Dynamics and phase transitions in non-reciprocal and kinetically constrained open quantum many-body systems

Pietro Brighi

Qopen kick-off meeting
Lisbon, 27/04/2026

COST Action CA24109 – QOpen https://cost.eu https://qopen.eu/

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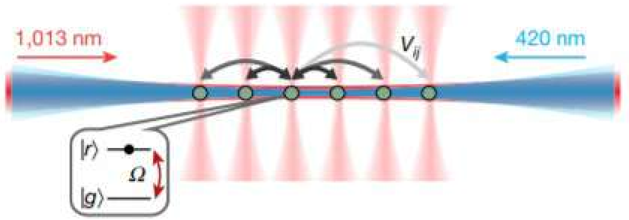
Quantum simulation platforms

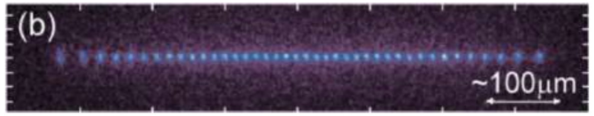
Artificial Matter:

Rydberg atoms arrays



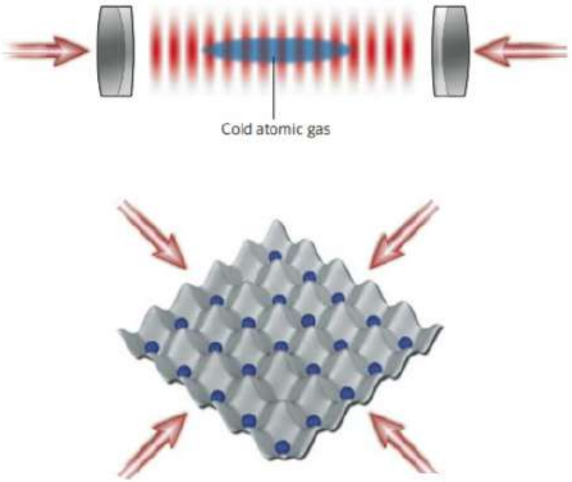
[Bernien,…,Lukin, Nature '17]

Trapped ions



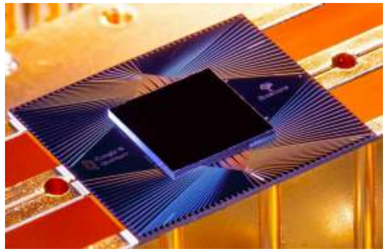
[Schneider *et al*., '12]

Ultracold atoms



[Bloch, Nature '08]

Superconducting Circuits



[Google Quantum AI]

Isolated Highly Tunable

Far From Equilibrium

Exotic Phases of Matter

Interesting Dynamics

Thermalization and ergodicity breaking

Quantum many-body dynamics

Drive

$$\hat{H}(t)$$

$$|\psi(t)\rangle = \hat{T}\int_0^t e^{-i\hat{H}(\tau)\tau}|\psi_0\rangle$$

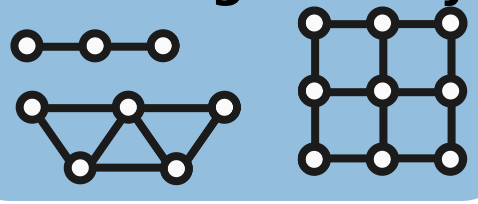
Dissipation

$$\hat{L}_j \qquad \rho(t) = e^{\mathcal{L}t}\rho_0$$

$$\hat{H} = \hat{H}_0 + \hat{H}_{int}$$

$$|\psi(t)\rangle = e^{-i\hat{H}t}|\psi_0\rangle$$

Lattice geometry



Degrees of freedom

$$[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij} \qquad \{\hat{c}_i, \hat{c}_j^\dagger\} = \delta_{ij}$$

$$[\sigma_i^\alpha, \sigma_j^\beta] = 2i\,\delta_{ij}\,\epsilon_{\alpha\beta\gamma}\,\sigma_j^\gamma$$

Few analytical methods
e.g. Bethe ansatz for
integrable models

Numerical techniques

Exact
Limited in
system size

Approximate
Krylov, tensor networks,
mean-field-like...
Method-specific limitations

Quantum many-body dynamics

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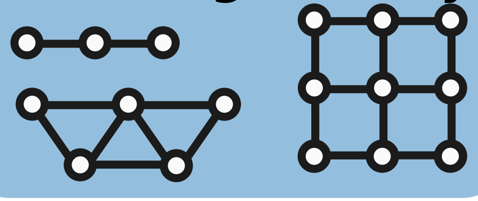

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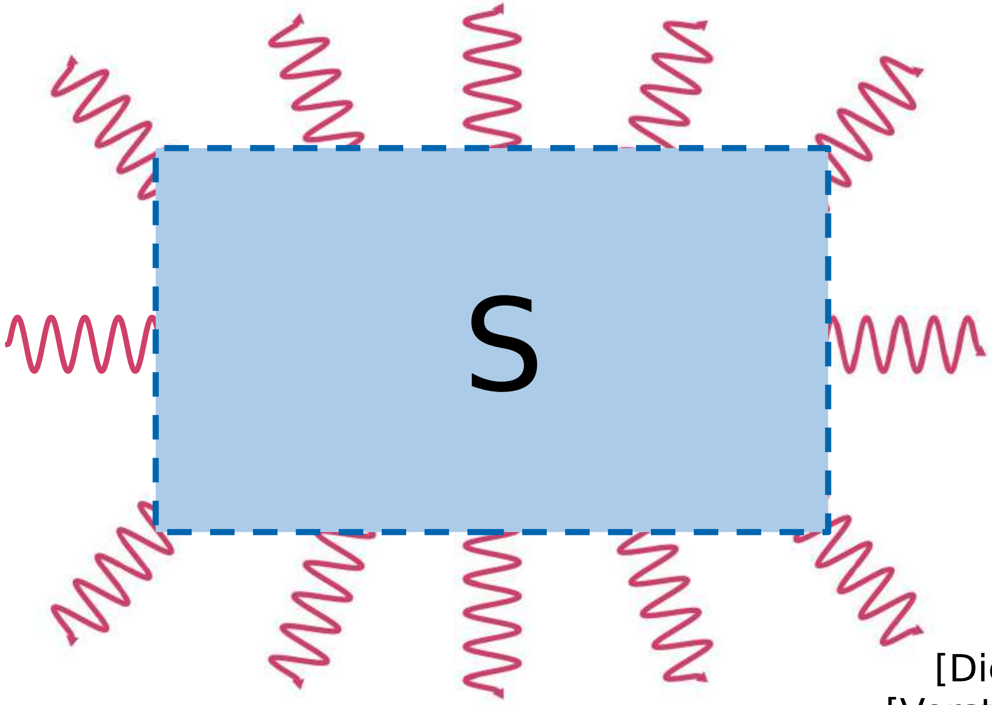
Reservoir engineering

Use the coupling with the **environment as a resource**

Generic coupling → Decoherence (bad!)



[Poyatos *et al*. PRL ‘96]

Engineered bath-system interactions

State preparation

[Diehl *et al*. Nat. Phys. ‘08]
[Verstraete *et al*. Nat. Phys. ‘09]
…
[Mi *et al*. Science ‘24]

Out-of-equilibrium phases

[Diehl *et al*. PRL ‘10]
[Tomadin *et al*. PRA ‘12]
…

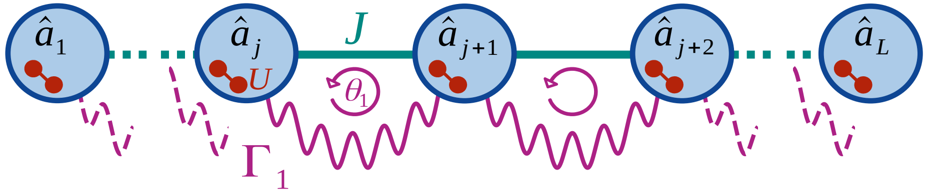
Entanglement control

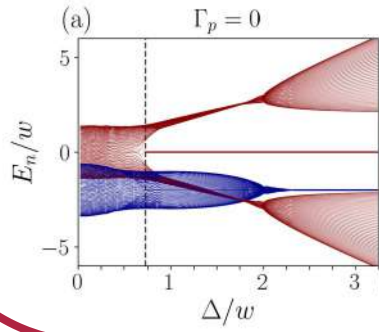
[Parkins *et al*. PRL ‘06]
[Krauter *et al*. PRL ‘11]
…

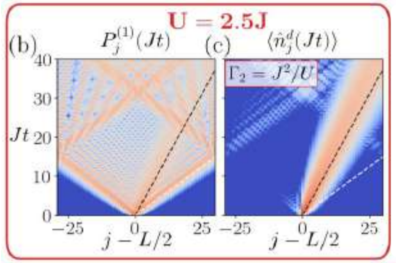
Outline

Non-reciprocal systems

- Reservoir engineering
- Non-reciprocal chains and interplay with interactions
- Novel phases

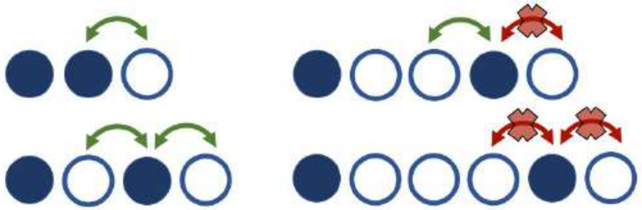


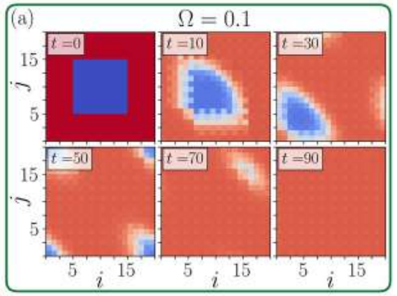




Kinetically constrained models

- Dynamical rules
- Bistability and dynamical heterogeneity
- Slow dynamics





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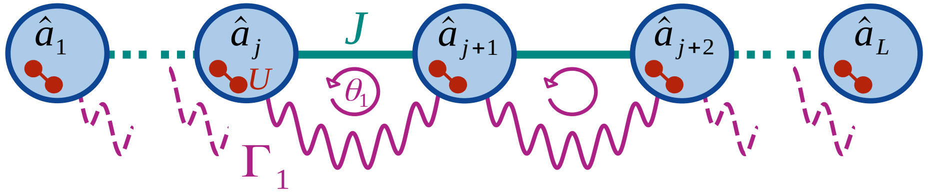
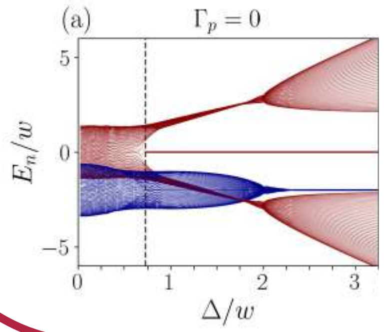
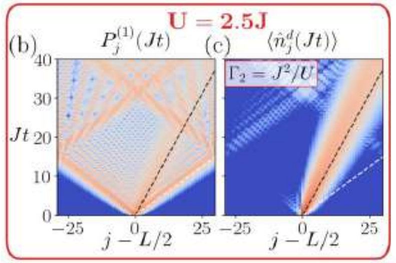






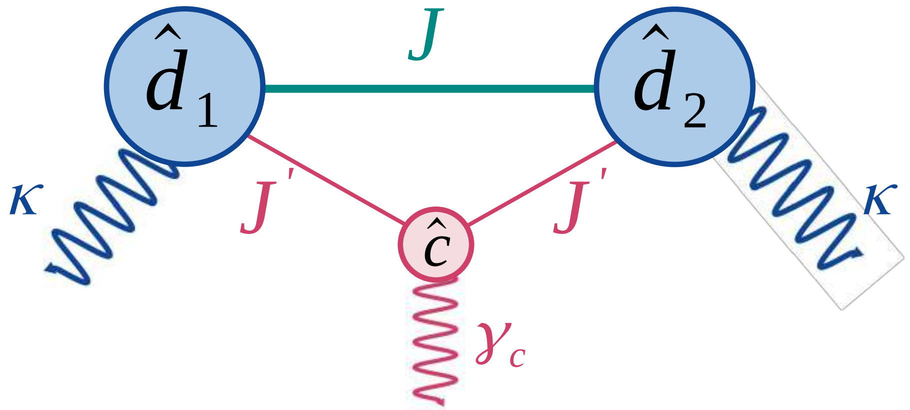
A. Nunnenkamp

[**PB** and A. Nunnenkamp, PRA '24]
[**PB** and A. Nunnenkamp, arXiv '25]

Engineering non-reciprocal couplings

[Metelmann and Clerk, PRX '15]

