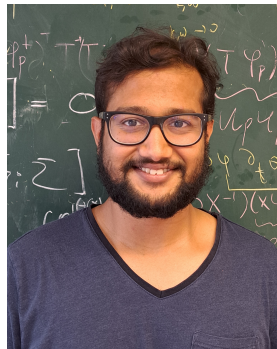
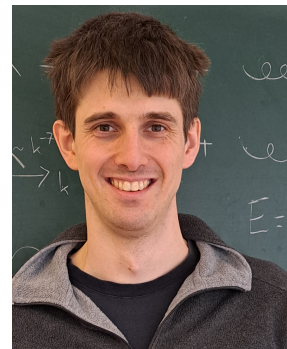


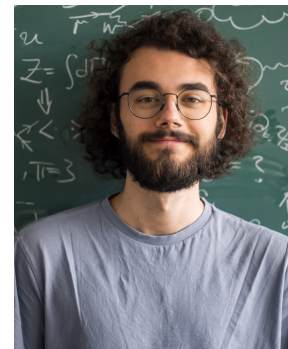
# Fermion quantum criticality far from equilibrium



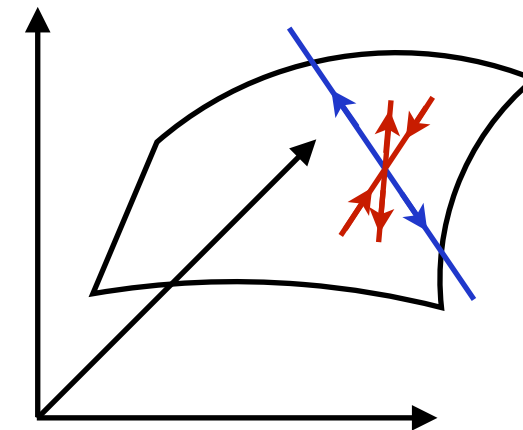
R. Mittal



J. Lang



T. Zander



Mittal, Zander, Lang, SD, PRX (2026) — Viewpoint in Physics by P. Zhang

Sebastian Diehl

Institute for Theoretical Physics,  
University of Cologne



Funded by  
the European Union



**QOpen**  
Many-body Open Quantum Systems

COST Action CA24109 – QOpen

<https://cost.eu>

<https://qopen.eu/>

# Motivation

## active matter

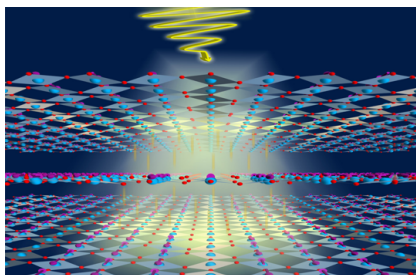
- many mobile agents:  
flocking birds, colonies of bacteria, traffic,...
- loose definition:
  - (1) emergent collective behavior
  - (2) no detailed balance on a microscopic level



## active condensed / synthetic matter?

- systems with broken detailed balance on microscopic level: **driven & open quantum systems**

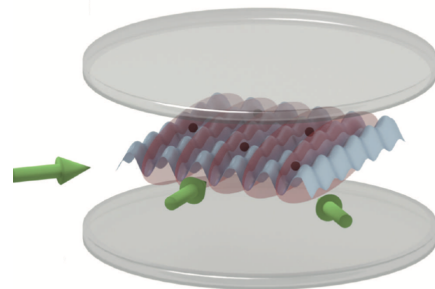
solid state



light induced superconductors/  
ferromagnets

Mitrano et al. Nature (2016)  
Bloch et al. Nature (2022)

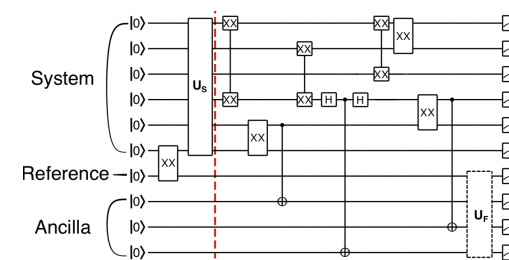
cold atoms



quantum gases and  
materials in optical cavities

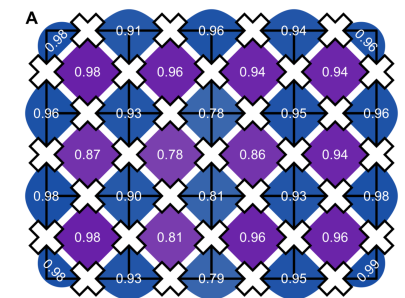
Baumann et al., Nature (2010)  
Hübener et al. Nat, Mat. (2020)

NISQ platforms



trapped ion quantum  
simulators

Noel et al. Nat. Phys. (2022)  
Monroe et al. RMP (2021)  
Blatt, Roos, Nature (2012)



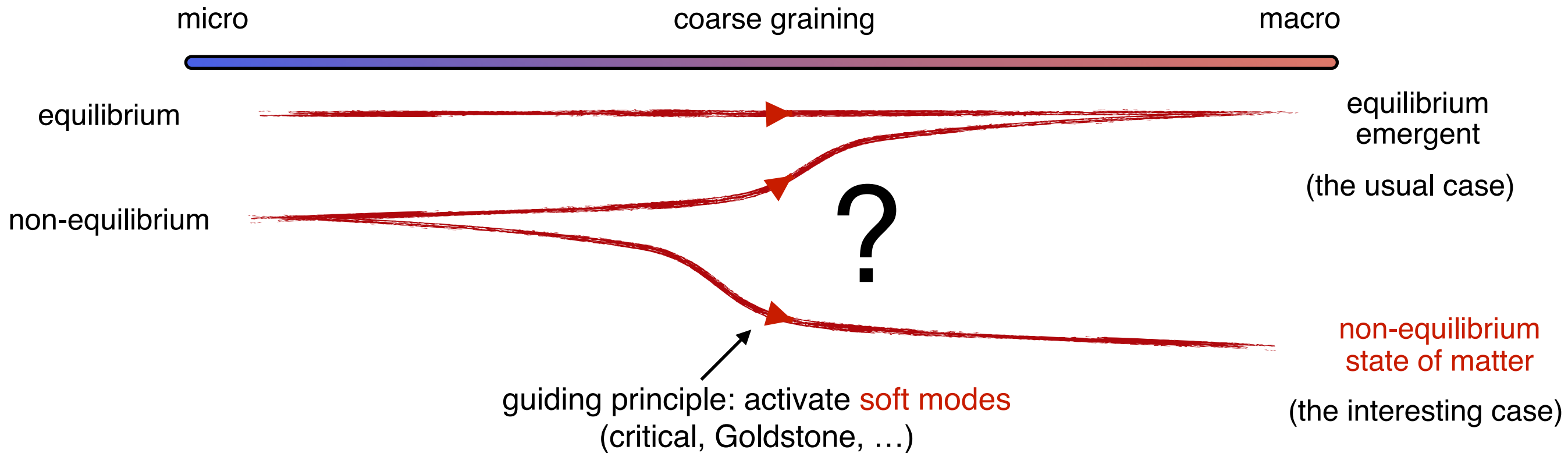
superconducting circuits /  
Rydberg arrays

Satzinger et al. Science (2021)  
Semeghini et al. Science (2021)

**What are the novel states of matter & phenomena?  
Are there universal quantum effects?**

# The question, refined: equilibrium vs. non-equilibrium

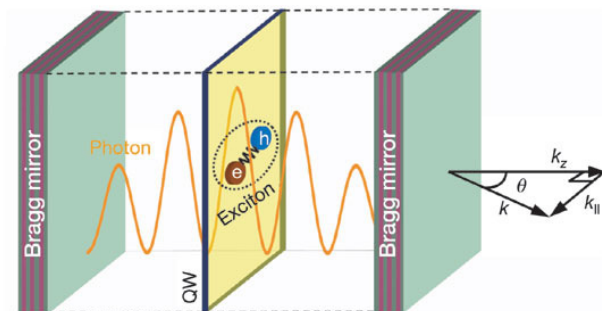
microscopic breaking of detailed balance  $\rightarrow$  universal macroscopic non-equilibrium phenomena



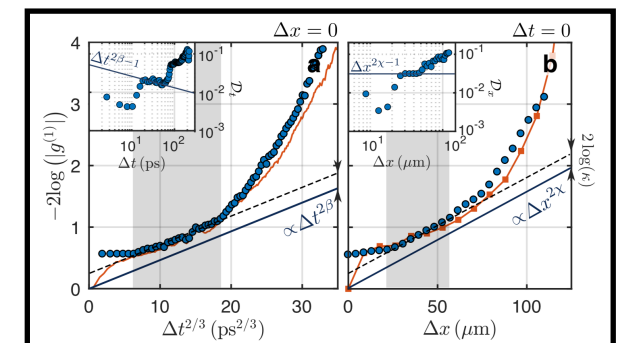
*Universality in driven open quantum matter,*  
Sieberer, Buchhold, Marino, SD, Rev. Mod. Phys. (2025)

working example

exciton-polariton driven condensates  $\rightarrow$  Kardar-Parisi-Zhang (KPZ) universality class



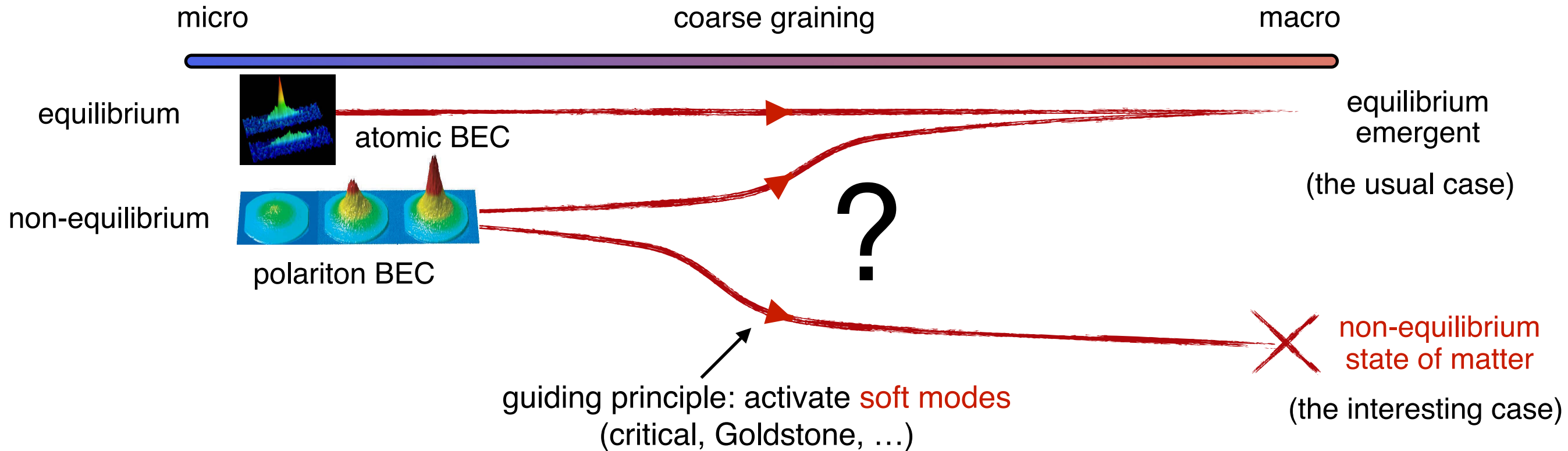
Kasprzak et al., Nature (2006)



Fontaine et al., Nature (2022)  
Th: Altman, Chen, Sieberer, SD, Toner, PRX (2015); He, Sieberer, SD, PRL (2017)

