Family–Vicsek scaling in dissipative XXZ chains: Unitary versus Lindbladian evolution

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Overview

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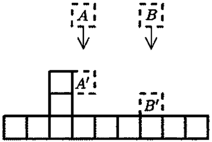
5. Noninteracting limit $\Delta = 0$
6. Interacting case $\Delta \neq 0$
7. Integrability breaking ($J_2 \neq 0$)
8. Summary

FV in classical statistical physics

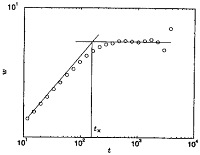
- Scaling and universality: powerful tools for organizing complex dynamics.
- **Family–Vicsek scaling**: a phenomenological ansatz for growth processes with application to surface roughening, interface growth, etc.



Surface growth example

- heigh field $h(x, t)$ describes the interface growth
- roughness $W(\ell, t)$ measures the fluctuations of h in a segment of length ℓ at time t.

$$W(\ell, t) = \sqrt{\langle \frac{1}{\ell} \sum_{x=1}^{\ell} (h(x, t) - \langle h(x, t) \rangle)^2 \rangle}.$$



- β: growth exponent, $W(\ell, t) \sim t^{\beta}$ for $t \ll t^*$.
- α: saturation (spatial) exponent, $W(\ell, t) \sim \ell^{\alpha}$ for $t \gg t^*$.

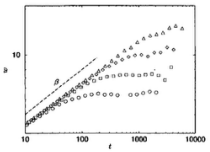
- The scaling exponents are not independent.

$$W(\ell,t)=\sqrt{\langle\frac{1}{\ell}\sum_{x=1}^{\ell}(h(x,t)-\langle h(x,t)\rangle)^2\rangle}.$$

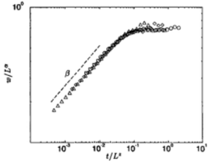
- β: growth exponent, $W(\ell,t)\sim t^{\beta}$ for $t\ll t^*$.
- α: saturation (spatial) exponent, $W(\ell,t)\sim \ell^{\alpha}$ for $t\gg t^*$.
- z: dynamical exponent, $z=\alpha/\beta$.

$$W(\ell,t)\sim \ell^{\alpha}\, f\left(\frac{t}{\ell^{z}}\right).$$

$$f(x\ll 1)\sim\begin{cases}x^{\beta}, & \text{growth regime}\\ \text{const}, & \text{saturation regime}\end{cases}$$



Surface growth example



[Barabási & Stanley, *Fractal concepts in surface growth*]

Why Family–Vicsek scaling in quantum chains?

- Goal: organize far-from-equilibrium fluctuations via a scaling collapse.
- Observable: magnetization fluctuations in a segment of length ℓ.
- Key question: how do transport and dissipation control the dynamical exponent z?
- Novelty: FV scaling has not been systematically explored in quantum many-body systems, especially in the presence of dissipation.
- Method: quantum generating function (QGF) for two-time correlations, evaluated via matrix product state techniques.
- Setting: XXZ chain with bulk gain/loss (Markovian Lindblad dynamics), starting from an exactly known product NESS.

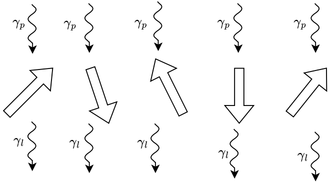
Driven–dissipative XXZ chain

Lindblad master equation:

$$\begin{aligned}\partial_t \rho(t) &= \mathcal{L}[\rho(t)] \\ &= -i[H, \rho(t)] + \mathcal{D}[\rho(t)].\end{aligned}$$

XXZ Hamiltonian:

$$H = \frac{J}{4} \sum_{j=-L/2}^{L/2-1} \left(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta\, \sigma_j^z \sigma_{j+1}^z \right).$$



