# PT Symmetry Breaking in an LMG Dimer

Peter Kirton

University of Strathclyde Glasgow

Lisbon, April 2026

cost EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

Funded by the European Union

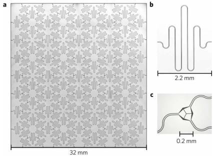
QOpen Many-body Open Quantum Systems

COST Action CA24109 – QOpen https://cost.eu https://qopen.eu/

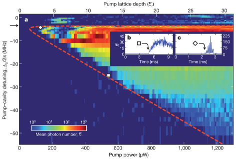
COST (European Cooperation in Science and Technology) is a funding agency for research and innovation networks. Our Actions help connect research initiatives across Europe and enable scientists to grow their ideas by sharing them with their peers. This boosts their research, career and innovation.

# Open Quantum Systems

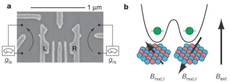
- Quantum systems are always coupled to the outside world



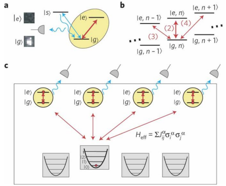

Circuit QED lattices [Houck et al. Nat. Phys. '12]



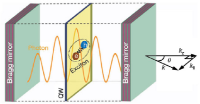

Atoms in cavities [Baumann et al. Nat. '10]



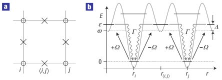

Quantum dots [Johnson et al. Nat. '05]




Trapped ions [Blatt et al. Nat. Phys. '12]




Polariton and photon condensates




Cold atoms [Diehl et al. Nat. Phys. '08]

- Focus on toy models to understand how this changes behavior

# Open Quantum Systems

- Quantum systems are always coupled to the outside world

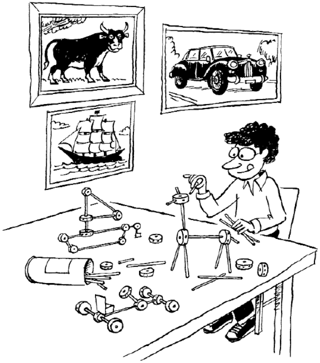


Illustration by Dick Codor.
[Auerbach, Interacting Electrons (Springer, 1998)]

- Focus on toy models to understand how this changes behavior

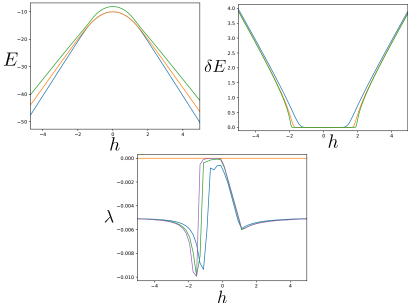
# Overview

# Dissipative Phase Transitions

- Markovian open quantum system with master equation:

$$\dot{\rho} = \mathcal{L}\rho = -i[H, \rho] + \mathcal{D}[\rho]$$

| Quantum PT | Dissipative PT |
|---|---|
| $H\left\|\psi_i\right\rangle = E_i\left\|\psi_i\right\rangle$ | $\mathcal{L}\rho_i = \lambda_i\rho_i$ |
| $H = H^\dagger$ | $\mathcal{L} \neq \mathcal{L}^\dagger$ |
| $\left\|\psi_0\right\rangle = \min H\left\|\psi\right\rangle$ | $\mathcal{L}\rho_0 = 0$ |
| $E_0 - E_1 = 0$ | $\lambda_1 = 0$ |



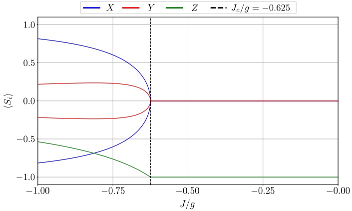

[Kessler et al. PRA '12, Minganti et al. PRA '18]

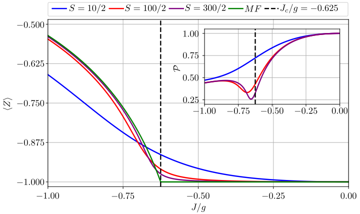
# Example: LMG model

- As an example of this consider the LMG model with dissipation

$$H_{LMG} = -\frac{J}{S}S_x^2 - gS_z$$

$$\dot{\rho} = -i[H_{LMG}, \rho] + \Gamma\mathcal{D}[S^-]$$



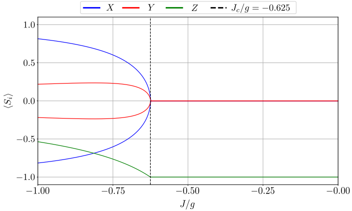
# Example: LMG model

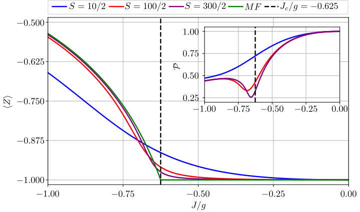
- As an example of this consider the LMG model with dissipation

