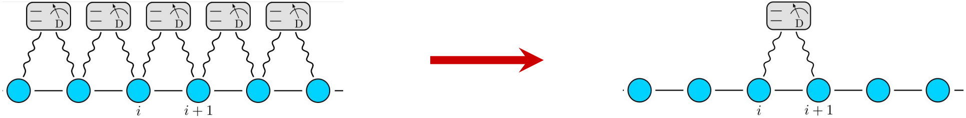
dilute "blind" cooling into many-body states



Langbehn, Snizhko, Morigi, Gefen, Koch, PRX Quantum 5, 030301 (2024)

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$\curvearrowright$ **"universal cooling" via stochastic interactions**

Langbehn, Mouloudakis, King, Menu, Gornyi, Morigi, Gefen, Koch, arXiv:2506.11964

control of open quantum systems

so far: 3 general rules

1. detrimental case: unwanted Markovian dynamics
   $\Longrightarrow$ do things as fast as you can
   or exploit symmetry protection
2. beneficial case: desired Markovian dynamics
   target must be fixed state of Liouvillian
   $\Longrightarrow$ quantum reservoir engineering
   $\Longrightarrow$ strong drives $\rightarrow$ faster decay
3. non-Markovian dynamics: beneficial & detrimental effects
   beneficial $\Longleftrightarrow$ few strongly coupled,
   sufficiently isolated modes

control of open quantum systems

open questions

Koch, arXiv:2512.04990 (lecture notes)

1. controllability of open quantum systems
   reachable states / operations
2. best strategies for dumping entropy
   strong drives vs strong coupling
   classical vs quantum controllers
3. conditions for non-Markovianity to be a resource for control