Use environment to engineer **non-reciprocal** couplings



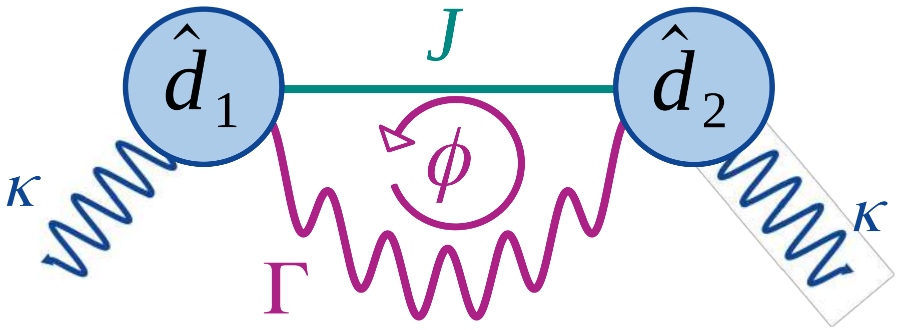
$$\hat{H} = J\hat{d}_1^\dagger \hat{d}_2 + J^* \hat{d}_2^\dagger \hat{d}_1 + [J'\hat{c}^\dagger(\hat{d}_1 + \hat{d}_2) + J'^* \hat{c}(\hat{d}_1^\dagger + \hat{d}_2^\dagger)]$$

$\gamma_c \gg J, J' \kappa$ ⟹ Adiabatic elimination

$$\Gamma \mathcal{D}[\hat{d}_1 + e^{i\phi}\hat{d}_2]$$

$$\frac{d}{dt}\langle \hat{d}_1 \rangle = -\frac{\kappa + \Gamma}{2}\langle \hat{d}_1 \rangle - \left[\frac{\Gamma}{2} + iJ\right]\langle \hat{d}_2 \rangle,$$

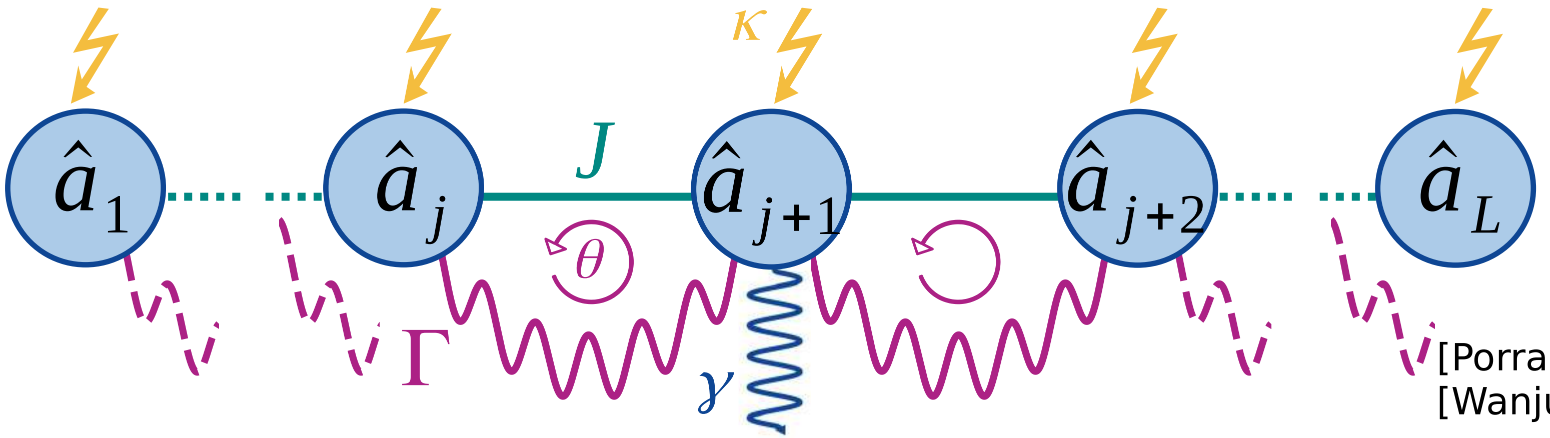
$$\frac{d}{dt}\langle \hat{d}_2 \rangle = -\frac{\kappa + \Gamma}{2}\langle \hat{d}_2 \rangle - \left[\frac{\Gamma}{2} + iJ^*\right]\langle \hat{d}_1 \rangle$$



Non-reciprocal equations of motion!

Non-reciprocal hopping chain

Coupling many cavities coherently and incoherently



$$\hat{H} = J \sum_j \hat{a}_j^\dagger \hat{a}_{j+1} + \mathcal{H.c.}$$

$$\Gamma \sum_j \mathcal{D}[\hat{a}_j + e^{i\theta} \hat{a}_{j+1}]\rho$$

[Metelmann and Clerk, PRX ‘15]

[Porras and Fernandez-Lorenzo, PRL ‘19]
[Wanjura, Brunelli and Nunnenkamp, Nat.Comm. ‘20]

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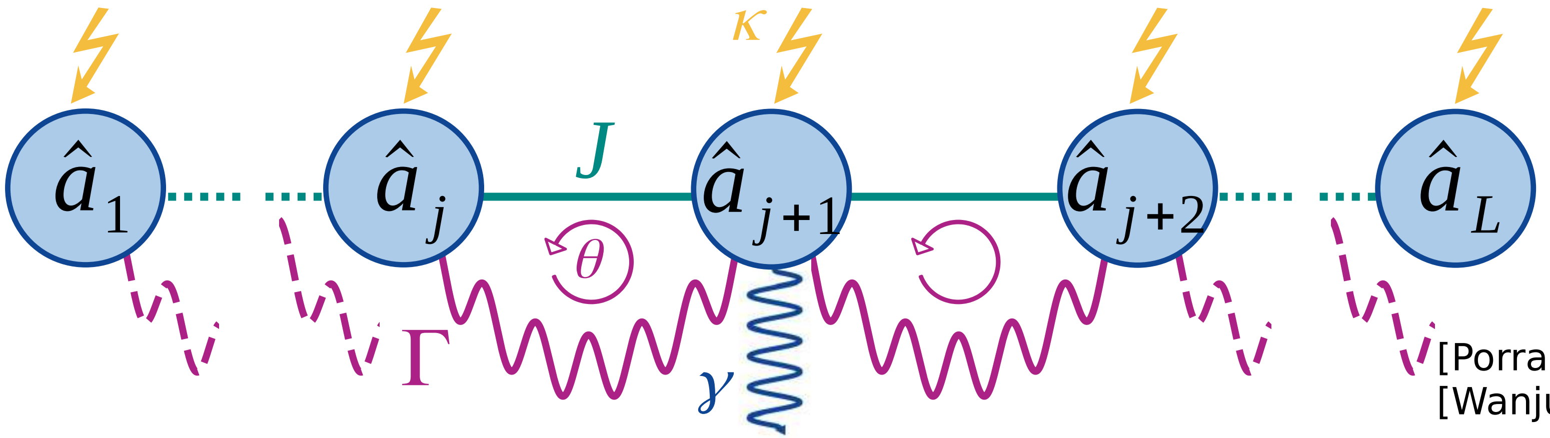

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[Metelmann and Clerk, PRX ‘15]

[Porras and Fernandez-Lorenzo, PRL ‘19]
[Wanjura, Brunelli and Nunnenkamp, Nat.Comm. ‘20]

$$\frac{d\rho}{dt} = -i[\hat{H},\rho] + \Gamma\sum_j \mathcal{D}[\hat{a}_j + e^{i\theta}\hat{a}_{j+1}]\rho + \gamma\sum_j \mathcal{D}[\hat{a}_j]\rho + \kappa\sum_j \mathcal{D}[\hat{a}_j^\dagger]\rho$$

Coherent evolution

Non-local dissipative coupling

Local loss

Local incoherent pump

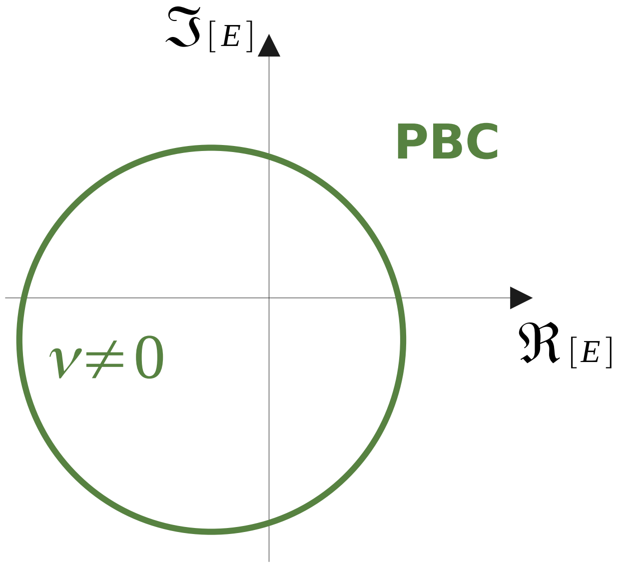
$$\langle \dot{\hat{a}}_j \rangle = \sum_\ell \boldsymbol{H}_{j\ell} \langle \hat{a}_\ell \rangle \qquad \boldsymbol{H} = \sum_j -\left(iJ + \frac{e^{i\theta}\Gamma}{2}\right)|j+1\rangle\langle j| - \left(iJ + \frac{e^{-i\theta}\Gamma}{2}\right)|j-1\rangle\langle j| + \frac{\kappa-\gamma-2\Gamma}{2}|j\rangle\langle j|$$

Non-reciprocal eq. of motion ⇨ **non-Hermitian** dynamical matrix

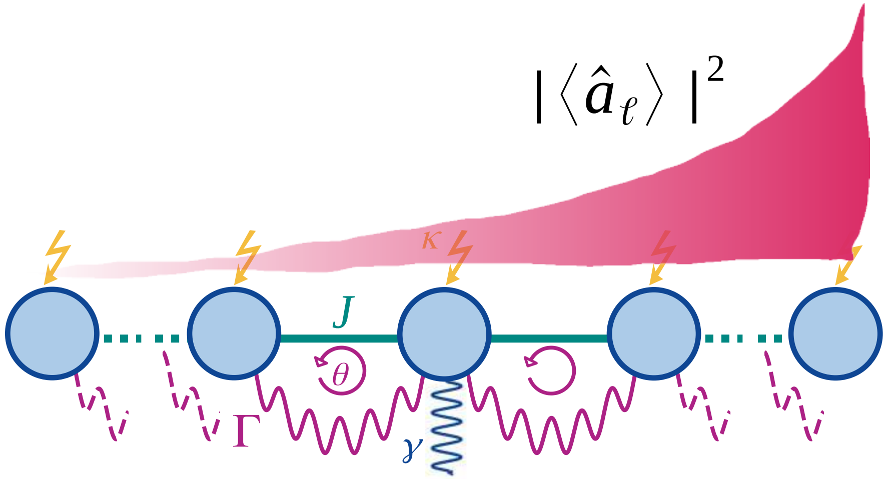
Non-Hermitian topology, amplification and NHSE

Non-reciprocity has dramatic consequences:

Non-Hermitian topology



Amplification



[Porras and Fernandez-Lorenzo, PRL ‘19]
[Wanjura, *et al.* Nat.Comm. ‘20]

Non-Hermitian skin effect

Localization of **all** eigenstates (in OBC)

[Lee, PRL ‘16]
[Yao and Wanf PRL ‘18]
...

Connection with **non-Hermitian** systems established thanks to **solvable** equations of motion

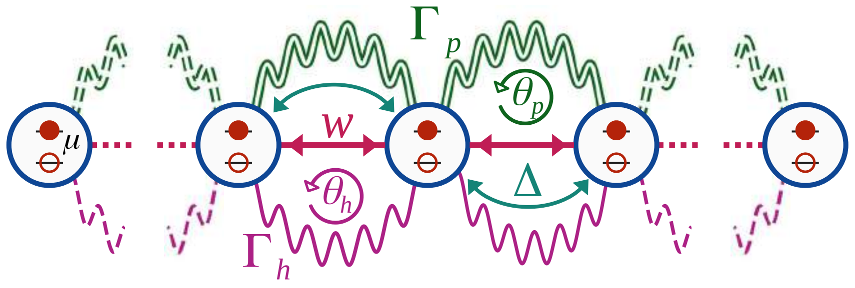
Non-reciprocal Kitaev chain

[**PB** and A. Nunnenkamp, arXiv '25]

Let's move to **Fermions!**

Kitaev chain: p-wave superconductor, hosting Majorana zero modes in the topological phase

$$\hat{H}=\sum_j -w\left(\hat{c}_j^\dagger \hat{c}_{j+1}+\mathcal{H}.c\right)+\left(\Delta\, \hat{c}_j^\dagger \hat{c}_{j+1}^\dagger+\mathcal{H}.c.\right)-\mu\, \hat{c}_j^\dagger \hat{c}_j$$



Engineered dissipation:

$$\boxed{\hat{L}_j^h=\sqrt{\Gamma_h}\left(\hat{c}_j+e^{i\theta_h}\hat{c}_{j+1}\right)} \qquad \boxed{\hat{L}_j^p=\sqrt{\Gamma_p}\left(\hat{c}_j+e^{i\theta_p}\hat{c}_{j+1}^\dagger\right)}$$