$$H_{LMG} = -\frac{J}{S} S_x^2 - g S_z$$

$$\dot{\rho} = -i[H_{LMG}, \rho] + \Gamma \mathcal{D}[S^-]$$

- Phase transition between different magnetic orders
- Small $|J|$ normal dissipation driven paramagnet
- Large $|J|$ symmetry broken ferromagnet
- Same transition as in ground state



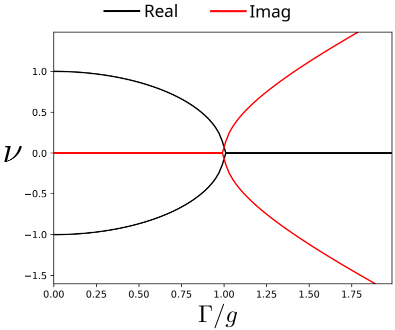



# Classical PT symmetry breaking

- Systems are Parity-time symmetric when they obey both
  - An inversion symmetry
  - Time reversal symmetry
- Consider a non-Hermitian Hamiltonian eg for a 2 mode system

$$H = \begin{pmatrix} i\Gamma_g & g \\ g & -i\Gamma_l \end{pmatrix}$$

- Balanced gain and loss requires $\Gamma_l = \Gamma_g$
- Eigenvalues change from real to imaginary at the exceptional point $\Gamma = g$



# PT symmetry in open quantum systems

- Consider master equation for pair of coupled oscillators

$$\dot{\rho} = \mathcal{L}[H, c_1, c_2]\rho = -i[H, \rho] + \mathcal{D}[c_1] + \mathcal{D}[c_2]$$

[Huber, PK, Rotter and Rabl, SciPost Phys (2020)]

# PT symmetry in open quantum systems

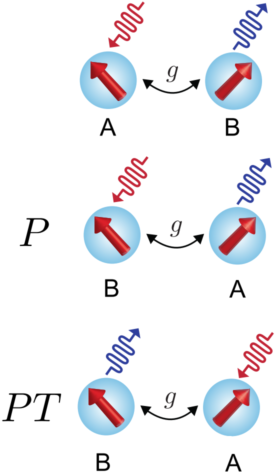
- Consider master equation for pair of coupled oscillators

$$\dot{\rho} = \mathcal{L}[H, c_1, c_2]\rho = -i[H, \rho] + \mathcal{D}[c_1] + \mathcal{D}[c_2]$$

- We propose this is PT symmetric if

$$\mathcal{L}[H, c_1, c_2] = \mathcal{L}[\mathbb{PT}(H), \mathbb{PT}(c_1), \mathbb{PT}(c_2)]$$

- $\mathbb{P}$ exchanges sites $a \leftrightarrow b$
- $\mathbb{T}$ takes the Hermitian conjugate $c \leftrightarrow c^\dagger$




[Huber, PK, Rotter and Rabl, SciPost Phys (2020)]

# PT symmetry in open quantum systems

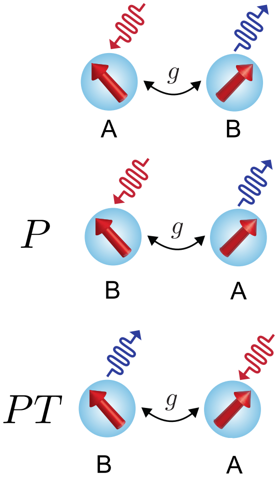
- Consider master equation for pair of coupled oscillators

$$\dot{\rho} = \mathcal{L}[H, c_1, c_2]\rho = -i[H,\rho] + \mathcal{D}[c_1] + \mathcal{D}[c_2]$$

- We propose this is PT symmetric if

$$\mathcal{L}[H, c_1, c_2] = \mathcal{L}[\mathbb{PT}(H), \mathbb{PT}(c_1), \mathbb{PT}(c_2)]$$

- $\mathbb{P}$ exchanges sites $a \leftrightarrow b$
- $\mathbb{T}$ takes the Hermitian conjugate $c \leftrightarrow c^\dagger$
- Can think of this as swapping over a pair of sites and exchanging gain for loss