# The question, refined: equilibrium vs. non-equilibrium

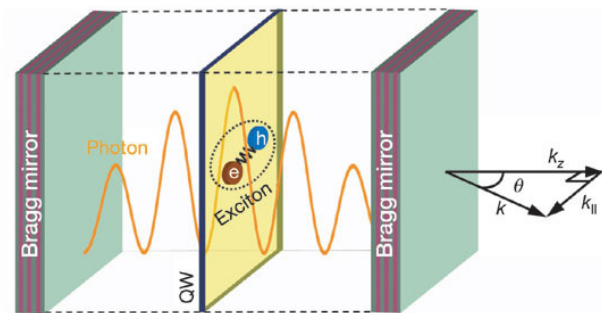
microscopic breaking of detailed balance  $\rightarrow$  universal macroscopic non-equilibrium phenomena



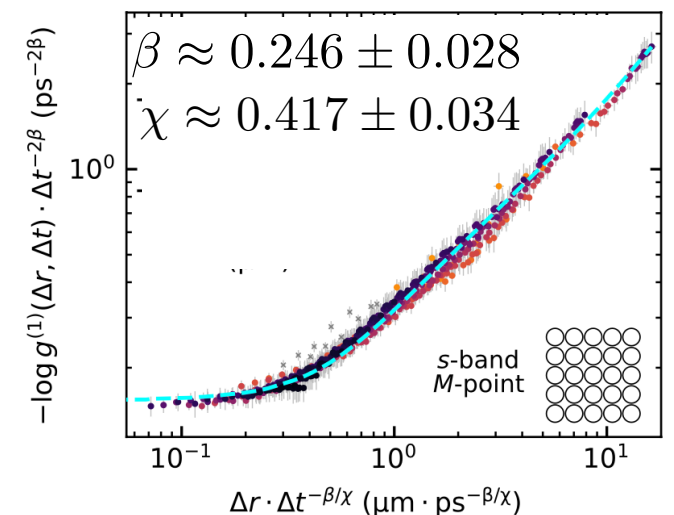
*Universality in driven open quantum matter,*  
Sieberer, Buchhold, Marino, SD, Rev. Mod. Phys. (2025)

working example

exciton-polariton driven condensates  $\rightarrow$  Kardar-Parisi-Zhang (KPZ) universality class



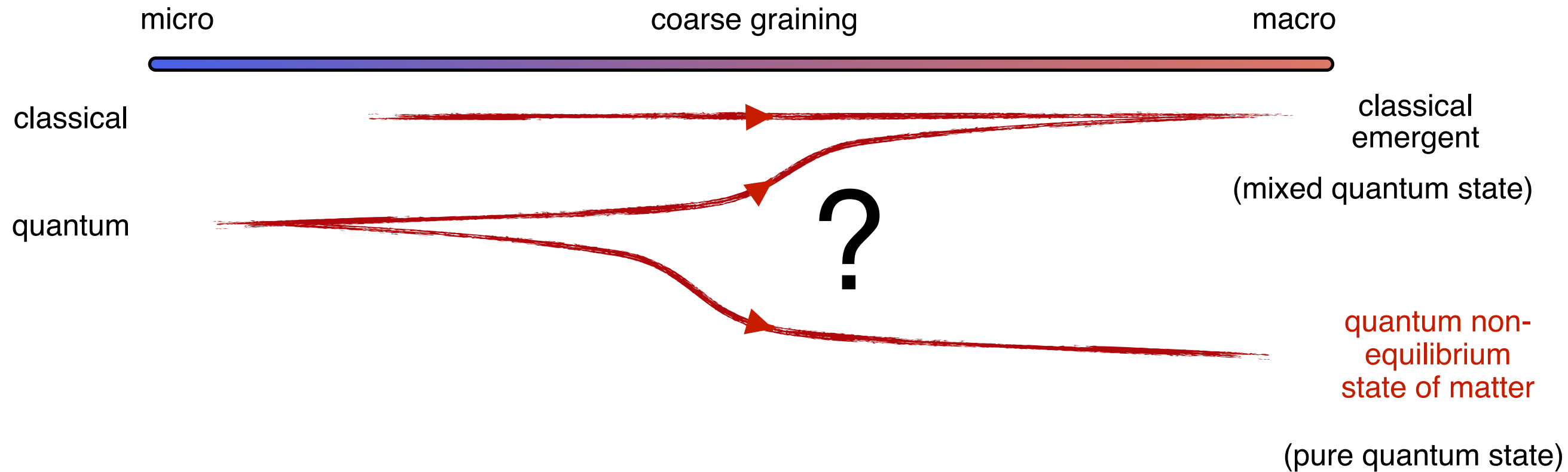
Kasprzak et al., Nature (2006)



2D KPZ quantitatively seen in polaritons!  
Dam et al. Science (2026)

# The question, refined: classical vs. quantum

microscopic breaking of detailed balance  $\rightarrow$  universal macroscopic non-equilibrium phenomena

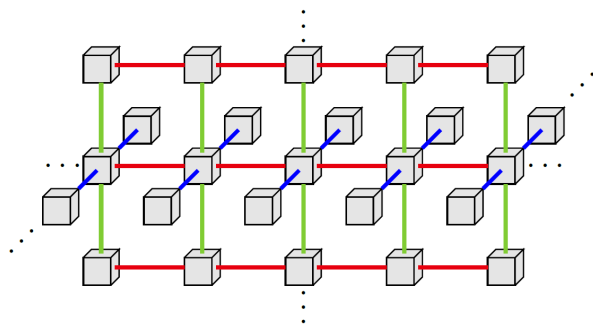


this talk:

Far from equilibrium & quantum

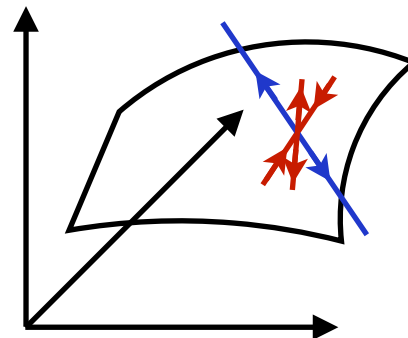
microscopic model

- fermion topological dark states
- competition and dark-to-dark transition



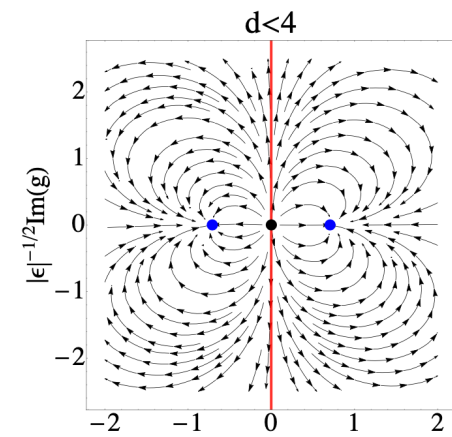
mesoscopic field theory

- fermion-boson action
- emergent symmetry protecting state's purity



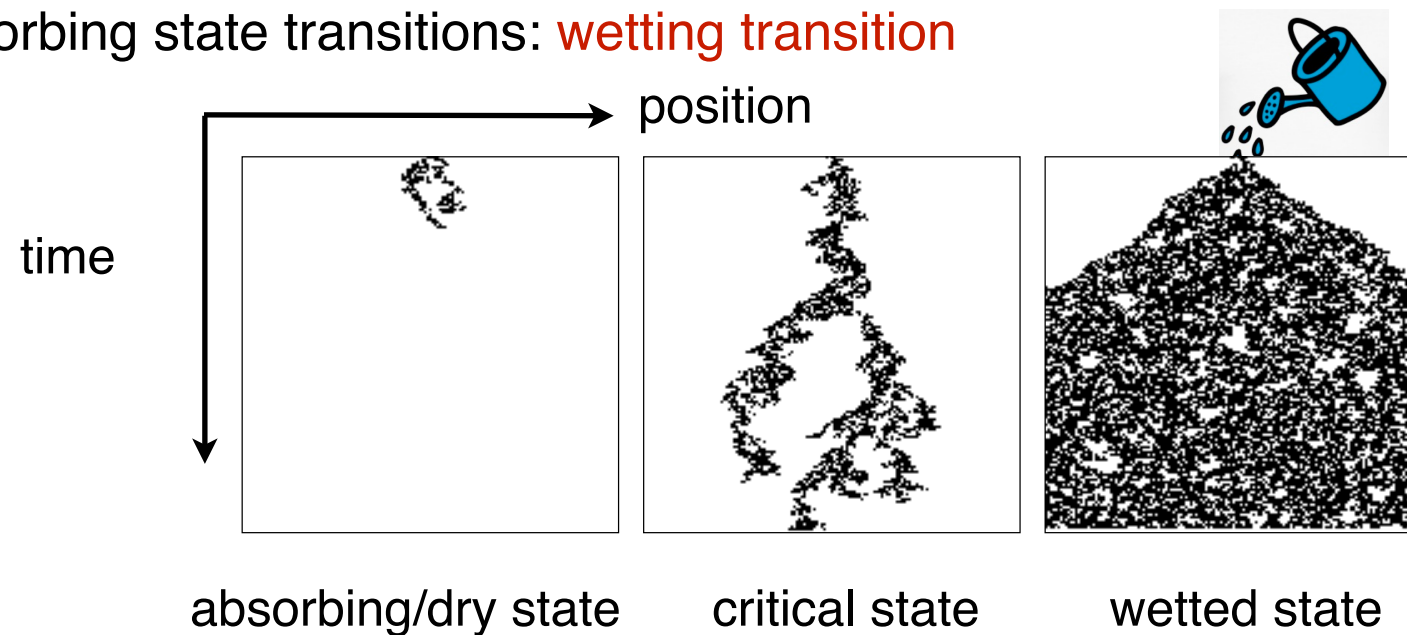
macroscopic phenomenology

- quantum critical fermions far from equilibrium
- new universality class



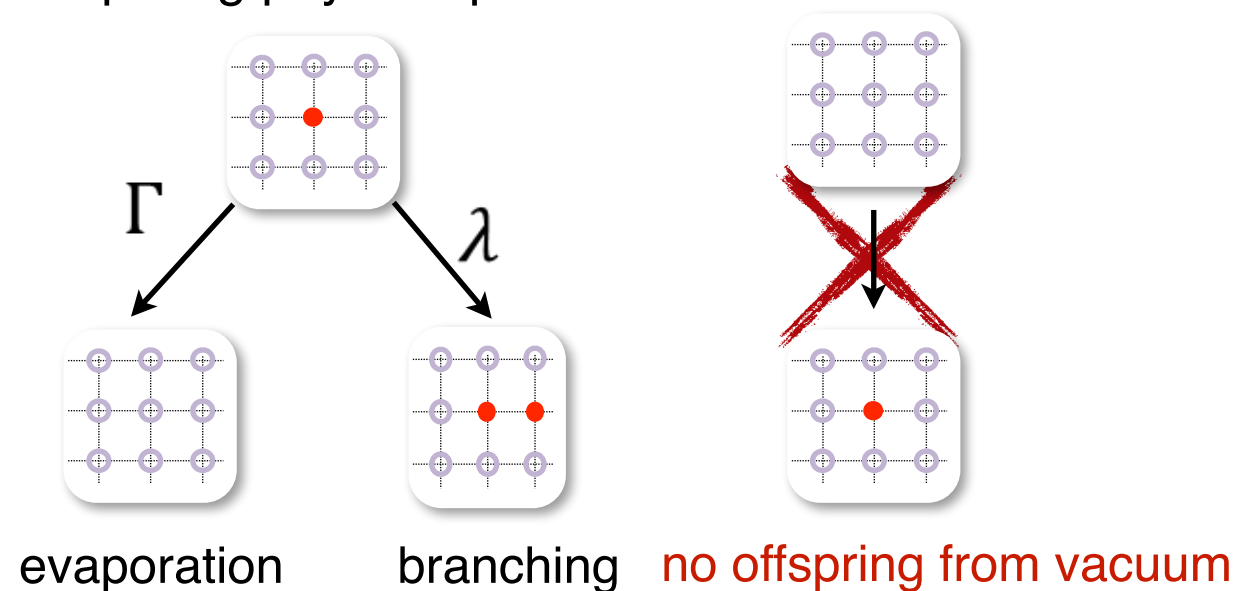
# Classical active matter scenario: Absorbing state transitions