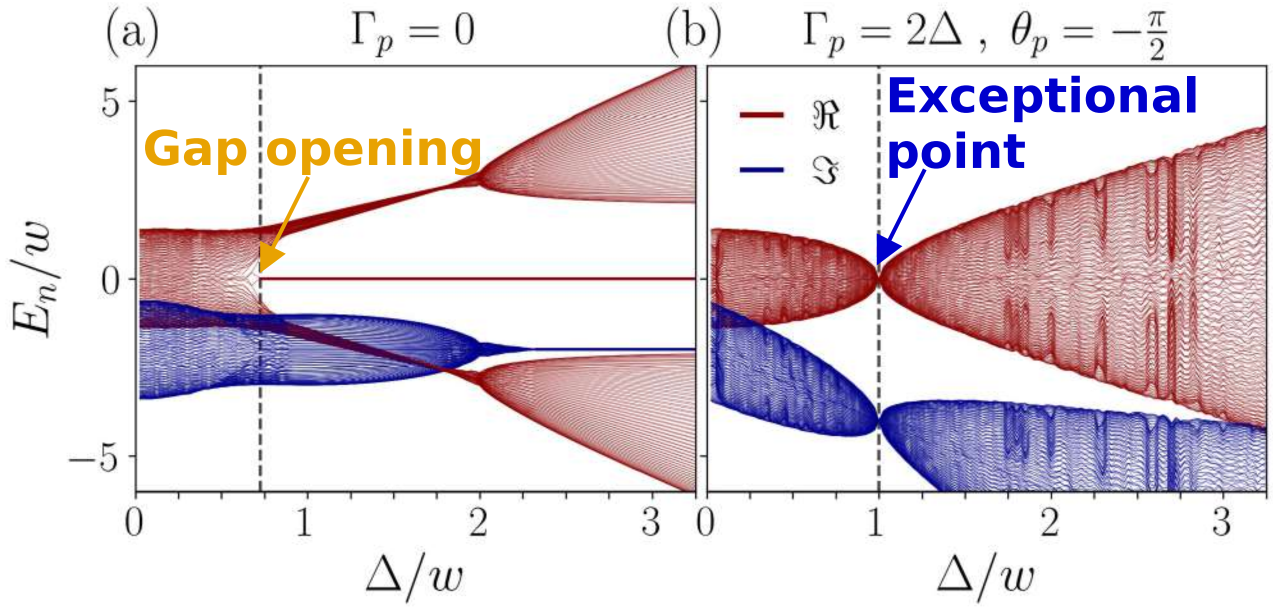
$$w_{R/L}=w-i\frac{\Gamma_h}{2}e^{\pm i\theta_h} \qquad \Delta_{R/L}=\Delta\mp i\frac{\Gamma_p}{2}e^{i\theta_p}$$

$$\mathcal{C}=\left(\begin{array}{c|c} \langle \hat{c}_\ell^\dagger \hat{c}_m\rangle & \langle \hat{c}_\ell^\dagger \hat{c}_m^\dagger\rangle \\ \hline -\langle \hat{c}_\ell \hat{c}_m\rangle & -\langle \hat{c}_\ell^\dagger \hat{c}_m\rangle \end{array}\right)$$

$$\Downarrow$$

$$i\dot{\mathcal{C}}=\left(\boldsymbol{H}\,\mathcal{C}-\mathcal{C}\,\boldsymbol{H}^\dagger\right)+\boldsymbol{F}$$

Reciprocal pairing **Non-reciprocal pairing**

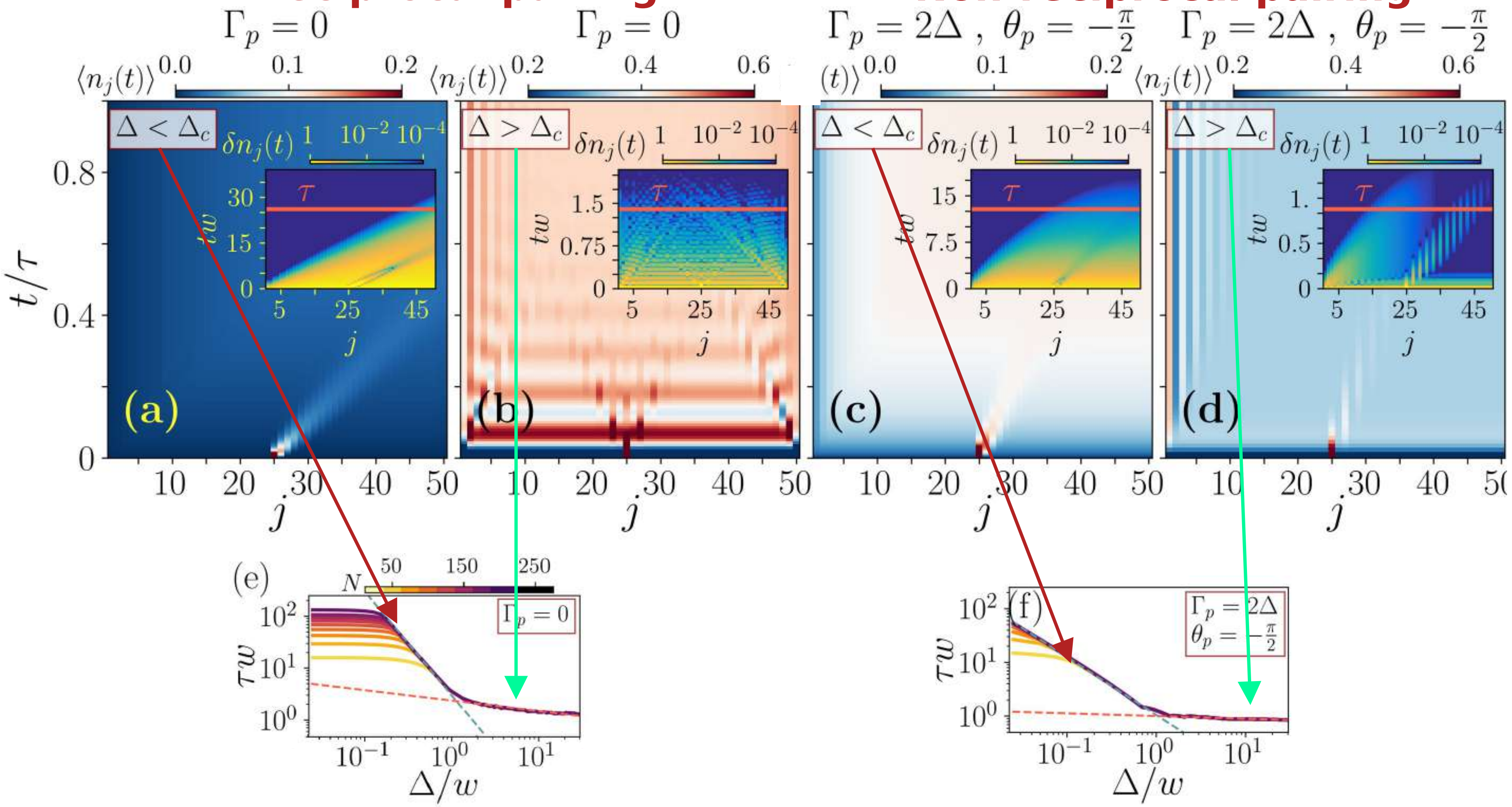


Phase transition in the dynamics

Pairing breaks non-reciprocity in the dynamics $\boxed{w_L = 0}$

Pairing term injects pairs of particles everywhere in the chain → competes with **non-reciprocity**

Reciprocal pairing **Non-reciprocal pairing**



$\Delta < \Delta_c$

- Directional dynamics
- Long relaxation times

$\tau \gg w$

$\Delta > \Delta_c$

- Density wave oscillations
- Fast relaxation

$\tau \approx O(w)$

Breakdown of non-reciprocity in the steady state

Pairing shapes steady state density distribution $\boxed{w_L = 0}$

Phase transition persists in the steady state

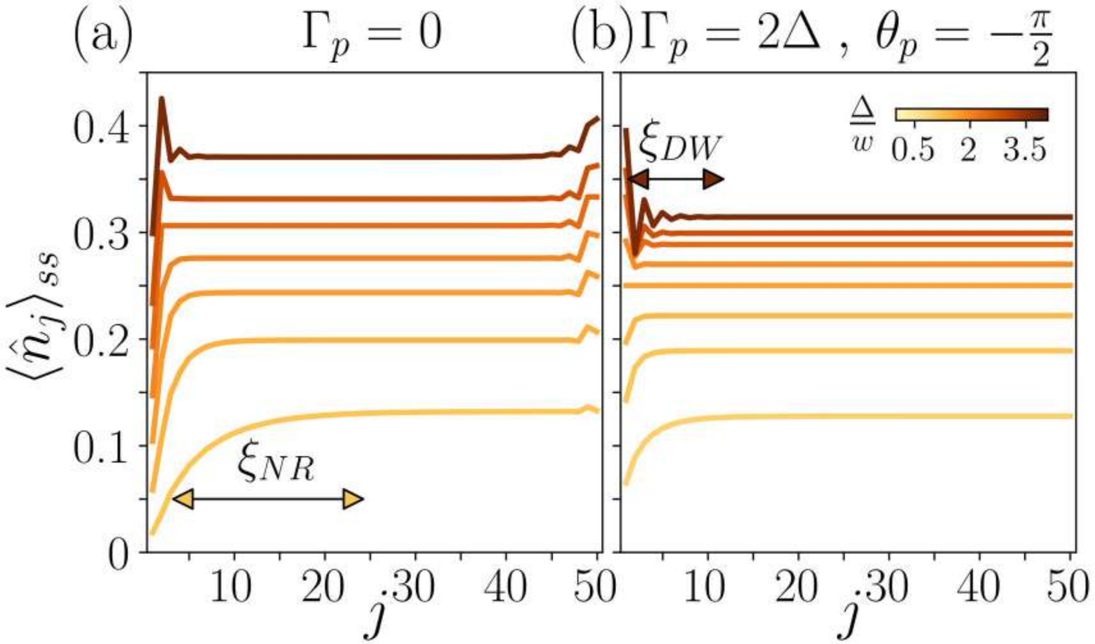
Non-reciprocal phase $\xrightarrow{\Delta}$ **Density wave phase**

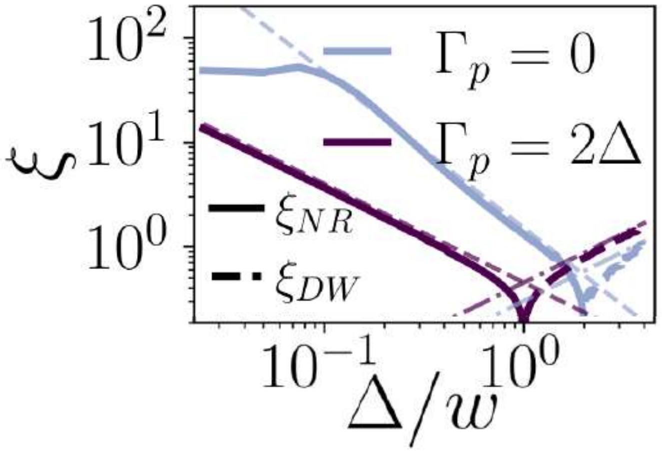
Non-reciprocal phase

- Inhomogeneous density
- Depletion close to the boundary
- ξ_{NR} characteristic lengthscale

Density wave phase

- Higher overall density
- Density wave close to boundaries
- ξ_{DW} characteristic lengthscale





Both lengthscales increase deep in the respective phases

$\xi_{NR} \propto \Delta^{-\alpha}$

$\xi_{DW} \propto \Delta$

They can be used to identify the different phases and the critical point

$$\Delta_c = w \ for\ \Gamma_p = 2\Delta \quad \Delta_c = 2w \ for\ \Gamma_p = 0$$

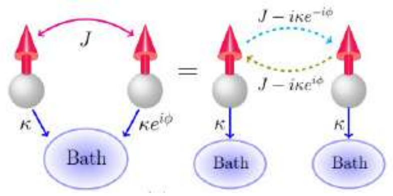
Interplay of interactions and non-reciprocity

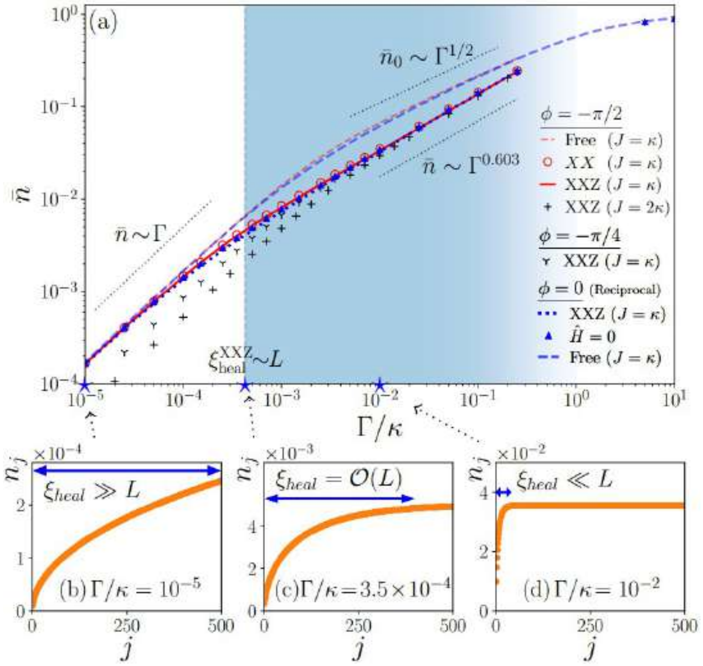
Interactions and non-reciprocity

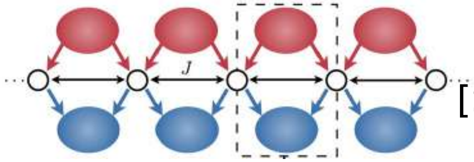
$$\hat{H} = \hat{H}_0 + \hat{H}_{int}$$

Equations of motion do **not** close! $\langle \dot{\hat{a}}_j \rangle = \sum_\ell \cancel{H_{j\ell}} \langle \hat{a}_\ell \rangle$