Jump operators (bulk gain/loss):

$$L_j^- = \sqrt{\gamma_l}\, \sigma_j^-, \qquad L_j^+ = \sqrt{\gamma_p}\, \sigma_j^+, \qquad \Gamma \equiv \gamma_l + \gamma_p.$$

Product nonequilibrium steady state (NESS)

Define fugacity

$$\zeta \equiv \frac{\gamma_p}{\gamma_l}.$$

Exact NESS (diagonal, product form):

$$\rho_{\mathrm{SS}} = \bigotimes_j \rho_j, \qquad \rho_j = \frac{1}{1+\zeta}\begin{pmatrix} \zeta & 0 \\ 0 & 1 \end{pmatrix}.$$

Consequences:

- No connected equal-time correlations at $t = 0$.
- Uniform magnetization

$$\langle \sigma_j^z \rangle_{\mathrm{SS}} = \frac{\zeta - 1}{\zeta + 1}.$$

- In the XX mapping, filling

$$\bar{n} = \frac{\zeta}{1+\zeta}.$$

- Aditional interactions do not change the NESS

$$H_{\mathrm{nnn}} = \frac{J_2}{4}\sum_{j=-L/2}^{L/2-2}\left(\sigma_j^x\sigma_{j+2}^x + \sigma_j^y\sigma_{j+2}^y + \Delta\,\sigma_j^z\sigma_{j+2}^z\right)$$

Segment magnetization and roughness

Segment magnetization (length ℓ):

$$\Sigma_\ell \equiv \frac{1}{2} \sum_{j \in \mathrm{seg}(\ell)} \sigma_j^z .$$

Transferred variable:

$$\Gamma(t) \equiv \Sigma_\ell(t) - \Sigma_\ell(0).$$

Roughness:

$$W(\ell, t) = \sqrt{\kappa_2(\ell, t)}.$$

Interpretation: W measures the growth and saturation of magnetization fluctuations in a finite segment.

Quantum generating function (QGF)

Counting-field twist operator:

$$R_\ell(\lambda) \equiv e^{i\lambda\Sigma_\ell}.$$

The initial twist operator is chosen as:

$$R^{(S^z)}(\lambda) = \mathbb{1} \otimes \dots \mathbb{1} \otimes \underbrace{e^{i\lambda S^z} \otimes \dots \otimes e^{i\lambda S^z}}_{\frac{L-l}{2} < j \leq \frac{L+l}{2}} \otimes \mathbb{1} \otimes \dots \mathbb{1}$$

Two-time generating function (unitary vs full Lindblad evolution):

$$G_\ell^{(\mathcal{L})}(\lambda, t) \equiv \mathrm{Tr}\left[R_\ell(\lambda)\, e^{\mathcal{L}t}\left(R_\ell^\dagger(\lambda)\rho_{\mathrm{SS}}\right)\right],$$

$$G_\ell^{(H)}(\lambda, t) \equiv \mathrm{Tr}\left[R_\ell(\lambda)\, e^{-iHt} R_\ell^\dagger(\lambda)\rho_{\mathrm{SS}}\, e^{iHt}\right].$$

Cumulants:

$$\kappa_n(\ell, t) = (-i)^n\, \partial_\lambda^n\, \ln G_\ell(\lambda, t)|_{\lambda=0}.$$

The second moment and cumulant in the small-λ limit:

$$\kappa_2^{(X)}(\ell, t) = \mu_2^{(X)}(\ell, t) \simeq \frac{2}{\lambda^2}\left(1 - \mathrm{Re}\, G_\ell^{(X)}(\lambda, t)\right) + \mathcal{O}(\lambda^2).$$

Unitary XX limit: analytic benchmark

- The XX limit ($\Delta = 0$) is exactly solvable via Jordan–Wigner transformation.
- The system is first prepared in the product NESS, then evolved unitarily (dissipation is turned off at $t = 0$).
- ρ_{SS} remains stationary under the unitary evolution ($\rho_{SS} \propto e^{\mu S^z}$, $[H, S^z] = 0$).
- Dynamical flucutions are incoded in the generating function $G_\ell^{(H)}(\lambda, t)$