- classical absorbing state transitions: **wetting transition**

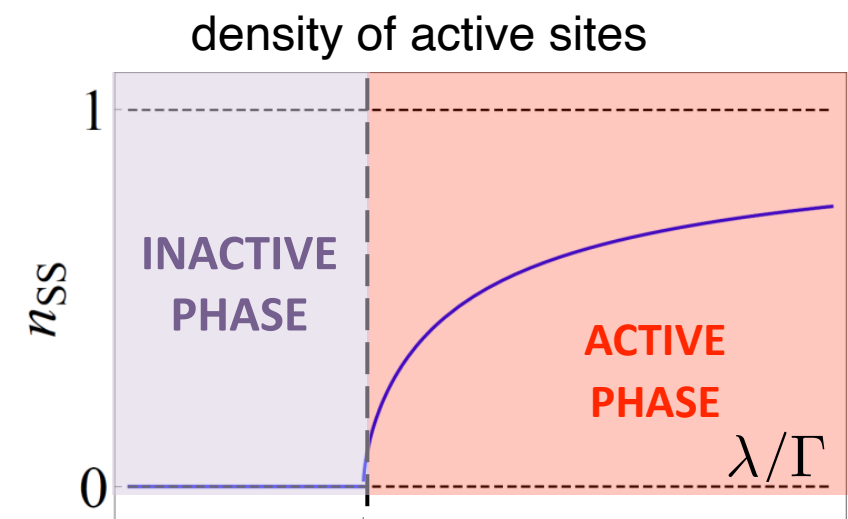


H. Hinrichsen, Adv. Phys. (2000);  
G. Odor, RMP (2004)

- competing physical processes



- phase transition:



- ➔ absorbing state = uncorrelated empty 'trivial vacuum'
- ➔ non-equilibrium dynamics: not every process comes with its reverse (no detailed balance)

# Classical active matter scenario: Non-equilibrium universality

- wetting transition in **directed percolation (DP) universality class**
- Grassberger-Janssen conjecture: all absorbing state transitions in DP class

Grassberger, Z. Phys. B (1982); Janssen, Z. Phys. B (1985)

## microscopic

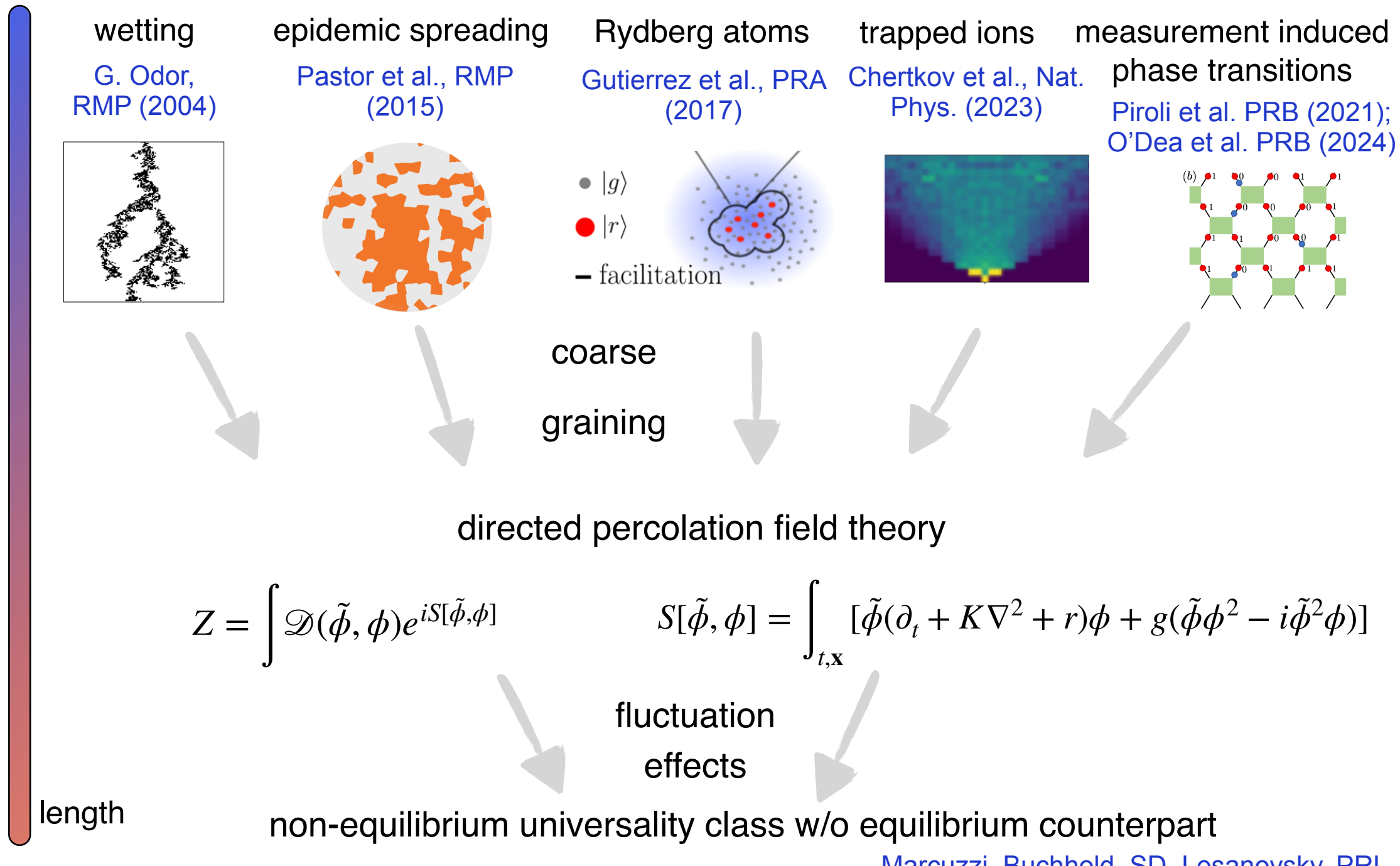
—> water, bacteria,  
driven atoms/ions

## mesoscopic

—> (bosonic) activity  
order parameter

## macroscopic

—> critical degrees  
of freedom



- ➔ emergent classical: quantum problems can map to classical DP
- ➔ clean & quantitative realizations possible ('non-equilibrium simulators')
- ➔ can we beat Grassberger-Janssen by quantum mechanics?

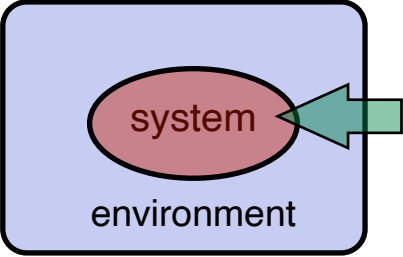
Marcuzzi, Buchhold, SD, Lesanovsky, PRL (2016); Carollo, Gillman, Weimer, Lesanovsky, PRL (2019)

# Quantum analog: Dark states in Lindblad equations

- Lindblad quantum master equation for driven open quantum system

$$\partial_t \hat{\rho} = \underbrace{-i[\hat{H}, \hat{\rho}]}_{\text{coherent evolution}} + \sum_i \underbrace{\gamma_i [2\hat{L}_i \hat{\rho} \hat{L}_i^\dagger - \hat{L}_i^\dagger \hat{L}_i \hat{\rho} - \hat{\rho} \hat{L}_i^\dagger \hat{L}_i]}_{\text{driven-dissipative evolution}}$$

spin, lattice site, ...



$$\equiv \hat{\mathcal{L}}[\hat{\rho}]$$

- interpretation:

$$\partial_t \hat{\rho} = \underbrace{-i(\hat{H})}_{\text{energy}} - \underbrace{i \sum_i \gamma_i \hat{L}_i^\dagger \hat{L}_i}_{\text{decay (dissipation)}} \hat{\rho} + \text{h.c.} + \underbrace{2 \sum_i \gamma_i \hat{L}_i \hat{\rho} \hat{L}_i^\dagger}_{\text{ensures probability conservation (fluctuation)}}$$

$$“E - i\Gamma”$$

$$\partial_t \text{tr} \hat{\rho}(t) = 0$$

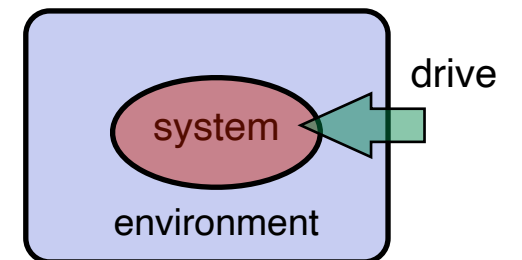
# Quantum analog: Dark states in Lindblad equations

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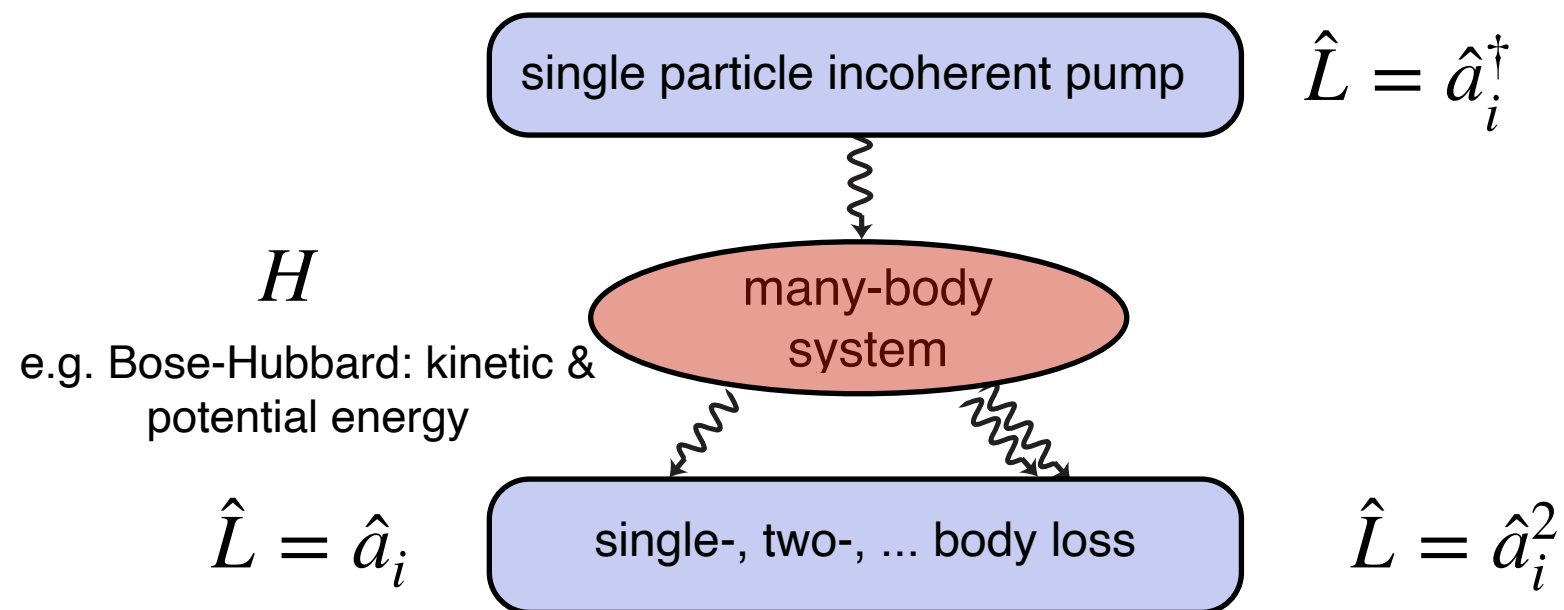
$$\partial_t \hat{\rho} = \underbrace{-i[\hat{H}, \hat{\rho}]}_{\text{coherent evolution}} + \sum_i \underbrace{\gamma_i [2\hat{L}_i \hat{\rho} \hat{L}_i^\dagger - \hat{L}_i^\dagger \hat{L}_i \hat{\rho} - \hat{\rho} \hat{L}_i^\dagger \hat{L}_i]}_{\text{driven-dissipative evolution}}$$

spin, lattice site, ...