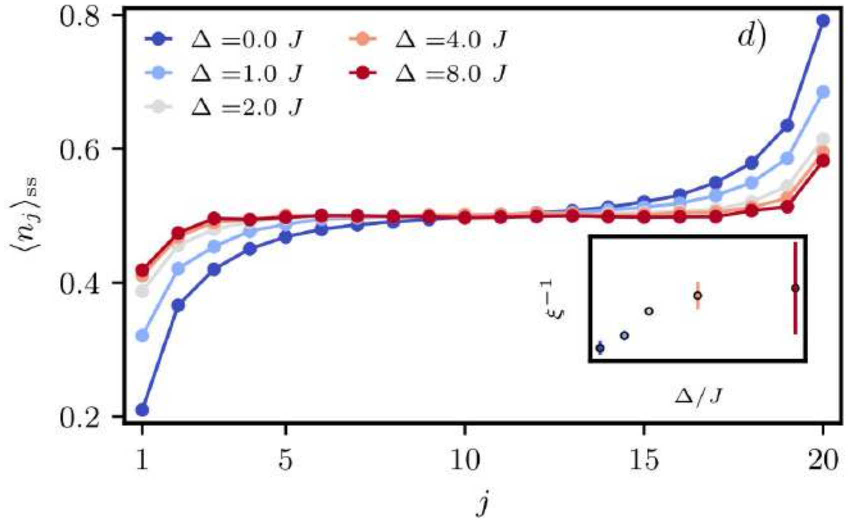
[Begg and Hanai, PRL '24]







[Soares, Brunelli and Schiro', arXiv '25]



XXZ chain:
Universality in the steady state
close to critical point
(Tensor network analysis)

Interacting fermions:
Steady state currents
Interaction-induced breakdown of
non-reciprocity

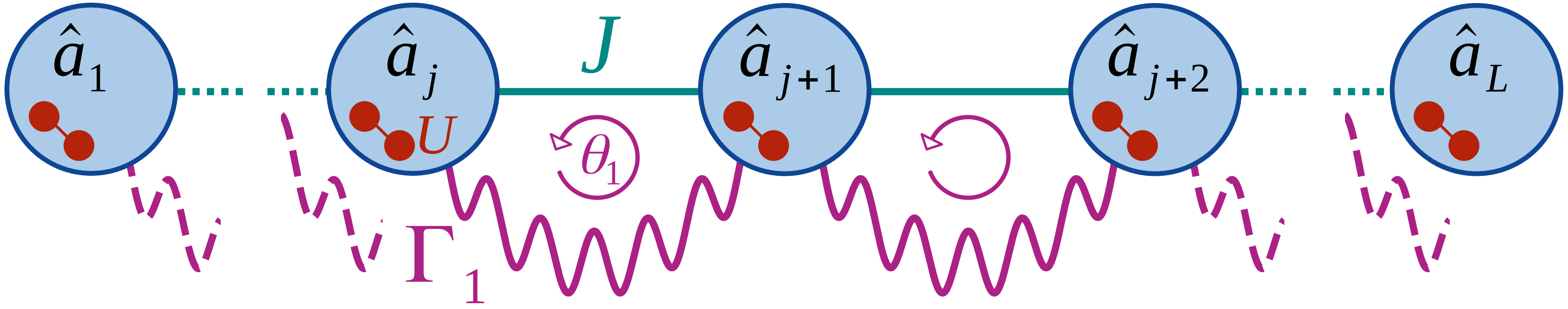
Bose-Hubbard model w/ engineered dissipation

Non-reciprocal interacting bosons

[**PB** and A. Nunnenkamp, PRA ‘24]

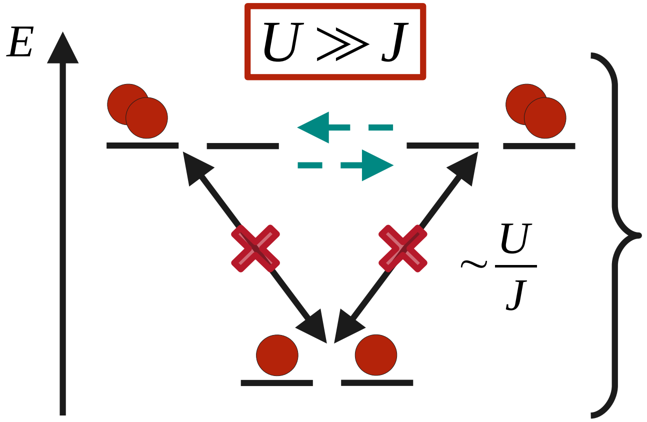
$$\hat{H} = J\sum_{j=1}^{L-1}\left(\hat{a}_j^\dagger \hat{a}_{j+1} + \mathcal{H.c.}\right) + \frac{U}{2}\sum_{j=1}^{L}\left(\hat{a}_j^\dagger\right)^2 \hat{a}_j^2$$

$$\hat{L}_j = \sqrt{\Gamma_1}\left(\hat{a}_j + e^{i\theta_1}\hat{a}_{j+1}\right)$$



$$\frac{d\rho}{dt} = -i[\hat{H},\rho] + \sum_j \hat{L}_j \rho \hat{L}_j^\dagger - \frac{1}{2}\{\hat{L}_j^\dagger \hat{L}_j, \rho\}$$

$$\boxed{\rho(t) = e^{\mathscr{L}t}\rho_0 \qquad |\psi_0\rangle = \frac{1}{\sqrt{2}}\left(\hat{a}_{L/2}^\dagger\right)^2 |vac\rangle}$$



Effective Hamiltonian:

$$\hat{H}_{eff} \propto \frac{J^2}{U}\sum_{j=1}^{L-1}\left(\hat{a}_j^\dagger\right)^2 \hat{a}_{j+1}^2 + \mathcal{H.c.}$$

Doublons effectively move as a single particle with larger mass

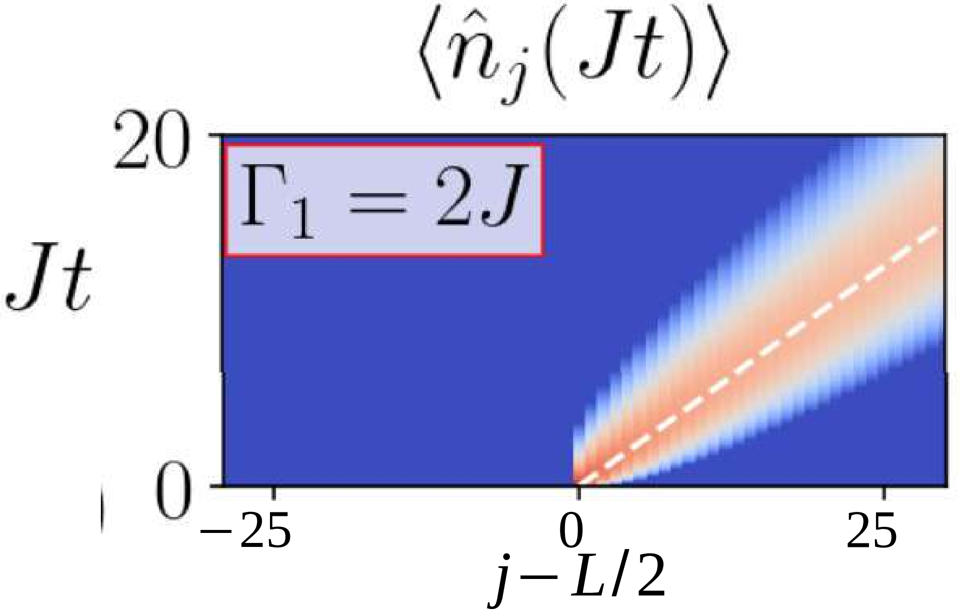
[Winkler *et al.* Nature ‘06]

Non-reciprocal doublons

Strong interaction effects $\boxed{U \gg J}$ and reservoir engineering $\boxed{\hat{L}_j = \sqrt{\Gamma_1}(\hat{a}_j + e^{i\theta_1}\hat{a}_{j+1})}$

Full non-reciprocity at $\boxed{\Gamma_1 = 2J}$, $\theta_1 = \pm\frac{\pi}{2}$

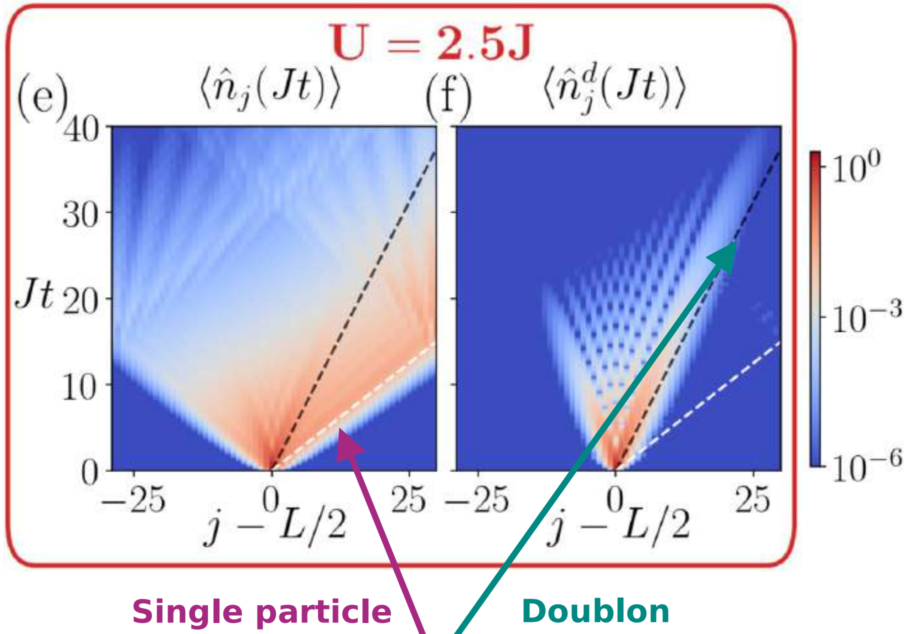
Strong loss



No qualitative difference between different values of U

Reducing the loss rate $\boxed{\Gamma_1 = 0.1J}$

Weak loss



$$\hat{n}_j^d = \frac{1}{2}(\hat{a}_j^\dagger)^2 \hat{a}_j^2$$

Two different **non-reciprocal lightcones!**

Non-reciprocal doublons

Strong interaction effects $\boxed{U \gg J}$ and reservoir engineering $\boxed{\hat{L}_j = \sqrt{\Gamma_1}(\hat{a}_j + e^{i\theta_1}\hat{a}_{j+1})}$

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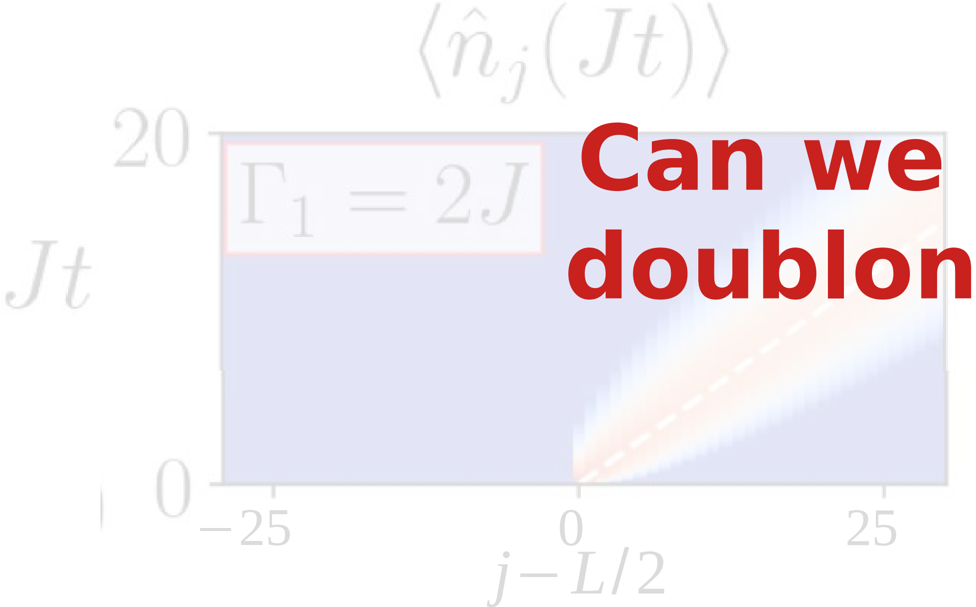

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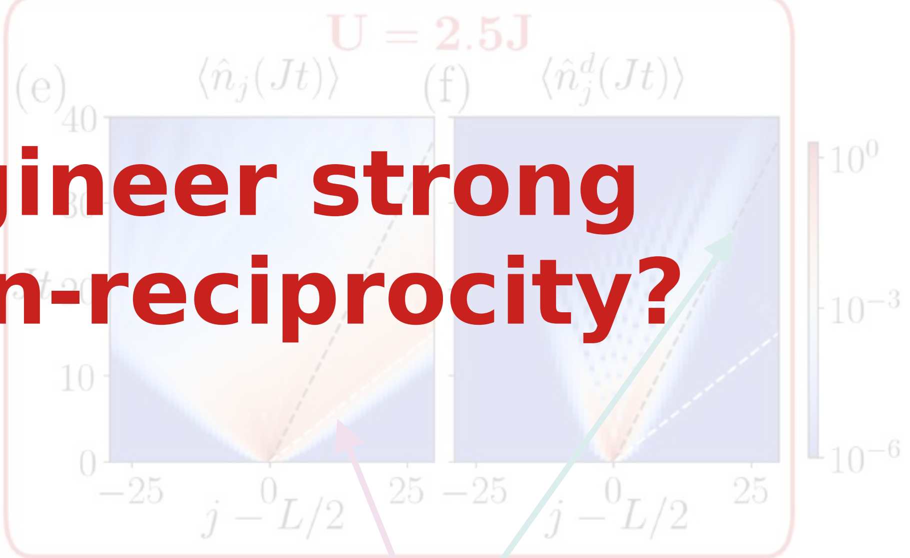


$$\hat{n}^d_j = \frac{1}{2}(\hat{a}^\dagger_j)^2 \hat{a}^2_j$$

Single particle

Doublon

Two different **non-reciprocal lightcones!**

Can we engineer strong doublon non-reciprocity?