At $\Delta = 0$ (XX chain), Jordan–Wigner gives a free-fermion hopping model. Second cumulant

(thermodynamic limit):

$$\mu_2^{(H)}(\ell, t) = 2\bar{n}(1-\bar{n}) \left[\ell - \sum_{r=-(\ell-1)}^{\ell-1} (\ell - |r|)\, J_r^2(Jt) \right].$$

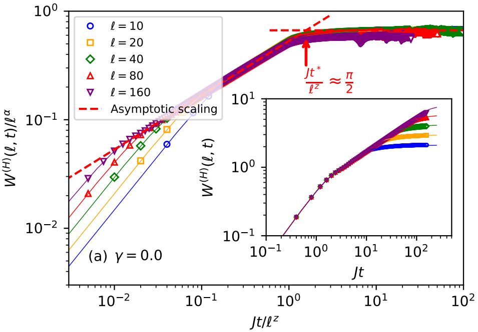
Asymptotics (FV growth and saturation):

$$W^{(H)}(\ell, t) \simeq \left(\frac{4}{\pi} \bar{n}(1-\bar{n})\, Jt \right)^{1/2}, \qquad 1 \ll Jt \ll \ell,$$

$$W_{\rm sat}^{(H)}(\ell) \simeq (2\bar{n}(1-\bar{n}))^{1/2}\, \ell^{1/2}, \qquad t \gg \ell/J.$$

Therefore $\alpha = \beta = 1/2$ and $z = 1$ (ballistic). A short-time transient gives $\kappa_2 \propto t^2$.

Unitary XX limit: FV collapse



$$(\alpha, \beta, z) = (\frac{1}{2}, \frac{1}{2}, 1) \quad \text{(Ballistic transport)}$$

Scaling function:

$$f_H(x) \sim \begin{cases} \left(\frac{4}{\pi}\bar{n}(1-\bar{n})\,x\right)^{1/2}, & x \ll 1 \\ \left(2\bar{n}(1-\bar{n})\right)^{1/2}, & x \gg 1 \end{cases}$$

Lindblad XX limit: dephasing-controlled saturation

In the same $\Delta = 0$ limit, two-time correlations are exponentially damped:

$$\mu_2^{(\mathcal{L})}(\ell,t) = 2\bar{n}(1-\bar{n}) \left[\ell - e^{-\Gamma t} \sum_{r=-(\ell-1)}^{\ell-1} (\ell - |r|)\, J_r^2(Jt) \right].$$

Two time scales compete:

- Coherent finite-size time: $t^* \sim \ell/J$.
- Dissipative time: $t_\Gamma \sim \Gamma^{-1}$.

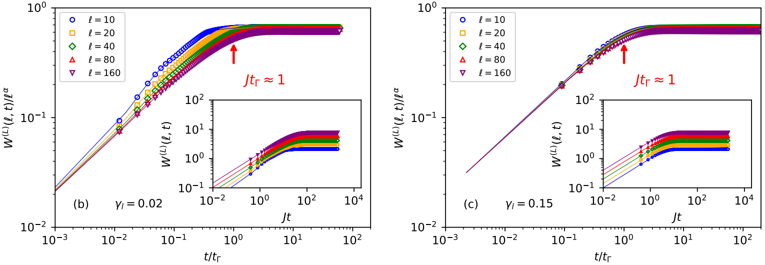
Weak dissipation limit: $t_\Gamma \gg t^*$, the roughness saturates at $t \sim t^*$, and a clean one-parameter FV collapse in Jt/ℓ can develop. Asymptotics:

$$W^{(\mathcal{L})}(\ell,t) \simeq \begin{cases} \left(\frac{4}{\pi}\bar{n}(1-\bar{n})\,Jt\right)^{1/2}, & 1 \ll Jt \ll \ell, \Gamma t \ll 1 \\ \left(2\bar{n}(1-\bar{n})\right)^{1/2} \ell^{1/2}, & t \gg \max\{\ell/J, \Gamma^{-1}\} \end{cases}$$