$$\equiv \hat{\mathcal{L}}[\hat{\rho}]$$



- Lindbladians break equilibrium conditions



- driven** open system: dissipation channels from independent microscopic processes
- system is 'confused' which bath to thermalise to
- sharply defined by **violation of thermal / KMS symmetry** of Lindblad-Keldysh path integral

Sieberer, ..., SD, PRB (2015); Haehl et al. JHEP (2016);

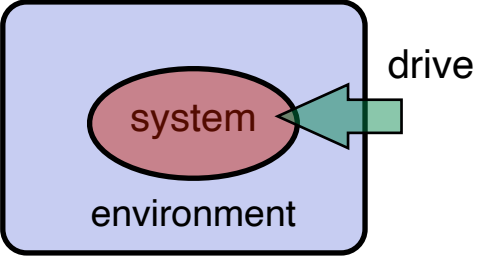
Crossley et al., JHEP (2016); Aron et al. SciPost (2018)

# Quantum analog: Dark states in Lindblad equations

- Lindblad quantum master equation for driven open quantum system

$$\partial_t \hat{\rho} = \underbrace{-i[\hat{H}, \hat{\rho}]}_{\text{coherent evolution}} + \underbrace{\kappa \sum_{\alpha} (\hat{L}_{\alpha} \hat{\rho} \hat{L}_{\alpha}^{\dagger} - \frac{1}{2} \{ \hat{L}_{\alpha}^{\dagger} \hat{L}_{\alpha}, \hat{\rho} \})}_{\text{driven-dissipative evolution}}$$

spin, lattice site, ...



$$\equiv \hat{\mathcal{L}}[\hat{\rho}]$$

- Dark states

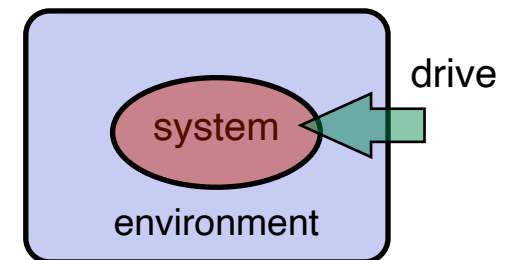
$$\hat{L}_{\alpha} |D\rangle = 0 \quad \forall \alpha \qquad H |D\rangle = E |D\rangle$$

→ time evolution stops when  $\rho = |D\rangle\langle D|$

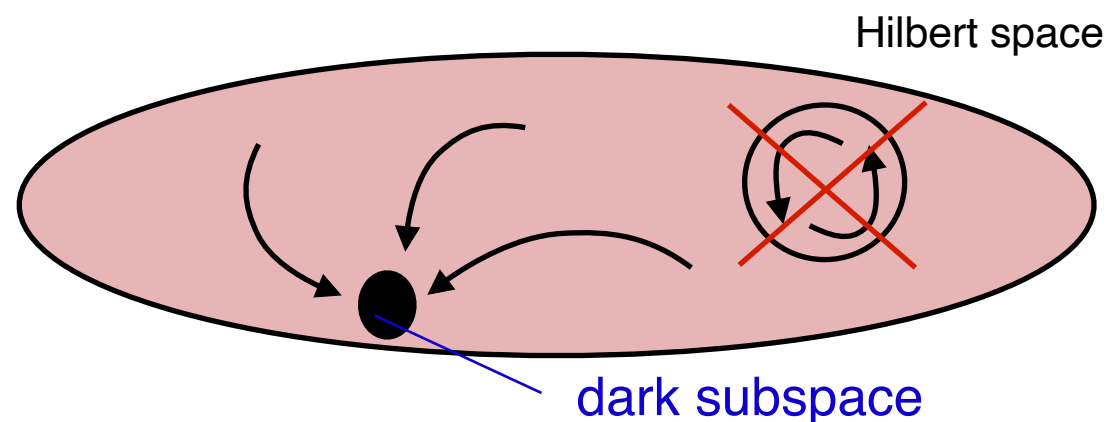
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$$\partial_t \hat{\rho} = \underbrace{-i[\hat{H}, \hat{\rho}]}_{\text{coherent evolution}} + \underbrace{\kappa \sum_{\alpha} (\hat{L}_{\alpha} \hat{\rho} \hat{L}_{\alpha}^{\dagger} - \frac{1}{2} \{ \hat{L}_{\alpha}^{\dagger} \hat{L}_{\alpha}, \hat{\rho} \})}_{\text{driven-dissipative evolution}}$$
$$\equiv \hat{\mathcal{L}}[\hat{\rho}]$$



- Interesting situation: **unique** dark state solution



➔ you can enter, but never leave (cf. absorbing state)

➔ directed motion in Hilbert space  $\rho \xrightarrow{t \rightarrow \infty} |D\rangle\langle D|$

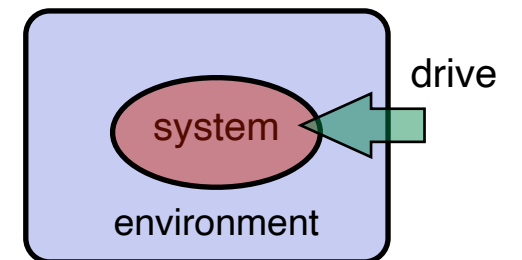
➔ **dissipation removes entropy, increases purity**

# Quantum analog: Dark states in *many-body* Lindblad equations

- Lindblad quantum master equation for driven open quantum system

$$\partial_t \hat{\rho} = \underbrace{-i[\hat{H}, \hat{\rho}]}_{\text{coherent evolution}} + \underbrace{\kappa \sum_{\alpha} (\hat{L}_{\alpha} \hat{\rho} \hat{L}_{\alpha}^{\dagger} - \frac{1}{2} \{ \hat{L}_{\alpha}^{\dagger} \hat{L}_{\alpha}, \hat{\rho} \})}_{\text{driven-dissipative evolution}}$$

$$\equiv \hat{\mathcal{L}}[\hat{\rho}]$$



- simple example: lossy cavity

$$\hat{H} = \omega_0 a^{\dagger} a, \quad \hat{L} = a \quad \Longrightarrow \quad |D\rangle = |0\rangle$$

→ ‘trivial vacuum’ empty state (like in classical absorbing states)

- many-body example (‘dissipation engineering’)

$$\hat{L}_i = (a_i^{\dagger} + a_{i+1}^{\dagger})(a_i - a_{i+1})$$

$\forall i$ , lattice sites

bosons

$$\Longrightarrow |D\rangle = |BEC, N\rangle$$

fixed number BEC

SD et al., Nat. Phys. (2008);  
Verstraete et al., Nat. Phys. (2009)

fermions

$$\Longrightarrow |D\rangle = |BCS, N\rangle$$

fixed number Kitaev chain

SD et al., Nat. Phys. (2011)

→ ‘non-trivial vacuum’ (particle non-empty, quantum correlated)

# Non-trivial vacuum: topological insulator dark state

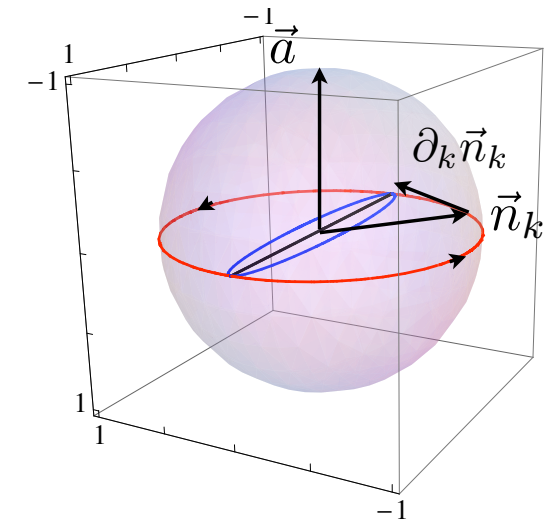
Goldstein, SciPost (2019); Shavit,  
Goldstein, PRB (2020)  
Tonielli, Budich, Altland, SD, PRL (2020)

- nontrivial vacuum state, example: dissipative chiral topological insulator 1D

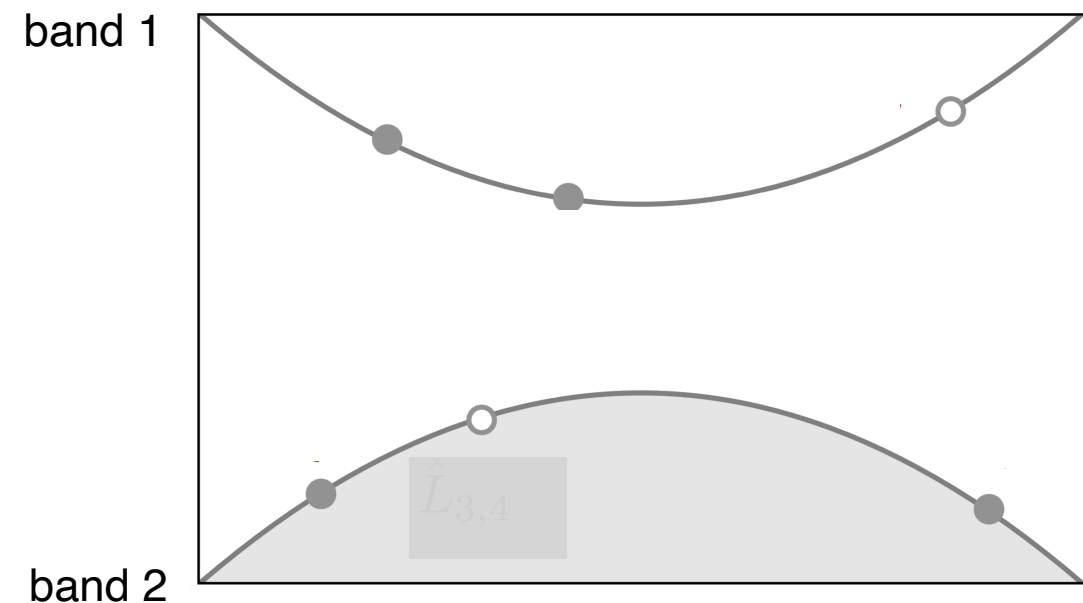
$$|D\rangle = \prod_{\mathbf{x}} l_{2,\mathbf{x}}^\dagger |0\rangle \quad d = 1: 2 \text{ bands, e.g. SSH state}$$