Stable doublon strong non-reciprocity

Now dissipator makes doublons **non-reciprocal!**

$$\hat{H} = -J\sum_{j=1}^{L-1}\left(\hat{a}_j^{\dagger}\hat{a}_{j+1} + \mathcal{H.c.}\right) + \frac{U}{2}\sum_{j=1}^{L}\left(\hat{a}_j^{\dagger}\right)^2\hat{a}_j^2$$

$$\xrightarrow{U \gg J} \hat{H}_{eff} \propto \frac{J^2}{U}\sum_{j=1}^{L-1}\left(\hat{a}_j^{\dagger}\right)^2\hat{a}_{j+1}^2 + \mathcal{H.c.}$$

$$\boxed{\hat{L}_j = \sqrt{\Gamma_2}\left(\hat{a}_j^2 + e^{i\theta_2}\hat{a}_{j+1}^2\right)}$$

Stable doublon strong non-reciprocity

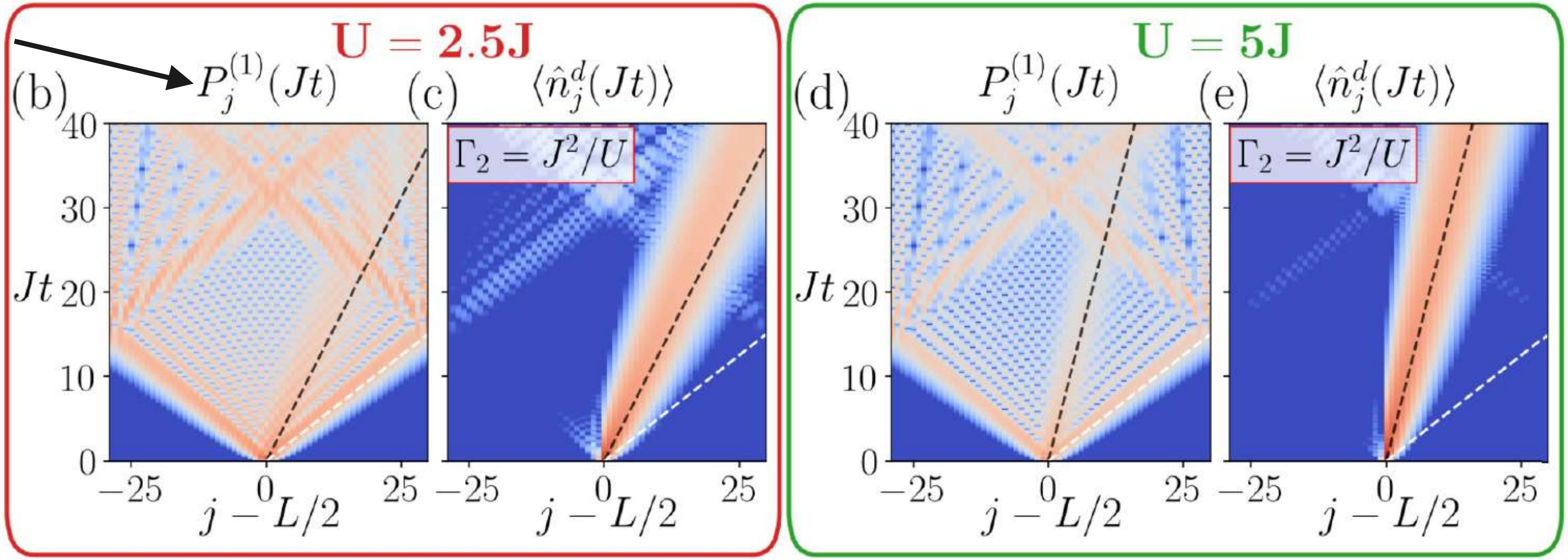
Now dissipator makes doublons **non-reciprocal!**

$$\hat{H}=-J\sum_{j=1}^{L-1}\left(\hat{a}_j^{\dagger}\hat{a}_{j+1}+\mathcal{H.c.}\right)+\frac{U}{2}\sum_{j=1}^{L}\left(\hat{a}_j^{\dagger}\right)^2\hat{a}_j^2 \xrightarrow{U\gg J} \hat{H}_{eff}\propto\frac{J^2}{U}\sum_{j=1}^{L-1}\left(\hat{a}_j^{\dagger}\right)^2\hat{a}_{j+1}^2+\mathcal{H.c.}$$

$$\boxed{\hat{L}_j=\sqrt{\Gamma_2}\left(\hat{a}_j^2+e^{i\theta_2}\hat{a}_{j+1}^2\right)}$$

Single-particle population

$$P_j^{(1)}=\hat{n}_j-2\hat{n}_j^d$$



Only **doublon** moves **non-reciprocally!**

Outline

Alberto Biella
(CNR-INO & UniTn)

[**PB** and A. Biella SciPost Phys. ‘26]

Gianluca Frazzei
(UniTn)

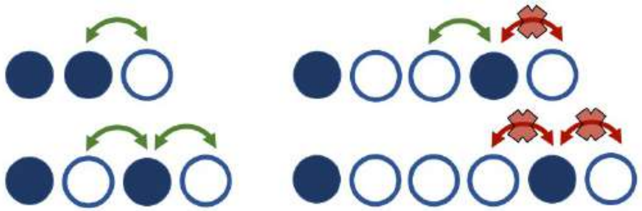
[ArXiv 26??]

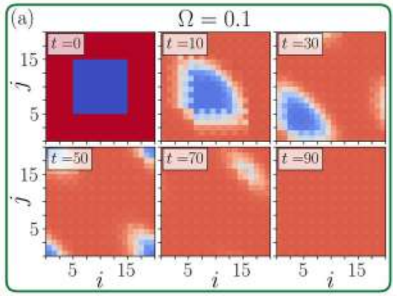
Igor Lesanovsky
(Uni Tubingen)

Juan P. Garrahan
(Uni Nottingham)

<u>Kinetically constrained models</u>

- Dynamical rules
- Bistability and dynamical heterogeneity
- Slow dynamics





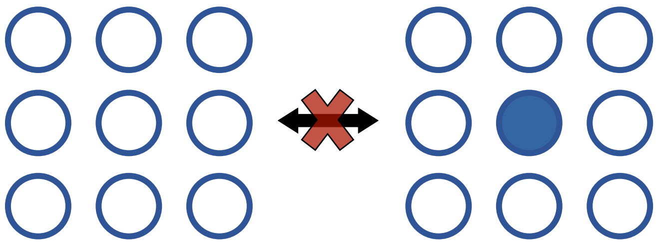
Kinetically constrained models

Dynamics have to satisfy a certain set of rules

Classic

Description of **glasses** and slow relaxation
[Ritort and Sollich Adv. Phys. ‘03]

Example: spin-facilitated Ising models
[Fredrickson and Andersen PRL ‘84]

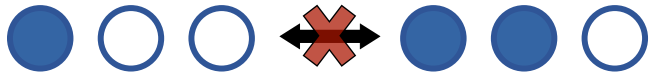


Quantum

Generalization to Hamiltonian dynamics
[Van Horsten *et al.* PRB ‘15]

Often appearing as effective description of strongly interacting models

Example: PXP Hamiltonian
[Turner *et al.* Nat. Phys. ‘18]



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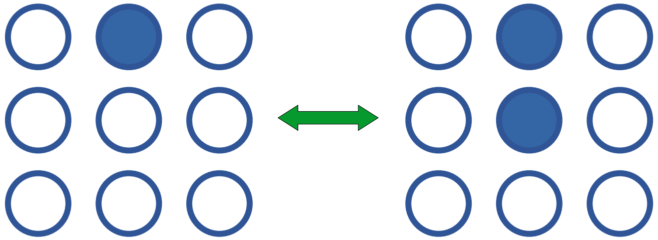

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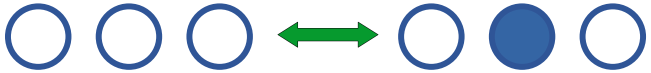
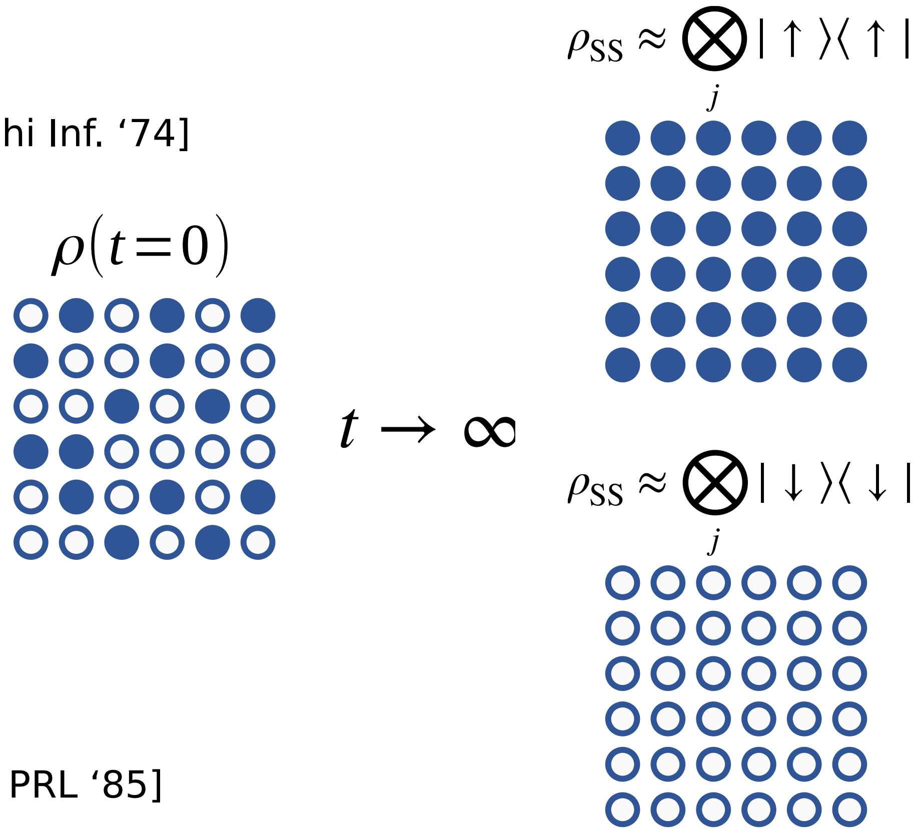


Very **rich dynamics!**

Bistability in classical KCMs

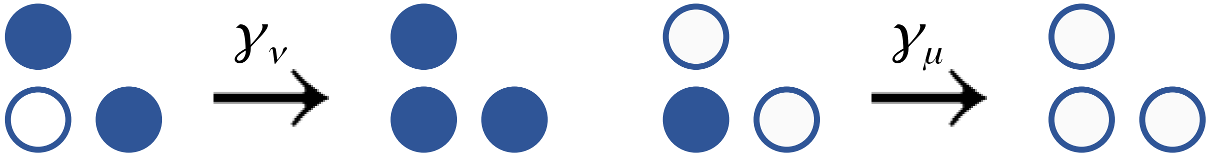
Classical North-East-Center (NEC) model

[Toom, Probl. Peredachi Inf. ‘74]



- 2d lattice majority vote process

Majority moves

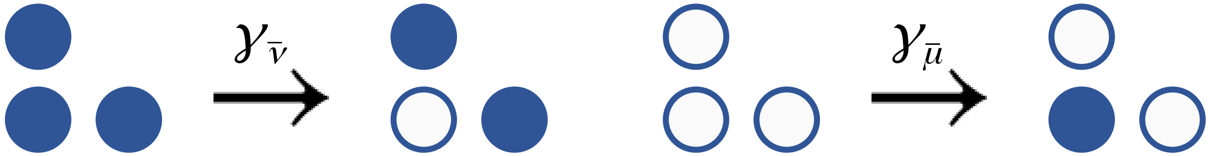


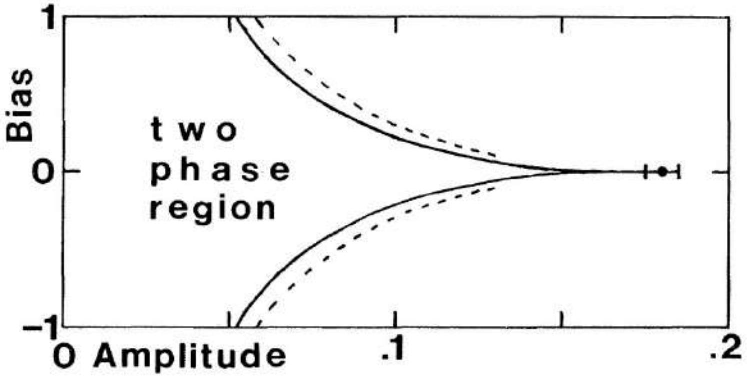
- Plaquette shape breaks reflection symmetry
- Finite bistable phase wrt noise amplitude and bias

$$T = \gamma_{\bar{v}} + \gamma_{\bar{\mu}} \qquad h = \frac{\gamma_{\bar{v}} - \gamma_{\bar{\mu}}}{T}$$

[Bennett *et al.*, PRL ‘85]

Classical noise





Quantum NEC model

[Kazemi and Weimer, arXiv ‘21]

Robustness with respect to coherent quantum dynamics (?)