[Huber, PK, Rotter and Rabl, SciPost Phys (2020)]

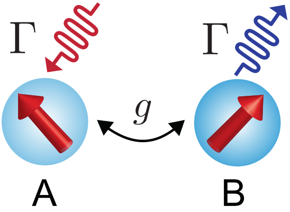
# PT Symmetric Dimer

- Consider a model with two spins

$$H = g(S_a^+ S_b^- + S_a^- S_b^+)$$

- Spin $a$ is driven. Spin $b$ is dissipative

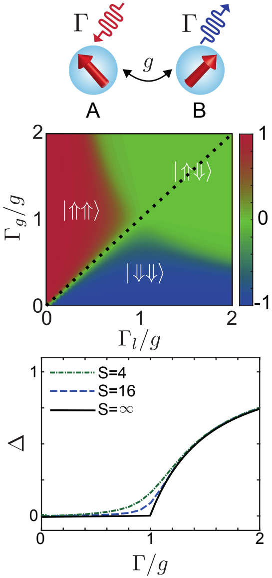
$$\dot{\rho} = -i[H, \rho] + \Gamma_g \mathcal{D}[S_a^+] + \Gamma_l \mathcal{D}[S_b^-]$$




- Ground state is trivial
- Physics of phase transition is dissipation driven

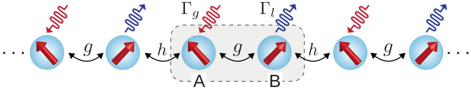
# PT Symmetric Dimer

- Phase diagram has different magnetic configurations
- PT symmetric when $\Gamma_g = \Gamma_l = \Gamma$
- If $g \ll \Gamma$ dominated by dissipation and fully polarised
- If $g \gg \Gamma$ eigenstates are mixed and driven to infinite temperate mixed state
- This is a generic feature of models with PT symmetry



# Generalisation: PT Chain

- Chain of spins with alternating pumping and decay



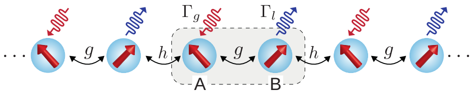

$$H = \sum_{n=1}^{N} g S^+_{a,n} S^-_{b,n} + h S^+_{b,n} S^-_{a,n+1} + \mathrm{H.c.}$$

$$\mathcal{D}[\rho] = \sum_{n=1}^{N} \Gamma_g \mathcal{L}[S^+_{a,n}] + \Gamma_l \mathcal{L}[S^-_{b,n}],$$

[Huber, PK and Rabl, PRA (2020)]

# Generalisation: PT Chain

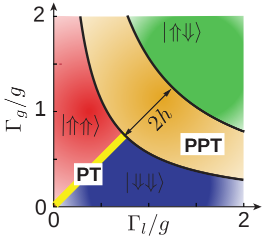
- Chain of spins with alternating pumping and decay




$$H = \sum_{n=1}^{N} g S^{+}_{a,n} S^{-}_{b,n} + h S^{+}_{b,n} S^{-}_{a,n+1} + \mathrm{H.c.}$$

$$\mathcal{D}[\rho] = \sum_{n=1}^{N} \Gamma_g \mathcal{L}[S^{+}_{a,n}] + \Gamma_l \mathcal{L}[S^{-}_{b,n}],$$

- Phase diagram - disconnected from equilibrium



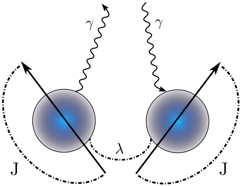

- First order transition without coexistance
- Second order transition without symmetry breaking

[Huber, PK and Rabl, PRA (2020)]

# Generalisation: LMG Dimer

- Pair of coupled LMG models

$$H = H_{LMG} \otimes I + I \otimes H_{LMG} + \lambda(S_a^+ S_b^- + S_a^- S_b^+)$$




[Kothe, PK arXiv:2504.18426]

# Generalisation: LMG Dimer

- Pair of coupled LMG models

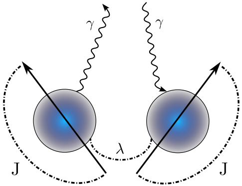
$$H = H_{LMG} \otimes I + I \otimes H_{LMG} + \lambda(S_a^+ S_b^- + S_a^- S_b^+)$$

- Nonlinear spin model

$$H_{LMG} = -\frac{J}{S} S_x^2 - g S_z$$

- One pumped one dissipative

$$\dot{\rho} = -i[H, \rho] + \Gamma \mathcal{L}[S_a^-] + \Gamma \mathcal{L}[S_b^+]$$