- quantum correlations, characterized by topological chiral winding number  $W \in \mathbb{Z}$

- e.g. SSH state: winding on grand circle of Bloch sphere



winding number in 1d



# Non-trivial vacuum: topological insulator dark state

- **nontrivial vacuum state, example:** dissipative chiral topological insulator 1D

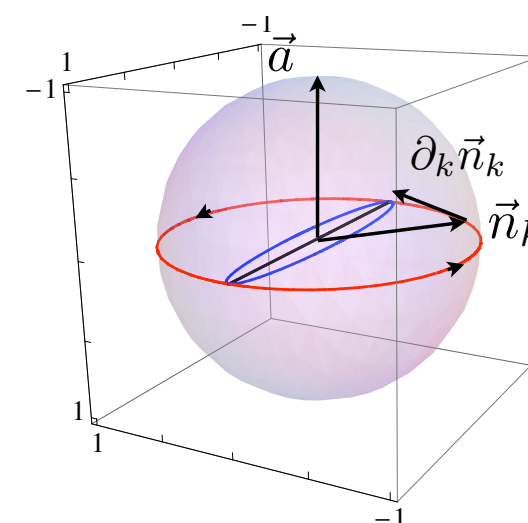
$$|D\rangle = \prod_{\mathbf{x}} l_{2,\mathbf{x}}^\dagger |0\rangle \quad d = 1:2 \text{ bands, e.g. SSH state}$$

- quantum correlations, characterized by topological chiral winding number  $W \in \mathbb{Z}$

- e.g. SSH state: winding on grand circle of Bloch sphere

- **parent Lindblad operators** cooling into this vacuum:

$$\hat{L}_{a,1} = \hat{\psi}_a^\dagger \hat{l}_1 \quad \hat{L}_{a,2} = \hat{\psi}_a \hat{l}_2^\dagger \quad a = 1,2 \text{ band}$$



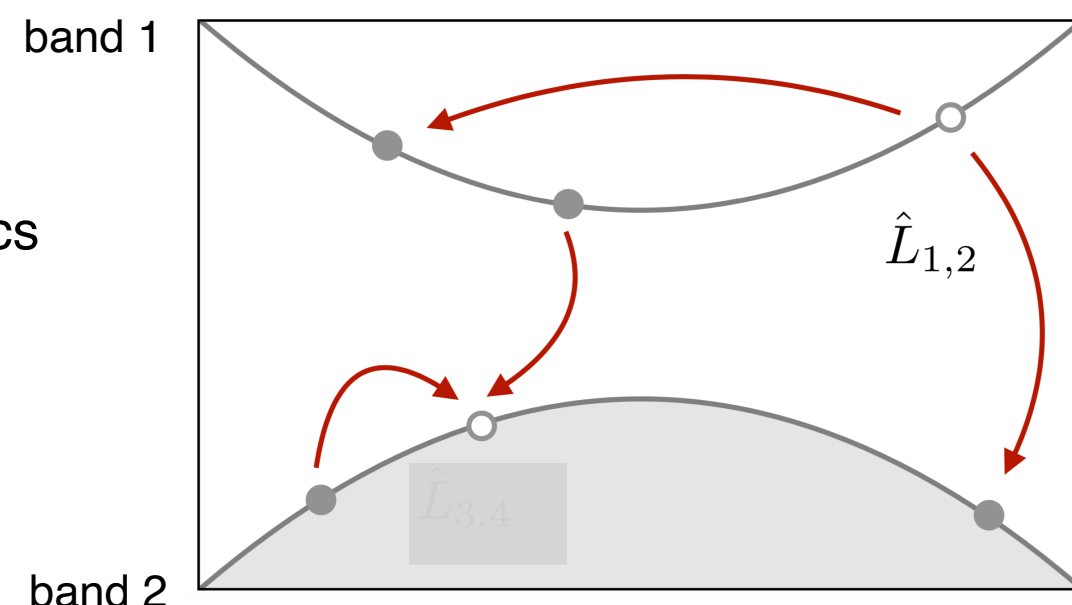
winding number in 1d

- interpretation: IF-THEN condition

IF band 1 empty  $\implies \hat{l}_1 |\psi\rangle = 0$   
 THEN stop dynamics  
 IF band 2 occupied  $\implies \hat{l}_2^\dagger |\psi\rangle = 0$

otherwise recycle into evolution

- ➔ dynamics stops once above dark state is reached



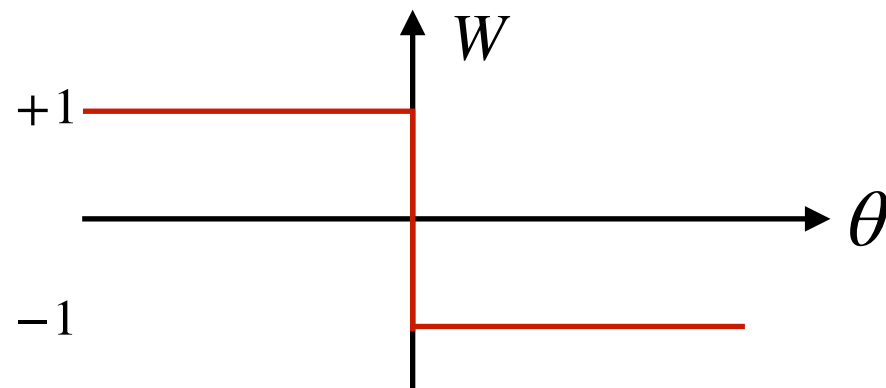
# Topological quantum phase transition far from equilibrium

- Competition by **superposition** of left and right winding Lindblad operators

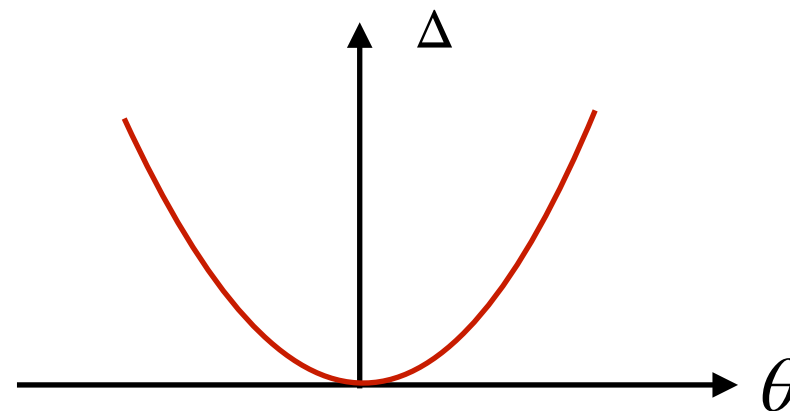
$$\hat{L}_i = \cos \theta \hat{L}_i^+ + \sin \theta \hat{L}_i^-$$

stabilises  $W = +1$ 
stabilises  $W = -1$

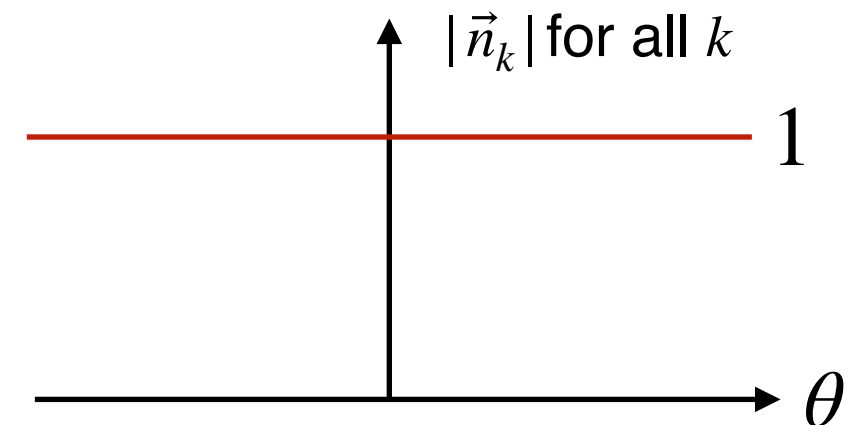
winding number



dissipative gap



purity



→ topological phase transition at  $\theta = \pi/4$  via gap closing

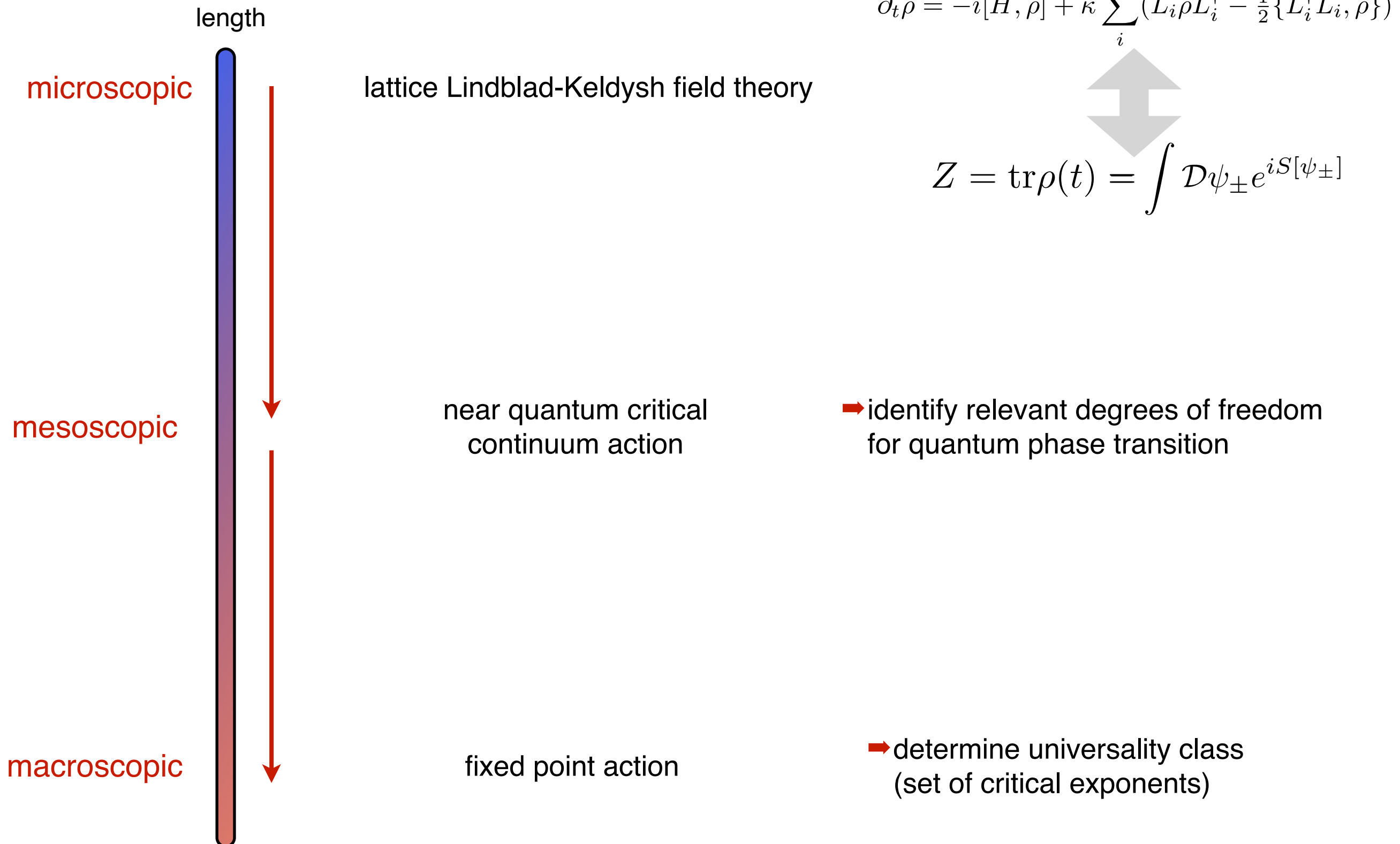
→ state pure across transition (canonical transformation)

→ **between two non-trivial vacua**: absorbing-to-absorbing state transition (cf. directed percolation: absorbing to active)

→ interacting (quartic) Lindbladian: quantum critical behavior far from equilibrium?