Jump operators implementing majority moves

$$L_{j,\nu} = \sqrt{\gamma_\nu}\sigma_j^+ \mathbb{P}^{\uparrow}_{j} \qquad L_{j,\bar{\nu}} = \sqrt{\gamma_{\bar{\nu}}}\sigma_j^- \mathbb{P}^{\uparrow}_{j}$$

$$L_{j,\mu} = \sqrt{\gamma_\mu}\sigma_j^- \mathbb{P}^{\downarrow}_{j} \qquad L_{j,\bar{\mu}} = \sqrt{\gamma_{\bar{\mu}}}\sigma_j^+ \mathbb{P}^{\downarrow}_{j}$$

$$\dot{\rho} = -\imath[H,\rho] + \sum_{j,\alpha}\left(L_{j,\alpha}\rho L^\dagger_{j,\alpha} - \frac{1}{2}\{L^\dagger_{j,\alpha}L_{j,\alpha};\rho\}\right)$$

Many-body 2d open quantum system 🤯

Cluster mean field 💡

Hamiltonian, both **kinetically constrained** and not

$$H^{\mathrm{PXP}} = \sum_j \Omega_1 \mathbb{P}^{\downarrow}_{j-e_x}\mathbb{P}^{\downarrow}_{j-e_y}\sigma^x_j\mathbb{P}^{\downarrow}_{j+e_x}\mathbb{P}^{\downarrow}_{j+e_y} + \Omega_2 \mathbb{P}^{\uparrow}_{j-e_x}\mathbb{P}^{\uparrow}_{j-e_y}\sigma^x_j\mathbb{P}^{\uparrow}_{j+e_x}\mathbb{P}^{\uparrow}_{j+e_y}$$

$$H^{\mathrm{PXP}} = \sum_j \Omega_1 \mathbb{P}^{\downarrow}_{j+e_y}\sigma^x_j\mathbb{P}^{\downarrow}_{j+e_x} + \Omega_2 \mathbb{P}^{\uparrow}_{j+e_y}\sigma^x_j\mathbb{P}^{\uparrow}_{j+e_x}$$

$$H_{\mathrm{X}} = \Omega\sum_j \sigma^x_j$$

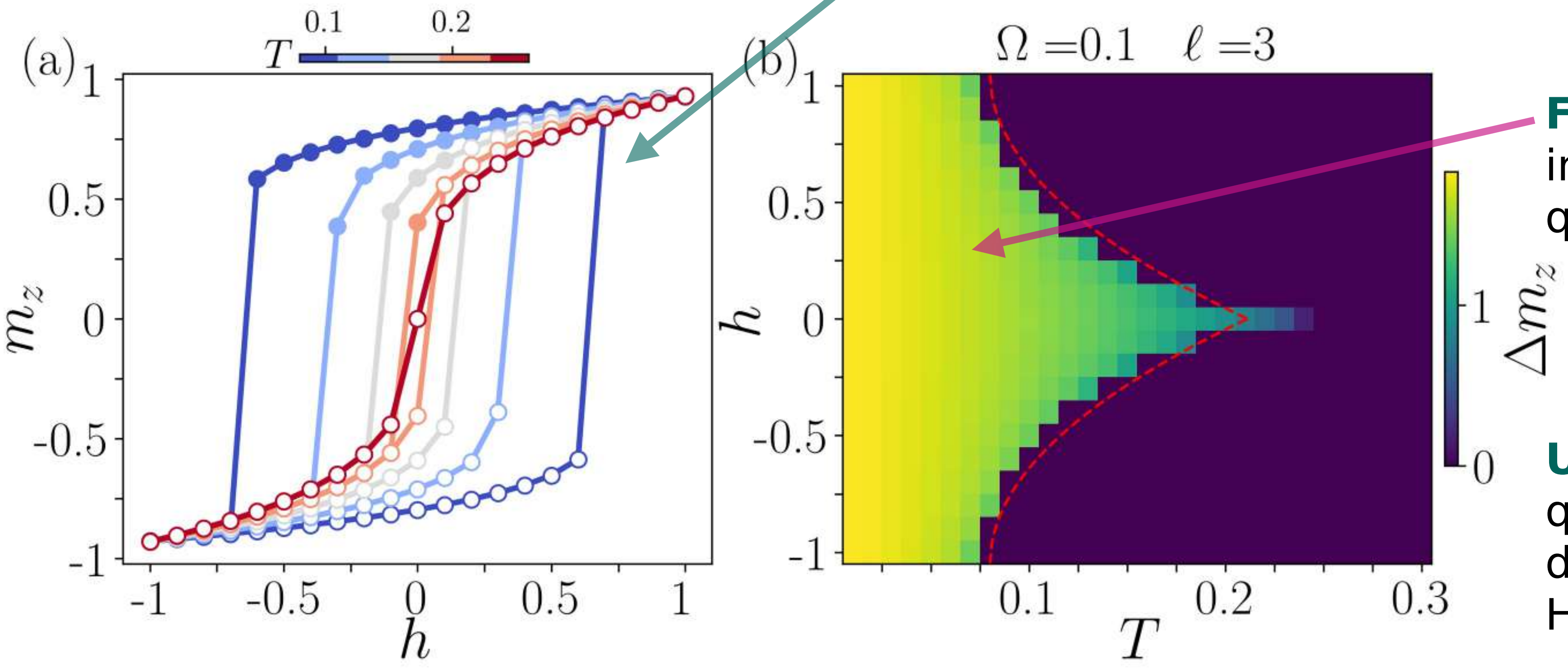
Bistability in the quantum NEC model

Steady state phase diagram

$$\lim_{t\to\infty} \rho(t),\ \dot{\rho}=0$$

Characterize magnetization through hysteresis cycle

$$\boxed{\Delta m_z = |m_z^{(f)} - m_z^{(b)}|}$$



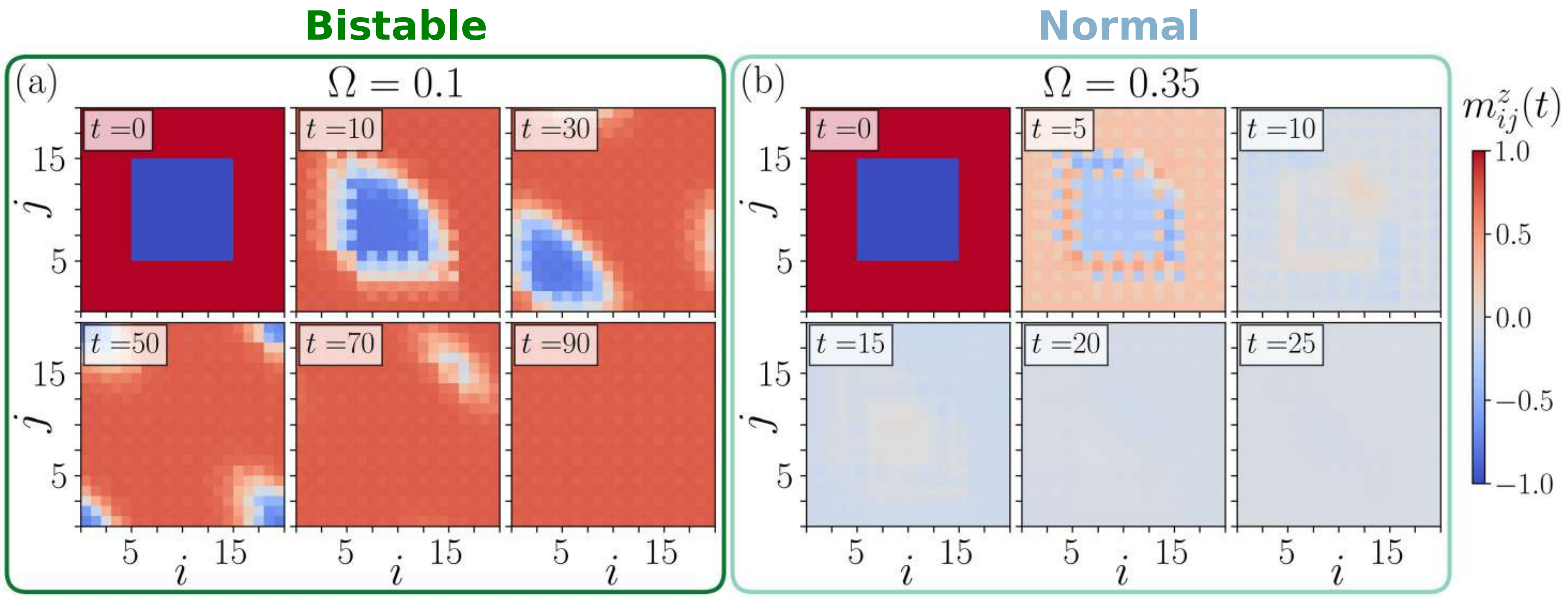
Finite bistable region in presence of quantum fluctuations!

Universal phase diagram qualitative feature do not depend on the specific Hamiltonian

What about dynamics?

Dynamics of minority islands

Bistability is rooted in the absorption of minority islands



Relaxation time $\propto$ island size: constant absorption velocity

Relaxation time is ~ constant: absorption velocity $\propto$ island size

QNEC

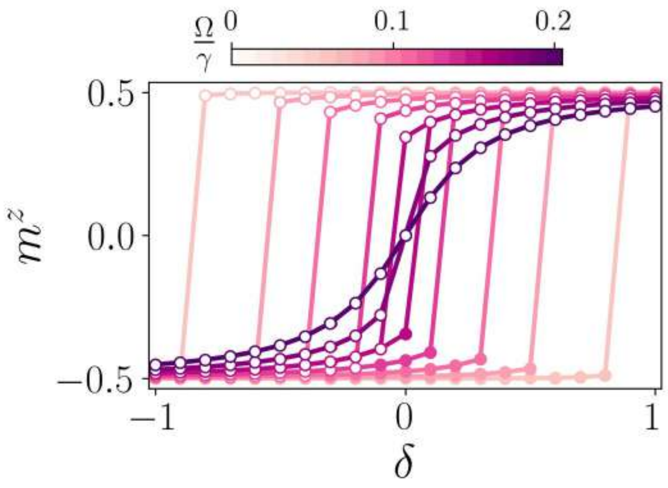
$$\frac{\mathrm{d}r}{\mathrm{d}t} = -C + Bh + DT + E\Omega$$

No **critical island size!**

Minority islands expand indefinitely

1d dissipative far-east

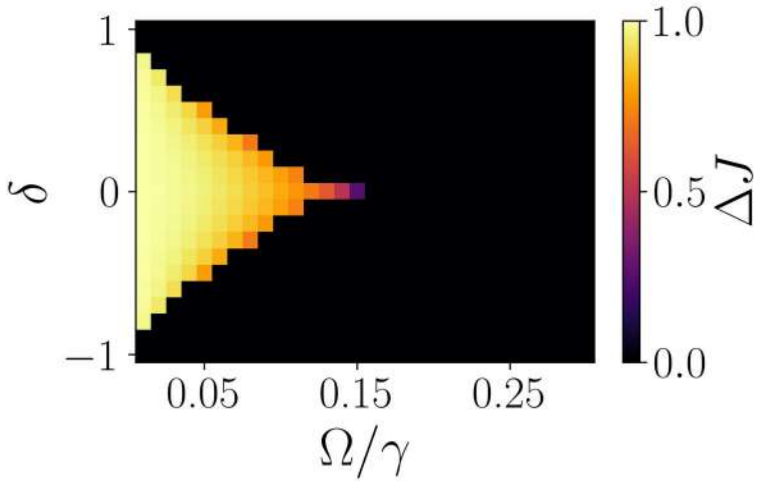
What about 1d systems?