Scaling function:

$$f_{\mathcal{L}}(x,y) \sim \begin{cases} \left(\frac{4}{\pi}\bar{n}(1-\bar{n})\,x\right)^{1/2}, & x \ll 1, y \ll 1 \\ \left(2\bar{n}(1-\bar{n})\right)^{1/2}, & x \gg 1, y \gg 1 \end{cases}$$

Lindblad XX limit: dissipation-dominated regime



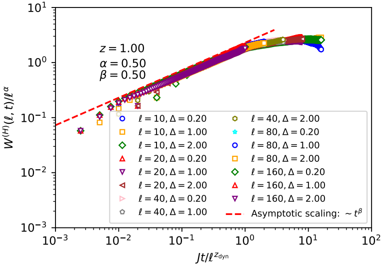
Dissipation-dominated saturation: $t_\Gamma \ll t^*$, the roughness saturates at $t \sim t_\Gamma$.

Failure of one-parameter FV collapse at strong dissipation: reservoirs impose a finite relaxation time t_Γ, for the dynamical fluctuations (relaxation scenario).

Unitary evolution at finite magnetization density ($\zeta \neq 1$)

Numerical TEBD + QGF shows a robust FV-type collapse consistent with ballistic transport:

$$W^{(H)}(\ell, t) \sim \ell^{1/2} f\left(\frac{t}{\ell}\right), \qquad z \simeq 1, \qquad (\zeta \neq 1).$$

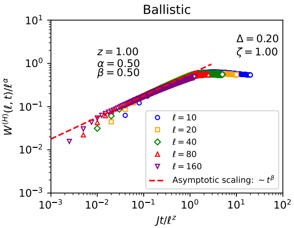


- Curves for different anisotropies Δ collapse onto a single scaling function
- For generic $\zeta \neq 1$, the FV scaling collapse is compatible with ballistic universality class, $z \simeq 1$.
- Interacting XXZ chains supports stable quasiparticles (magnons) leading to ballistic transport at finite magnetization density.

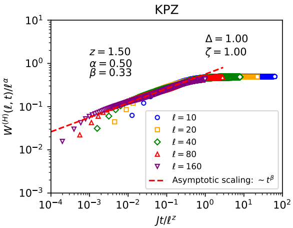
Unitary evolution at $T = \infty$ ($\zeta = 1$)

At $\zeta = 1$ (zero average magnetization), z reflects known XXZ transport regimes:

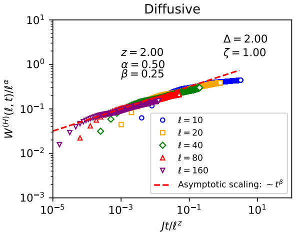
- $|\Delta| < 1$: ballistic, $z \simeq 1$.
- $\Delta = 1$: KPZ-like, $z \simeq 3/2$.
- $\Delta > 1$: diffusive, $z \simeq 2$.



$\Delta = 0.2$



$\Delta = 1.0$



$\Delta = 2.0$

Interacting Lindbladian evolution: breakdown of one-parameter FV collapse

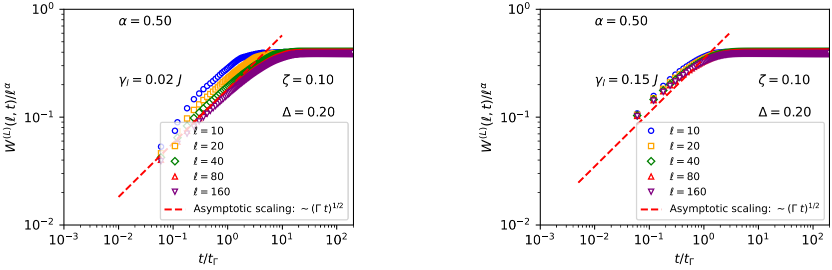
Bulk gain/loss introduces the microscopic relaxation time $t_\Gamma \sim \Gamma^{-1}$. A useful description is

two-parameter scaling:

$$W^{(\mathcal{L})}(\ell, t) = \ell^{1/2} f_{\mathcal{L}}\left(\frac{Jt}{\ell}, \Gamma t\right).$$