# Theoretical approach

- map dark state Lindbladian into equivalent Lindblad-Keldysh field theory (Keldysh = Feynman for density matrix)



# Microscopic fermion model

- chiral topological insulator models in all odd spatial dimensions (class AIII)

$$\hat{L}_{\alpha,\beta}^{(U)}(\mathbf{x}) = \hat{\psi}_{\alpha}^{\dagger}(\mathbf{x}) \hat{x}_{\beta}(\mathbf{x}), \quad \beta \in U, \alpha \in I, \quad I = (U, L)$$

$$\hat{L}_{\alpha,\beta}^{(L)}(\mathbf{x}) = \hat{\psi}_{\alpha}(\mathbf{x}) \hat{x}_{\beta}^{\dagger}(\mathbf{x}), \quad \beta \in L, \alpha \in I$$

upper, lower bands

superposition realising competition

$$\hat{x}_{\beta} = \cos(\theta) \hat{l}_{\beta} + \sin(\theta) \hat{r}_{\beta} = W_{\beta\delta}(\theta) \hat{\psi}_{\delta}$$

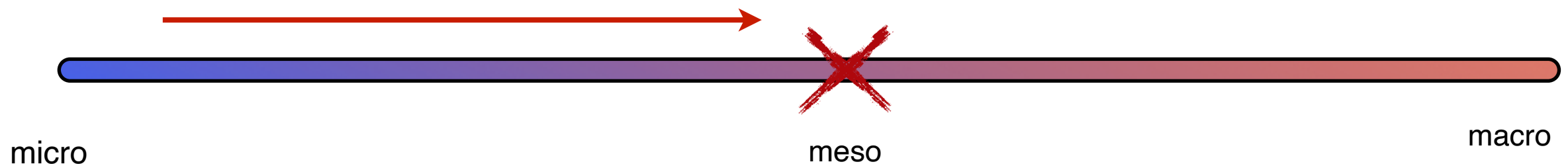
- Important feature: conserves total number of particles

→ expect diffusive slow mode to couple to fermions

- two ways to derive ‘mesoscopic’ near critical action

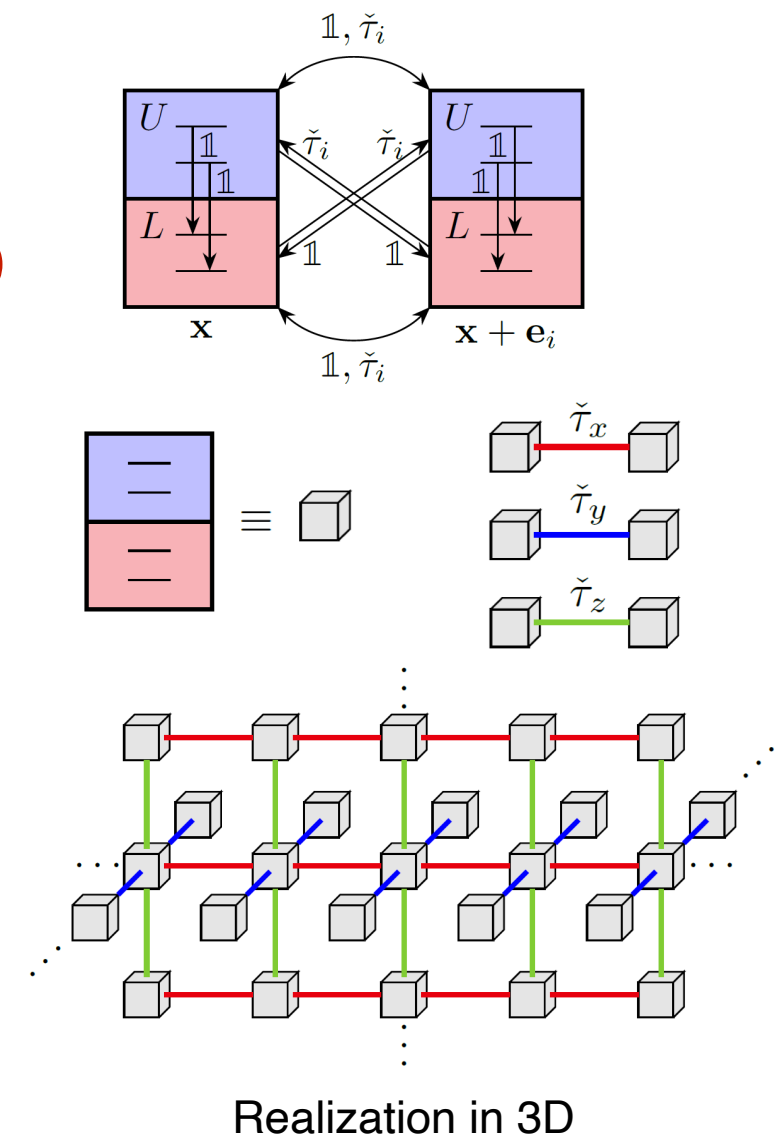
- direct calculation: Hubbard-Stratonovich decoupling

→ provides parameters & identify symmetries



- symmetry based construction

→ promote identified symmetries to construction principle for intermediate scale action



# Microscopic fermion model

- chiral topological insulator models in all odd spatial dimensions

$$\hat{L}_{\alpha,\beta}^{(U)}(\mathbf{x}) = \hat{\psi}_{\alpha}^{\dagger}(\mathbf{x}) \hat{x}_{\beta}(\mathbf{x}), \quad \beta \in U, \alpha \in I, \quad I = (U, L)$$

$$\hat{L}_{\alpha,\beta}^{(L)}(\mathbf{x}) = \hat{\psi}_{\alpha}(\mathbf{x}) \hat{x}_{\beta}^{\dagger}(\mathbf{x}), \quad \beta \in L, \alpha \in I$$

upper, lower bands

superposition realising competition

$$\hat{x}_{\beta} = \cos(\theta) \hat{l}_{\beta} + \sin(\theta) \hat{r}_{\beta} = W_{\beta\delta}(\theta) \hat{\psi}_{\delta}$$

- Important feature: conserves total number of particles

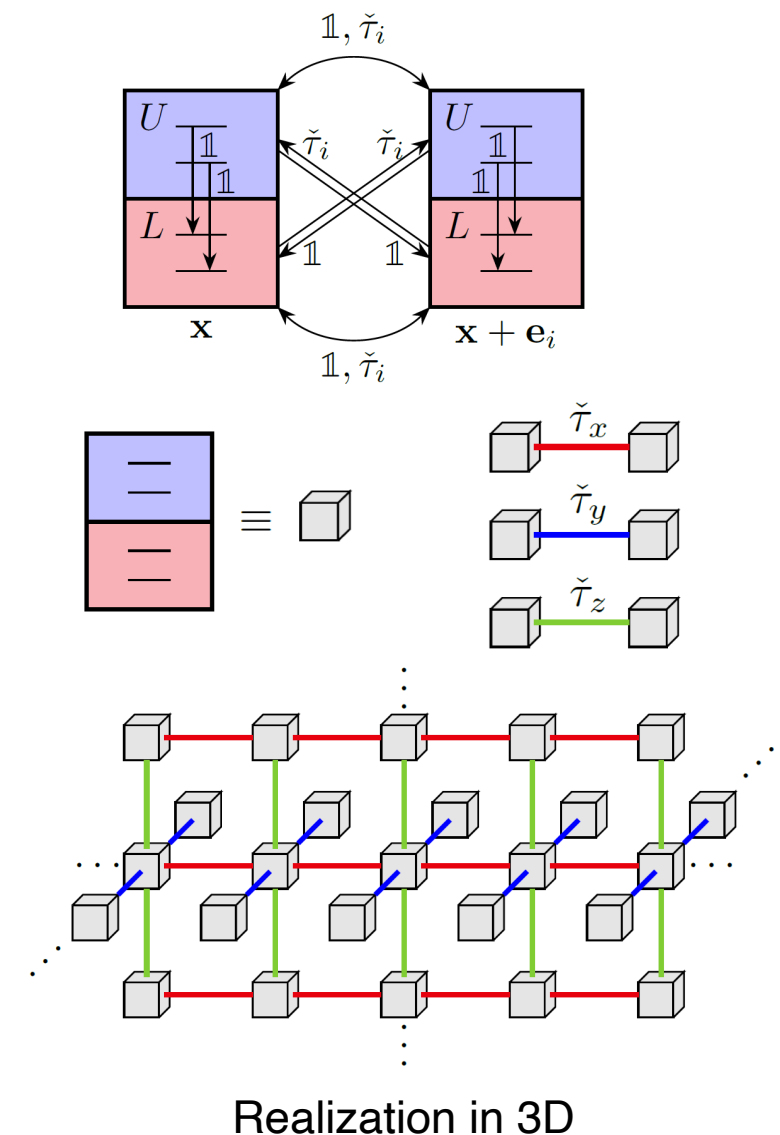
➔ expect diffusive slow mode to couple to fermions

- symmetries (in part emergent in continuum limit):

- unitary symmetries

- weak Pin(d) symmetry (spatial subgroup of Lorentz transformations)
- chiral symmetry

- new anti-unitary symmetry: fermion dark state symmetry



dissipative Dirac insulators

# Symmetry based derivation: fermion sector

micro

- near critical effective action in d+1 dimensions in diagonal basis

$$S = S_F + S_B + S_Y$$

(1) unitary symmetries (weak Pin(d) and chiral)

(2) **fermion dark state symmetry**: emergent dynamical symmetry with two components

- anti-unitary dynamical involution (squares to 1)

meso

$$\begin{aligned}\chi(\mathbf{x}, t) &\rightarrow T_F \chi(\mathbf{x}, -t) \\ \bar{\chi}(\mathbf{x}, t) &\rightarrow \bar{T}_F \bar{\chi}(\mathbf{x}, -t)\end{aligned} \quad i \rightarrow -i$$

$$T_F = \begin{pmatrix} -\mathbb{1} & 0 \\ F & \mathbb{1} \end{pmatrix}, \quad \bar{T}_F = \begin{pmatrix} -\mathbb{1} & 0 \\ -F & \mathbb{1} \end{pmatrix}$$

only possible for Grassmann fields

- weak global U(1)

$$F = F^\dagger, \quad F^2 = \mathbb{1}$$

$$\chi \rightarrow \mathcal{U}_z(\theta) \chi \quad \mathcal{U}_z(\theta) = \exp(i\theta \tau_z)$$

macro

➡ cooling into Dirac insulator pure state

➡ action has **single relevant parameter!**

$$S_F = \int_{\mathbf{q}, \omega} \bar{\chi}^\top(\omega, \mathbf{q}) \begin{pmatrix} 0 & \omega - K(\mathbf{q}) \\ \omega - K^\dagger(\mathbf{q}) & 2iP(\mathbf{q}) \end{pmatrix} \chi(\omega, \mathbf{q})$$

$$K(\mathbf{q}) = H(\mathbf{q}) + iD(\mathbf{q})$$

$$D(\mathbf{q}) = (\Delta_d + K_d \mathbf{q}^2) \mathbb{1}$$

$$H(\mathbf{q}) = (\Delta_c + K_c \mathbf{q}^2) \mathbb{1}$$

$$P(\mathbf{q}) = D(\mathbf{q}) \tau_z,$$

in diagonal basis, in line with  
microscopic derivation



# Result: Quantum critical fermion-boson action

- near critical effective action in d+1 dimensions

$$S = S_F + S_B + S_Y$$

- critical fermions: cooling into Dirac non-trivial vacuum

$$S_F = \int_{\mathbf{q}, \omega} \bar{\chi}^\top(\omega, \mathbf{q}) \begin{pmatrix} 0 & \omega - K(\mathbf{q}) \\ \omega - K^\dagger(\mathbf{q}) & 2iP(\mathbf{q}) \end{pmatrix} \chi(\omega, \mathbf{q})$$