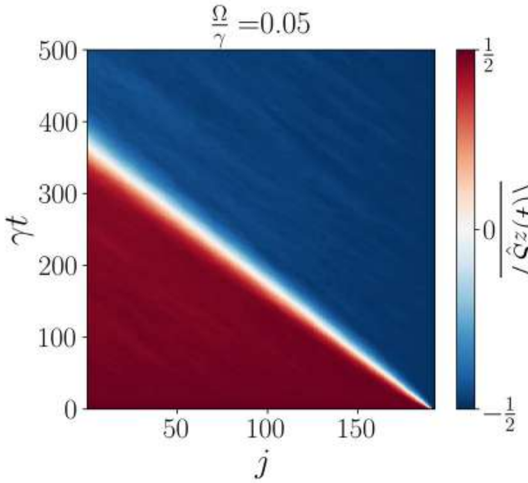


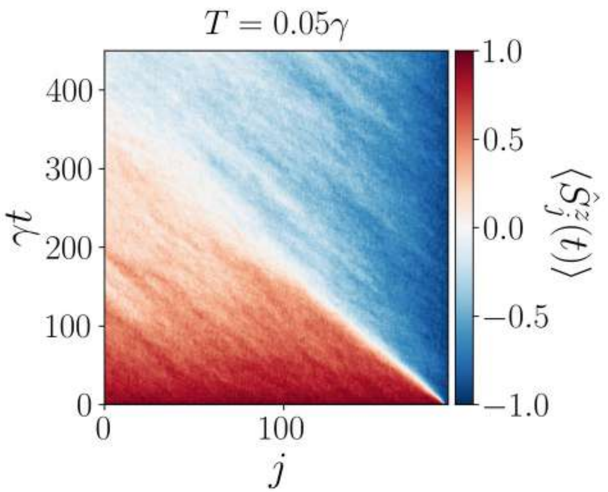
Visit Gianluca's poster!

Similar phenomenology, **but**

- Unstable to classical fluctuations
- Large scale exact dynamics
- Entanglement entropy signatures of dynamical heterogeneity



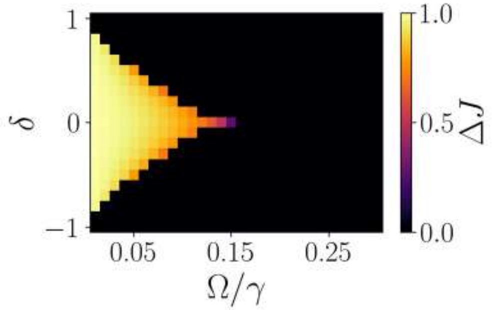


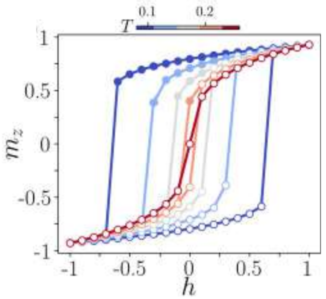


Summary

Kinetically constrained models

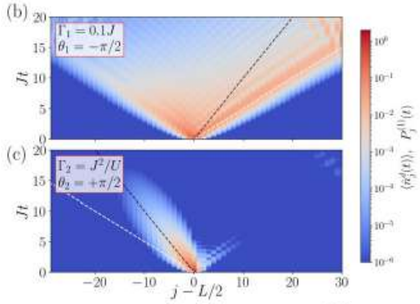
- Dissipative KCMs
- Bistability and non-ergodic dynamics
  in quantum NEC
- Long relaxation and dynamical heterogeneity

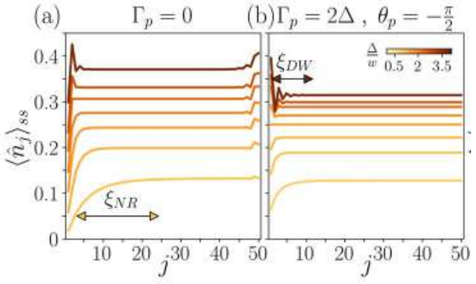


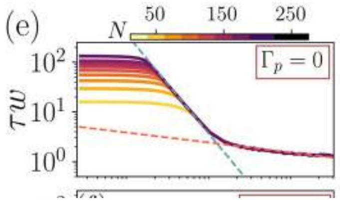


Non-reciprocal systems

- Non-reciprocity at the **Lindblad** level in interacting system
- Combining reservoir engineering with effective Hamiltonian
  strong doublon non-reciprocity
- Combining the two dissipators leads to interesting situations
- Pairing as an **exactly tractable** kind of interaction
- Phase transition and new density wave phase



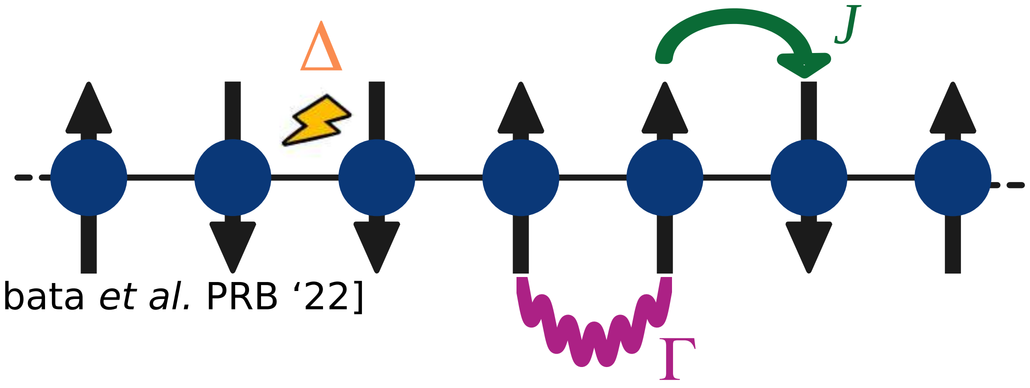


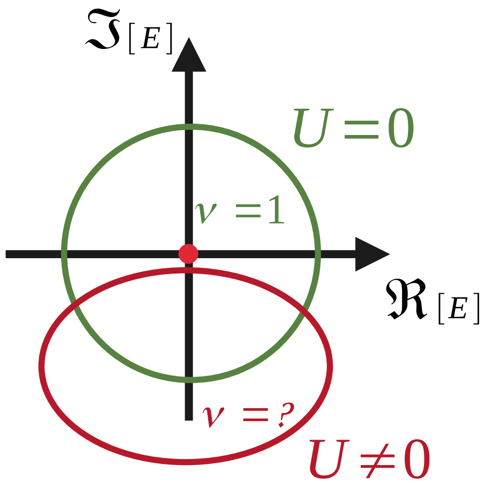


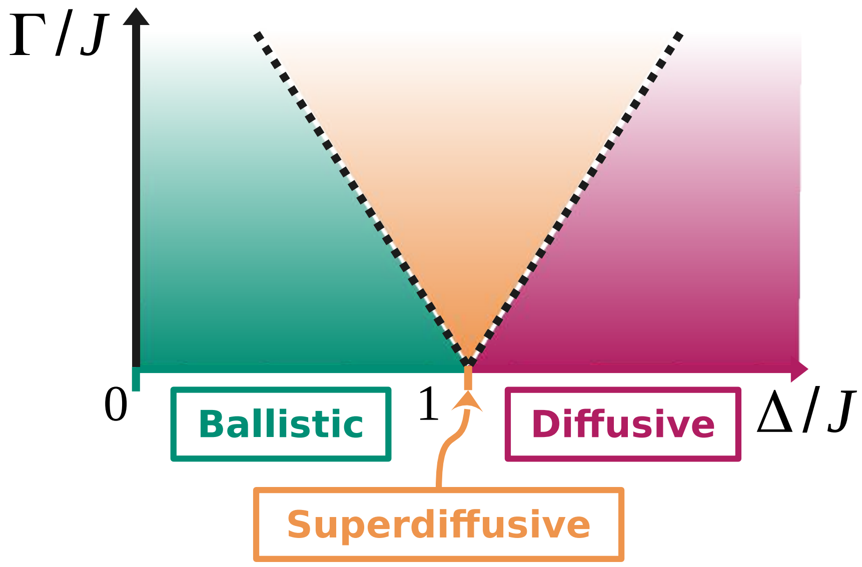
Outlook

Future directions

- Experimental implementation of kinetic constraints (Work in progress…)
- Non-reciprocal spin systems [Begg and Hanai, PRL ‘24]
- Non-Hermitian topology in interacting models [Kawabata *et al.* PRB ‘22]
- Transport in NR models (Work in progress…) [Minoguchi *et al.* arXiv ‘23] [Garbe *et al.* SciPost ‘24]
- Interplay of kinetic constraints and non-reciprocity
- Non-reciprocity in long-range systems [Nadonly *et al.* PRX ‘24]







Supplementary Material

Engineering unidirectional doublons

Introduce a new dissipator → stabilize doublon non-reciprocity

$\boxed{U \gg J}$

$$\hat{H}_{eff} \propto \frac{J^2}{U} \sum_{j=1}^{L-1} (\hat{a}_j^\dagger)^2 \hat{a}_{j+1}^2 + \mathcal{H.c.}$$

⇒

$$\boxed{\hat{L}_j = \sqrt{\Gamma_2}\left(\hat{a}_j^2 + e^{i\theta_2} \hat{a}_{j+1}^2\right)}$$

$\xrightarrow[\text{particle sector}]{\text{Within 2}}$

$$i \frac{d\langle(\hat{a}_\ell^\dagger)^2 \hat{a}_m^2\rangle}{dt} = \sum_j \boldsymbol{H}_{m,j} \langle(\hat{a}_\ell^\dagger)^2 \hat{a}_j^2\rangle - \boldsymbol{H}_{j,\ell}^\dagger \langle(\hat{a}_j^\dagger)^2 \hat{a}_m^2\rangle$$

At strong interactions the equations of motion are effectively solvable in **doublon space!**

$$\boxed{\boldsymbol{H} = \sum_j J_R |j+1\rangle_2 \langle j|_2 + J_L |j-1\rangle_2 \langle j|_2 - 2i\Gamma_2 |j\rangle_2 \langle j|_2}$$

Non-Hermitian dynamical matrix, supporting non-Hermitian skin effect of doublons

Tensor-networks: a powerful ansatz

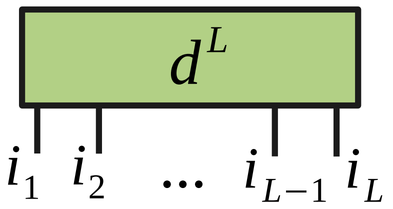
Tensor-networks are a clever **compression** of the wavefunction

Reviews:
[Schollwöck, Ann. Phys. ‘11]
[Orus, Ann. Phys. ‘14]
[Montangero, Springer ‘18]

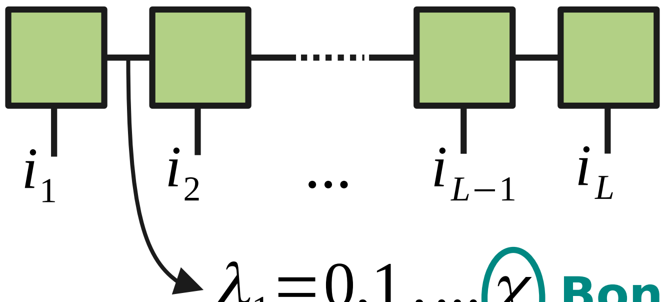
$$|\psi\rangle = \sum_{\{i_1, i_2, \dots, i_L\}} C_{i_1, i_2, \dots, i_L} |i_1, i_2, \dots, i_L\rangle \qquad i_n = 0, 1, .. d$$

e.g. d=2 for spin-1/2

$C_{i_1, i_2, \dots, i_L}$



Singular-value decompositions →



$\lambda_1 = 0, 1, \dots \chi$ **Bond dimension**

If state is not too complex (i.e. weakly entangled)