[Kothe, PK arXiv:2504.18426]

# Generalisation: LMG Dimer

- Pair of coupled LMG models

$$H = H_{LMG} \otimes I + I \otimes H_{LMG} + \lambda(S_a^+ S_b^- + S_a^- S_b^+)$$

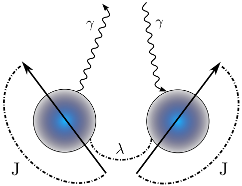
- Nonlinear spin model

$$H_{LMG} = -\frac{J}{S} S_x^2 - g S_z$$

- One pumped one dissipative

$$\dot{\rho} = -i[H, \rho] + \Gamma \mathcal{L}[S_a^-] + \Gamma \mathcal{L}[S_b^+]$$

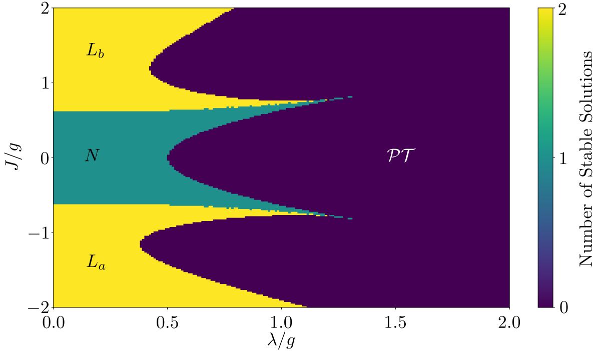
- Phase diagram is surprisingly complex



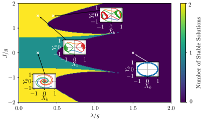

[Kothe, PK arXiv:2504.18426]

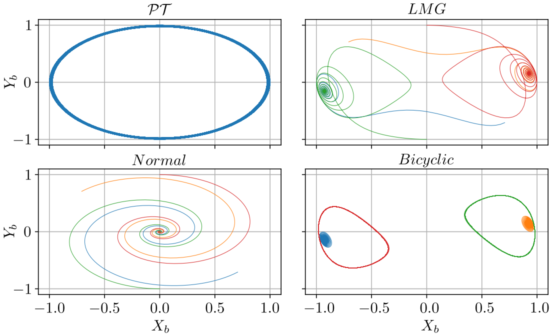
# Mean-Field Phase Diagram

- Find solutions of mean-field equations
- Check how many of them are stable



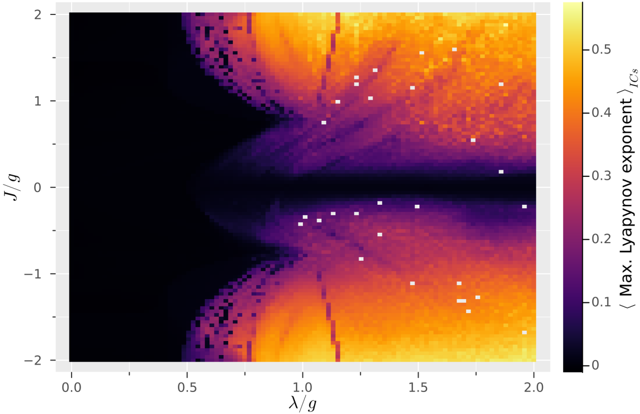
# Mean-Field Dynamics



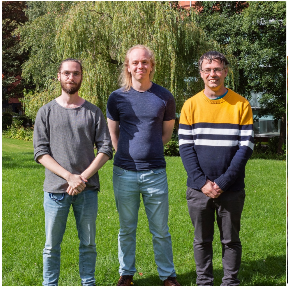
# Lyapunov Exponent

- Can calculate largest Lyapunov exponent to diagnose chaos

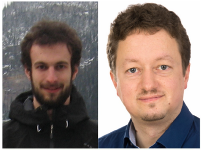


# Acknowledgements

SOQS Group



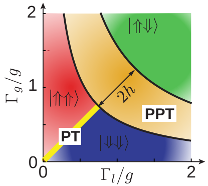
Simon Kothe Chris Oliver

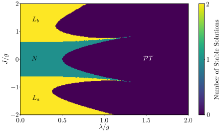


Julian Huber Peter Rabl

# Conclusions

- Dissipative phase transitions are more complex than in equilibrium
- PT symmetry can be defined at the level of microscopic quantum models
- Lots of interesting physics in these models!




[Huber, PK, Rotter, Rabl SciPost Phys (2020)
Kothe, PK arXiv:2504.18426 (2025)]