- gapless bosons: diffusive modes of conserved particle number

$$S_B = \int_{t, \mathbf{x}} (n_c, \theta_q) \begin{pmatrix} 0 & \partial_t - D\partial_x^2 \\ \partial_t + D\partial_x^2 & -i2DT\partial_x^2 \end{pmatrix} \begin{pmatrix} n_c \\ \theta_q \end{pmatrix}$$

- cubic fermion-boson coupling

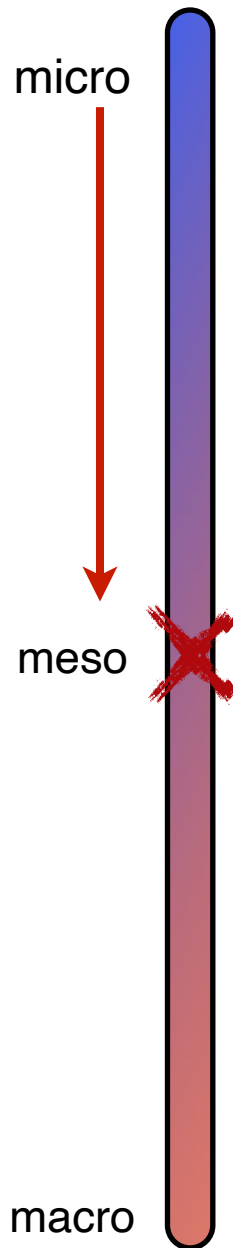
$$S_Y = \int_{\mathbf{x}, t} n_c(\mathbf{x}, t) \bar{\chi}^\top(\mathbf{x}, t) \begin{pmatrix} 0 & g\mathbf{1} \\ g^*\mathbf{1} & i(g^* - g)\tau_z \end{pmatrix} \chi(\mathbf{x}, t)$$

- compare non-linearities:

$$n_c \bar{\chi}_c \chi_q + h.c. + n_c \bar{\chi}_q \chi_q$$

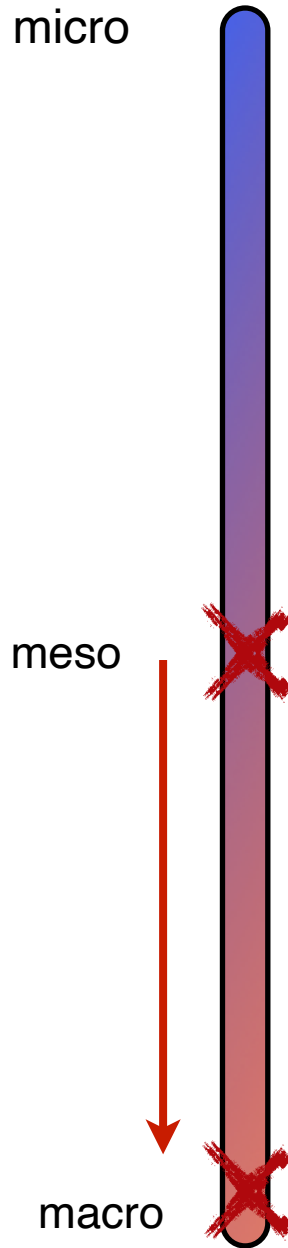
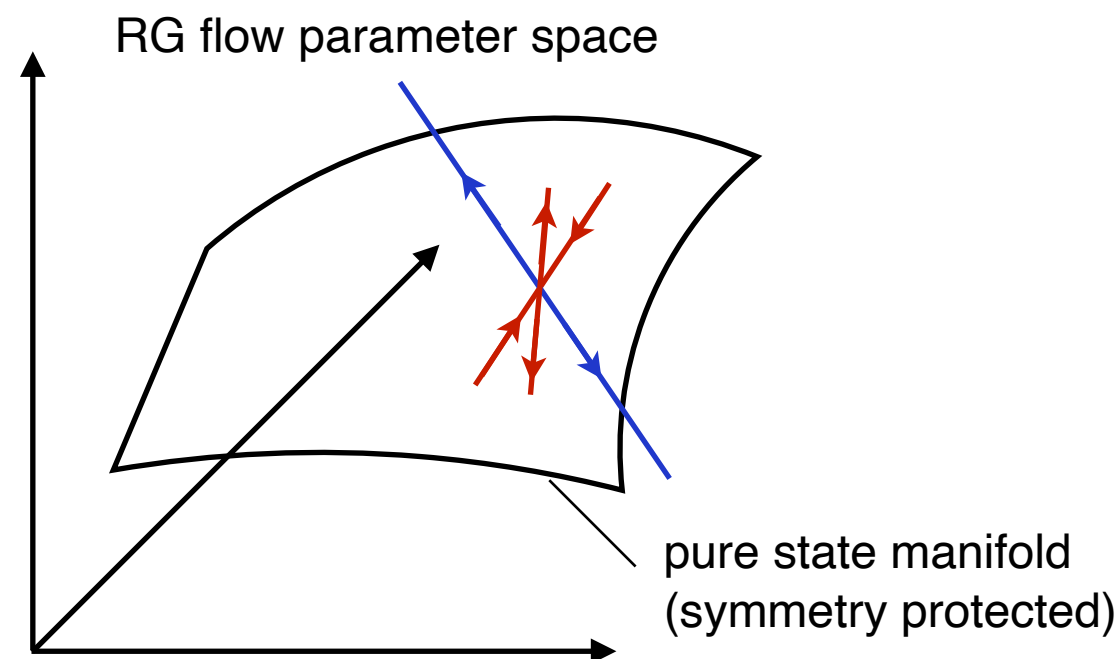
vs. directed  
percolation

$$\phi_c \phi_c \phi_q + \phi_c \phi_q \phi_q$$



# Towards macroscopic physics I: symmetry principle for pure states micro

- analysis guided by fermion dark state symmetry
- implication I: **protecting** the fermion state's purity
  - Ward identity: **non-thermal fluctuation-dissipation relation**  $G^K \stackrel{\text{FDS}}{=} F(G^R - G^A)$   
correlation response
  - equal-time:  $G^K(t, t) = \Gamma$ ,  $F^2 = 1$   
 $\implies$  full renormalized covariance matrix  $\Gamma^2 = 1$ , pure state
- implication II: **constraining** RG flow
  - forbids activation of a relevant direction (noise)

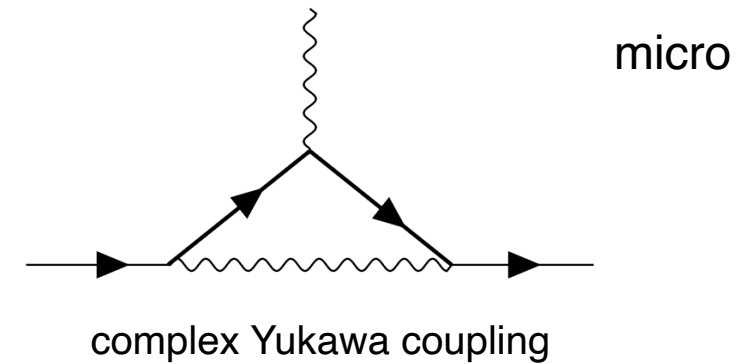
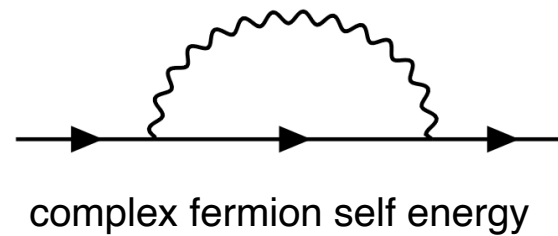


- non-Gaussian fixed point reached by **single fine tuning**
- unlike previous instances of non-equilibrium quantum criticality (bicritical points)

non-Markovian: Dalla Torre et al. Nat. Phys. (2012)  
Markovian: Marino, SD, PRL (2016)

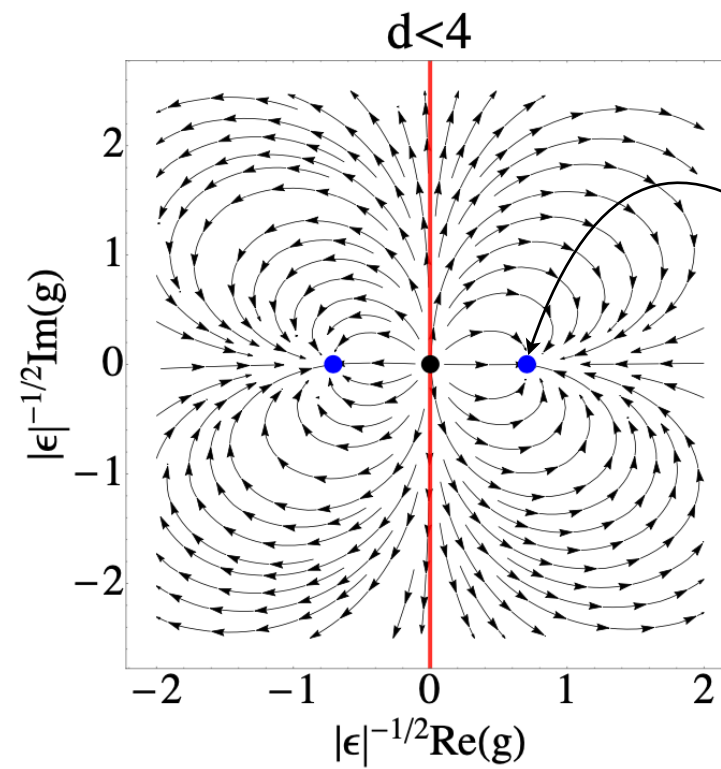
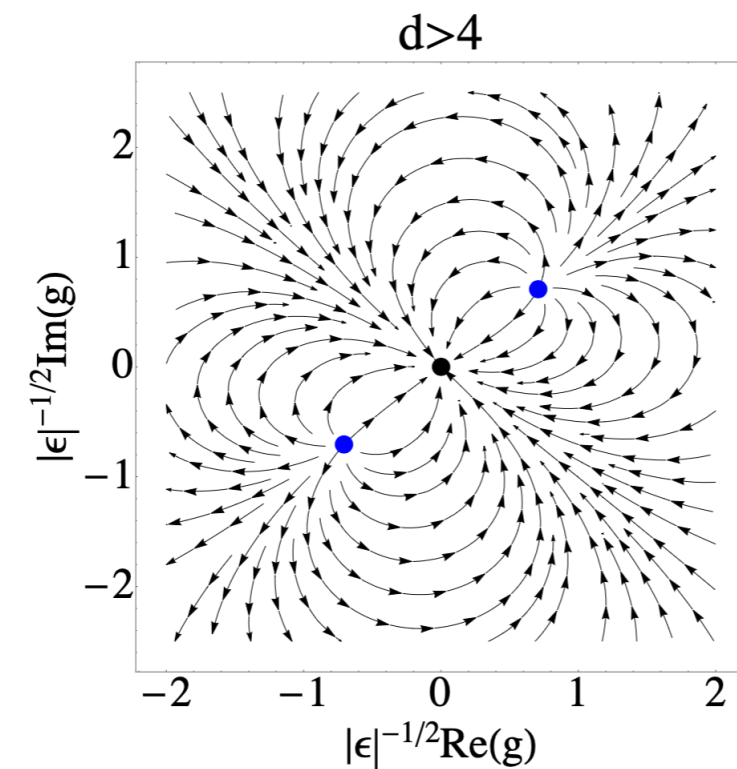
# Towards macroscopic physics II: RG analysis