Strong dissipation ($\Gamma t \gg 1$) implies a relaxiation scenario:

$$W^{(\mathcal{L})}(\ell, t) = \ell^{1/2} g(\Gamma t) \tag{1}$$



$\Delta = 0.2,\ \zeta_0 = 0.1,\ \gamma_l = 0.02J$ $\Delta = 0.2,\ \zeta_0 = 0.1,\ \gamma_l = 0.15J$

Next-nearest-neighbor coupling

Add an integrability-breaking perturbation (still $U(1)$ symmetric):

$$H_{\rm nnn} = \frac{J_2}{4} \sum_l \left(\sigma_l^x \sigma_{l+2}^x + \sigma_l^y \sigma_{l+2}^y + \Delta\, \sigma_l^z \sigma_{l+2}^z \right).$$

At $\Delta = 0$ it remains free-fermionic; FV exponents stay ballistic while the effective velocity changes.

$$U_r(t) = \int_{-\pi}^{\pi} \frac{dk}{2\pi}\, e^{-i\varepsilon(k)t}\, e^{ikr}.$$

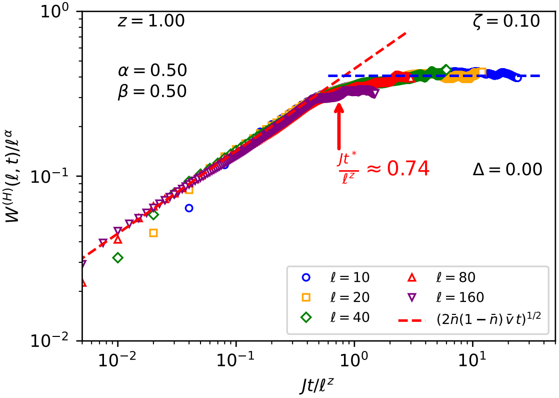
Example ($J_2 = J$):

$$\bar{v}\big|_{J_2=J} = \frac{17}{4\pi} J \simeq 1.35J, \qquad t^* \sim \ell/\bar{v}.$$

Asymptotics:

$$W^{(H)}(\ell, t) \simeq \begin{cases} \left(2\bar{n}(1-\bar{n})\right)^{1/2} \ell^{1/2}, & t \gg \ell/\bar{v} \\ \left(2\bar{n}(1-\bar{n})\, \bar{v}\, t\right)^{1/2}, & 1 \ll \bar{v}t \ll \ell \end{cases}$$

$\Delta = 0$, unitary with $J_2 = J$: ballistic collapse with renormalized velocity

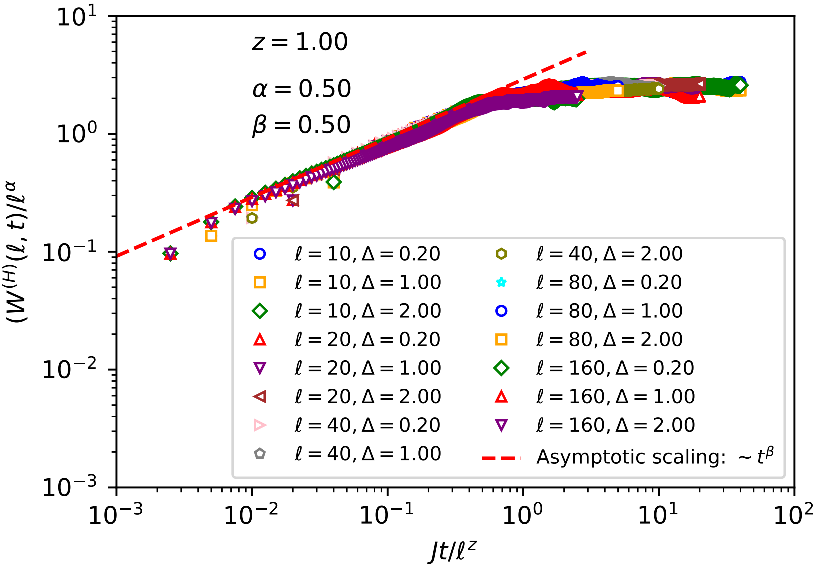


$$(\alpha, \beta, z) = (1/2, 1/2, 1)$$

Scaling function:

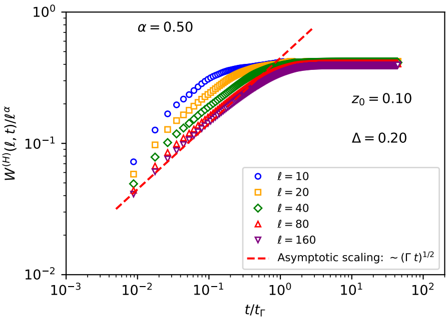
$$f_H(x) \sim \begin{cases} (2\bar{n}(1-\bar{n})\,\bar{v}\,x)^{1/2}, & x \ll 1 \\ \left(2\bar{n}(1-\bar{n})\right)^{1/2}, & x \gg 1 \end{cases}$$

$\Delta \neq 0$, unitary with $J_2 = J$: robustness at $\zeta \neq 1$

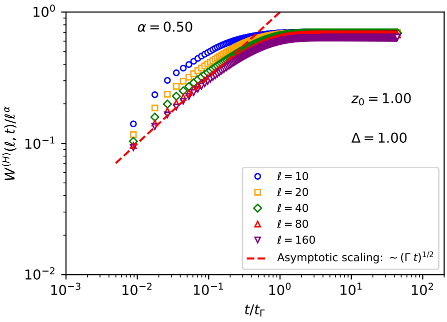


Lindbladian with $J_2 = J$: dissipation-dominated scaling persists

Integrability breaking does not qualitatively change the Lindblad scenario: saturation is still controlled by t_Γ.



$\Delta = 0.2,\ \zeta_0 = 0.1$



$\Delta = 1.0,\ \zeta_0 = 1.0$

Conclusions

- Starting point: exactly known product NESS of an XXZ chain with bulk gain/loss.
- Unitary $\Delta = 0$: analytic FV scaling with $\alpha = \beta = 1/2$ and $z = 1$ (ballistic), plus a short-time $\kappa_2 \propto t^2$ transient.
- Lindblad $\Delta = 0$: ballistic light-cone survives, but memory decays as $e^{-\Gamma t}$; saturation can be set by $t_\Gamma \sim \Gamma^{-1}$, and FV collapse fails at strong dissipation.
- Interacting unitary: robust FV collapse at $\zeta \neq 1$ (ballistic); at $\zeta = 1$, z tracks ballistic/KPZ/diffusive regimes.
- Interacting Lindblad: one-parameter coherent FV collapse typically fails; dissipation controls the crossover.
- Integrability breaking via J_2 does not qualitatively change the picture.
- Overall, FV scaling provides a useful framework for organizing far-from-equilibrium fluctuations in quantum chains, and highlights the interplay between transport and dissipation.

Outlook

- Map out the two-parameter scaling function $f_{\mathcal{L}}(Jt/\ell, \Gamma t)$ in regimes where t_Γ and t^* are comparable.
- Explore spatially inhomogeneous dissipation or boundary-driven setups.
- Extend FV analysis to other conserved quantities / higher cumulants.
- Extend to other models. (prelimiary results are available for the Fermi-Hubbard chain with bulk gain/loss, showing a similar phenomenology to the XXZ case).

Thank you!

Questions?