- flow equations at one-loop order
- fixed point structure

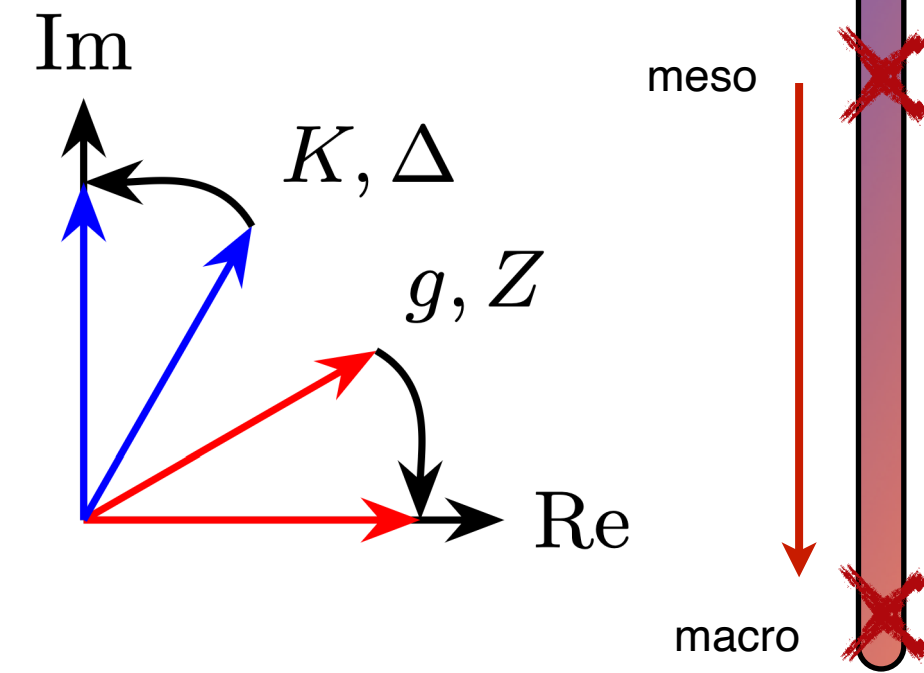


Flow of Yukawa coupling

fixed point with reversible and irreversible couplings



IR stable  
WF fixed  
point

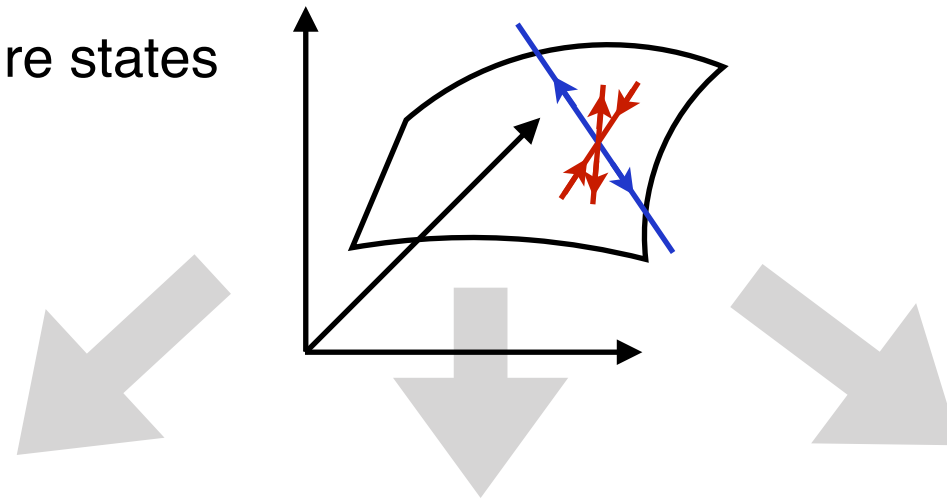


- set of one-loop critical exponents for new universality class

|                             |                                          |
|-----------------------------|------------------------------------------|
| correlation length exponent | $\nu = \frac{1}{2} - \frac{\epsilon}{4}$ |
| dynamical exponent          | $z = 2 + \frac{\epsilon}{2}$             |
| anomalous dimension         | $\eta = -\frac{\epsilon}{4}$             |

# Discussion: related symmetry structures at critical points

- symmetry **constrains flow** in pure states



in quantum systems:

Marcuzzi et al., PRL (20016); Piroli et al. PRB (2021), O'Dea et al., PRB (2024)

quantum directed percolation:

Carollo, Gillman, Weimer, Lesanovsky, PRL (2019); Thompson, Kamenev, SciPost (2024)

## quantum criticality

- thermal/KMS symmetry for  $T = 0$

Sieberer, ..., SD, PRB (2015); Haehl et al. JHEP (2016); Crossley et al., JHEP (2016); Aron et al. SciPost (2018)

➡ further commonalities:



## fermion dark state transitions

- fermionic dark state symmetry



## absorbing state transitions

- rapidity inversion

U.C. Täuber, Critical Dynamics (2014)



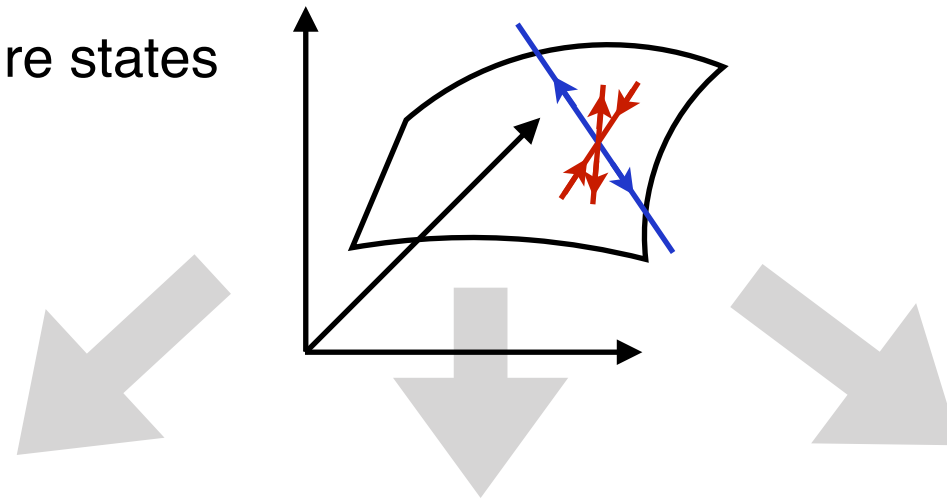
➡ implication: fluctuation-dissipation relations fixing system in **pure state**

➡ all symmetries reminiscent of time reversal:

- $\mathbb{Z}_2$  antiunitary involution
- involve  $t \rightarrow -t$

# Discussion: related symmetry structures at critical points

- symmetry **constrains flow** in pure states



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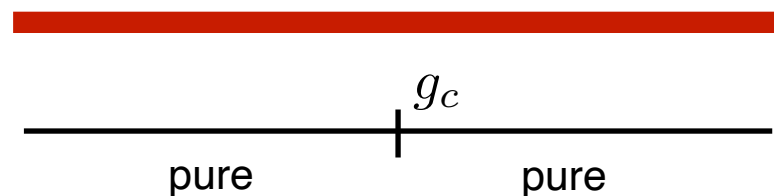
## absorbing state transitions

- rapidity inversion

U.C. Täuber, Critical Dynamics (2014)

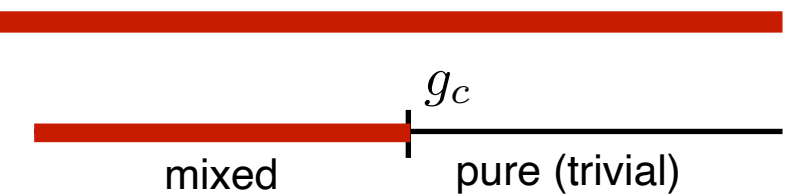
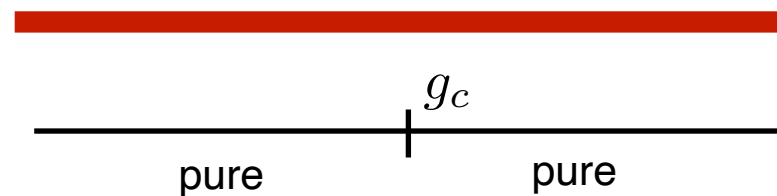
➔ further comparison:

equilibrium



quantum

non-equilibrium



classical

➔ new intersection: non-equilibrium quantum

➔ caveat: symmetry protection  $\neq$  physical protection

# Fermion quantum criticality far from equilibrium

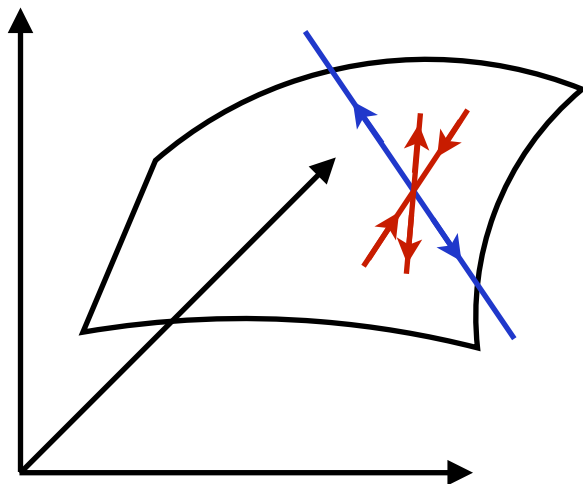
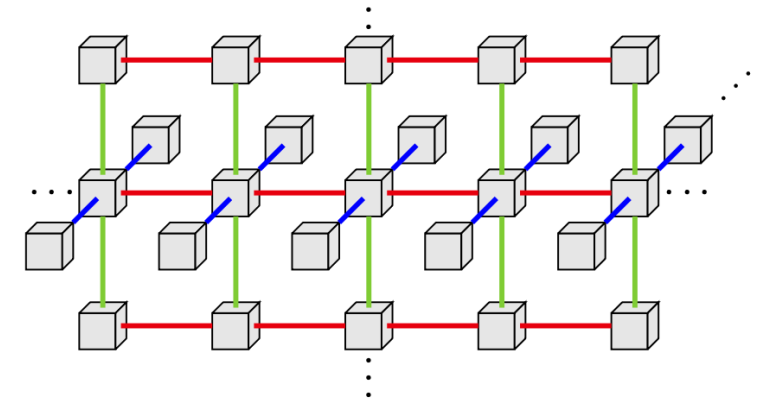
R. Mittal, T. Zander, J. Lang, SD, PRX (2026)

microphysics

macrophysics

- **microscopic ingredients:**

- fermion dark state, interactions, breaking of equilibrium, particle number conservation
- building blocks available in Floquet driven solid state platforms



- **mesoscopic symmetries:**

- protection of pure quantum states
- fermion-boson theory analogous to DP action

- **macroscopic physics:**

- new intersection, parallels to equilibrium quantum criticality and absorbing state transitions
- quantum criticality far from equilibrium