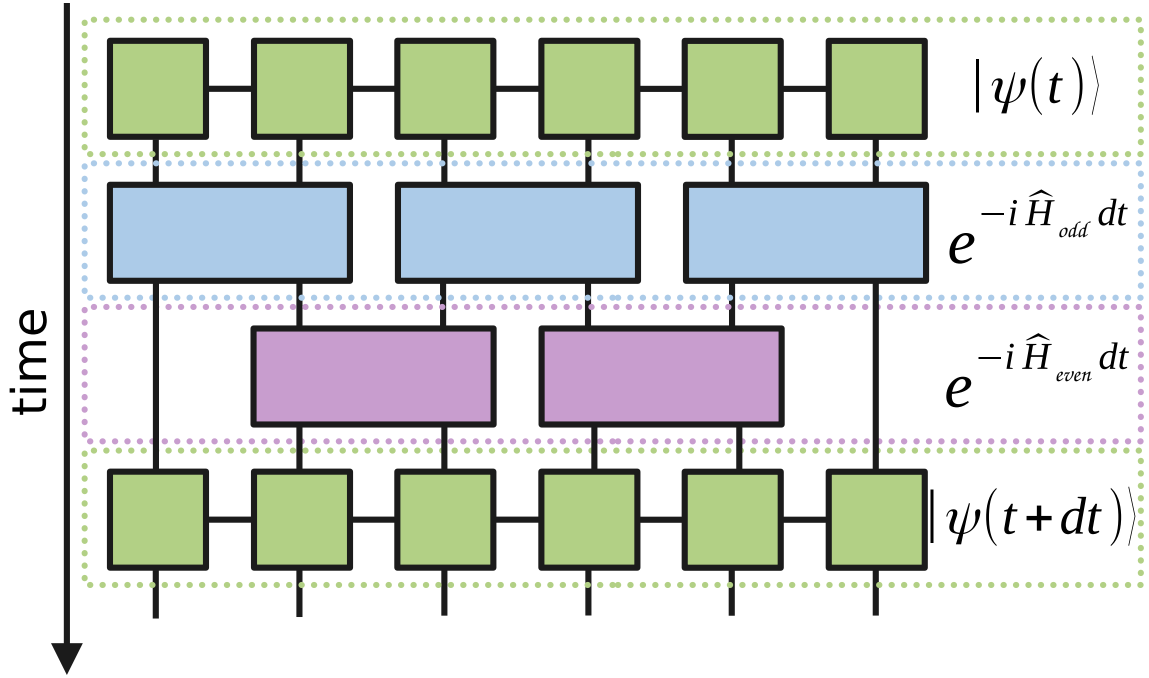
$d^L \rightarrow dL\chi$

Finite bond dimension captures
relevant physics

Performing time evolution

Efficient time evolution: time-evolving-block-decimation (TEBD) [Vidal, PRL '03]

$$|\psi(t)\rangle = e^{-i\hat{H}t}|\psi_0\rangle \qquad \hat{H} = \hat{H}_{even} + \hat{H}_{odd} \xrightarrow[\text{decomposition}]{\text{Trotter}} e^{-i\hat{H}dt} \approx e^{-i\hat{H}_{even}dt} e^{-i\hat{H}_{odd}dt} + O(dt^2)$$



Caveat

Error sources:

1) Finite timestep dt

2) Singular value decomposition at each timestep

Hence

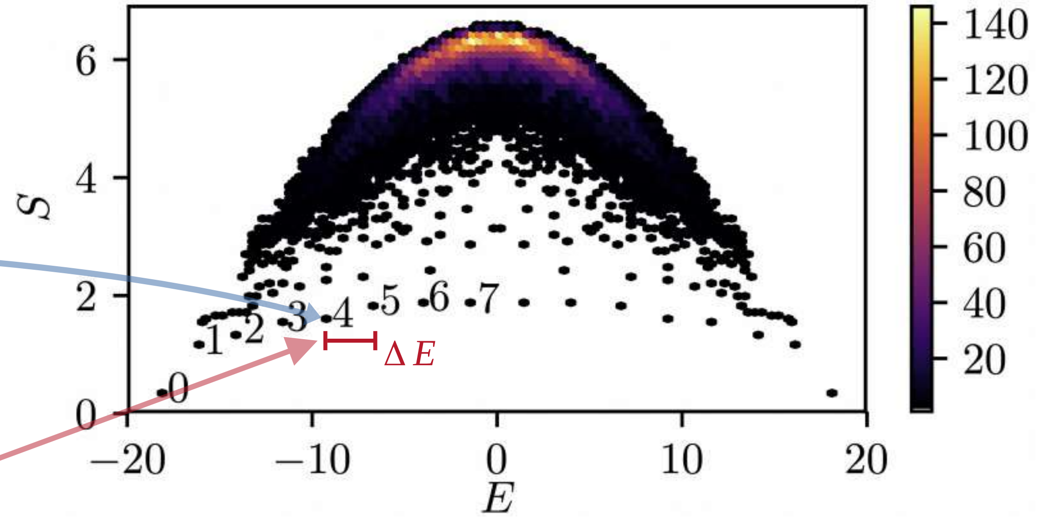
1) Small dt + higher order Trotter

2) Large bond dimension

Quantum many-body scars

Ergodicity breaking in few eigenstates

[Turner *et al.*, PRB ‘18]



[Turner *et al.*, Nat. Phys. ‘18]

PXP Hamiltonian $\widehat{H}_{PXP} = \sum_j P^{\downarrow}_{j-1} \sigma^x_j P^{\downarrow}_{j+1}$

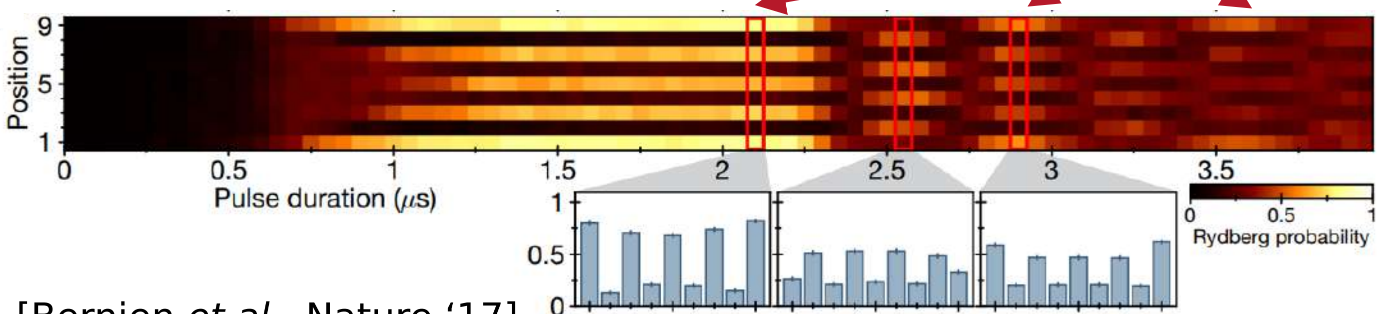
describing **Rydberg blockade**

Scars:Vanishing fraction of **non-thermal** eigenstates

Affect the dynamics of experimentally relevant states

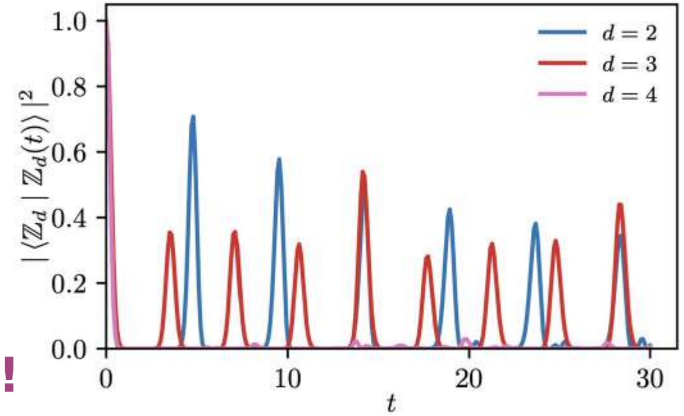
Néel state: $|\mathbb{Z}_2\rangle = |\bullet\circ\bullet\circ\ldots\rangle$

Initial configuration revives at a certain frequency



[Bernien *et al.*, Nature ‘17]

Fidelity revivals



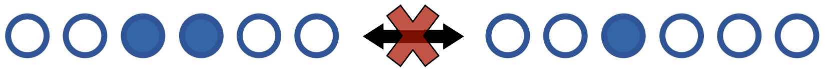
PXP model is kinetically constrained!

Hilbert space fragmentation

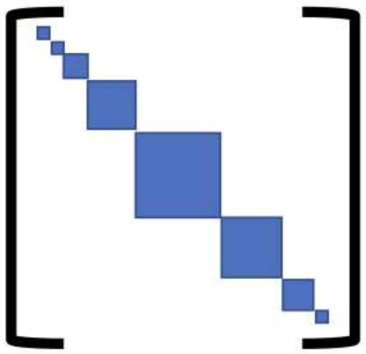
A different kind of ergodicity breaking

Kinetic constraints impose additional **conservation laws**

$$\widehat{H}_{PXP} = \sum_j P^{\downarrow}_{j-1} \sigma^x_j P^{\downarrow}_{j+1}$$



IF
Emergent conserved quantities define **exponentially many** diconnected sectors



THEN

Hilbert space fragmentation

[Sala *et al*., PRX '20]
[Khemani, Hermele and Nandkishore PRB '20]
…

[Moudgalya and Motrunich, PRX '22]

Classical
In product states basis

Quantum
In entangled states basis

Note that subsectors of H can be of **all** sizes: (O(1), O(L), O(exp(L))

and

may still have **thermal** behavior

Anomalous transport

Non-integrable KCMs are expected to be diffusive, but...

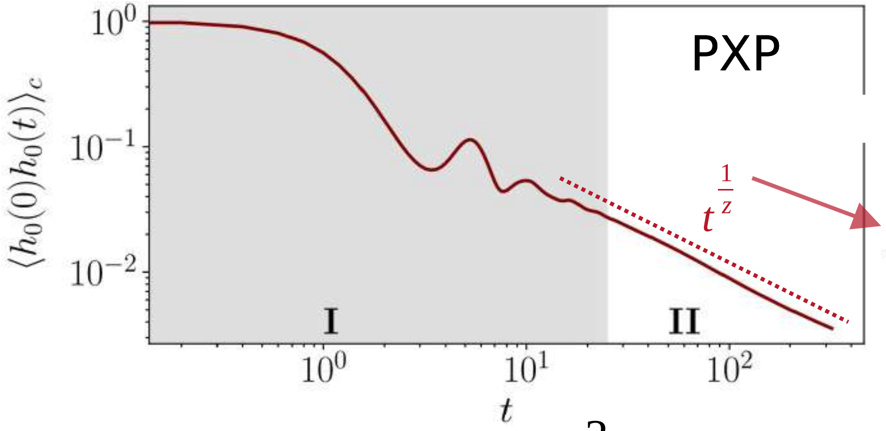
Within **thermal** subsectors, system is chaotic generically expected to have diffusive behavior

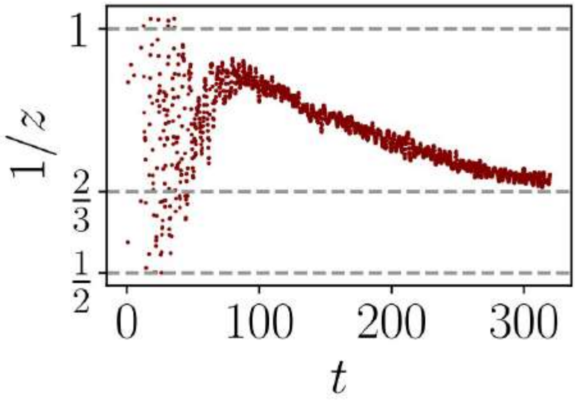
$$\Rightarrow \langle \hat{O}(0)\hat{O}(t)\rangle \propto \frac{1}{\sqrt{t}}$$

$z=1$: Ballistic
$1<z<2$: Superdiffusive
$z=2$: Diffusive
$z>2$: Subdiffusive

However

[Ljubotina *et al.*, PRX ‘23]



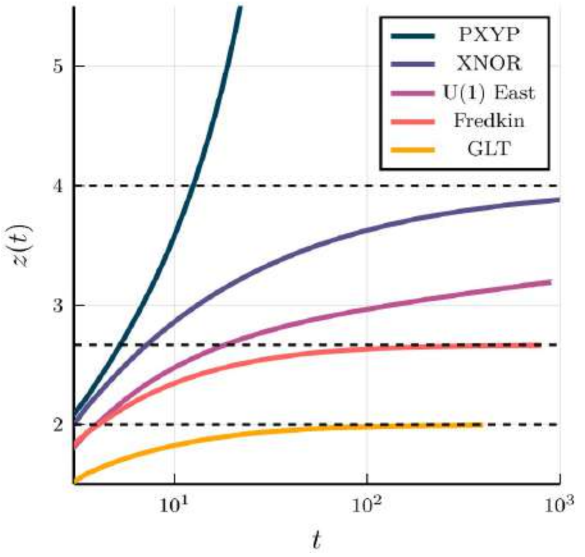


PXP: superdiffusive $z \rightarrow \frac{3}{2}$

Perhaps due to vicinity to integrable point?

[Singh *et al.*, PRL ‘21]

Subdiffusion When mixing different subsectors



Anomalous transport is ubiquitous in KCMs!

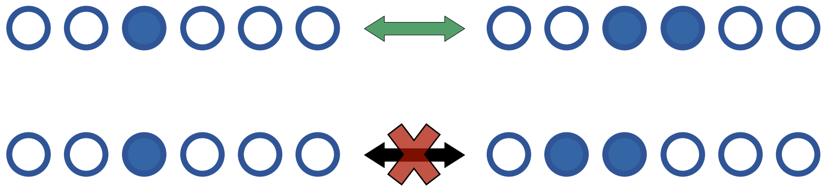
Chiral kinetically constrained models

Non-inversion-symmetric constraints

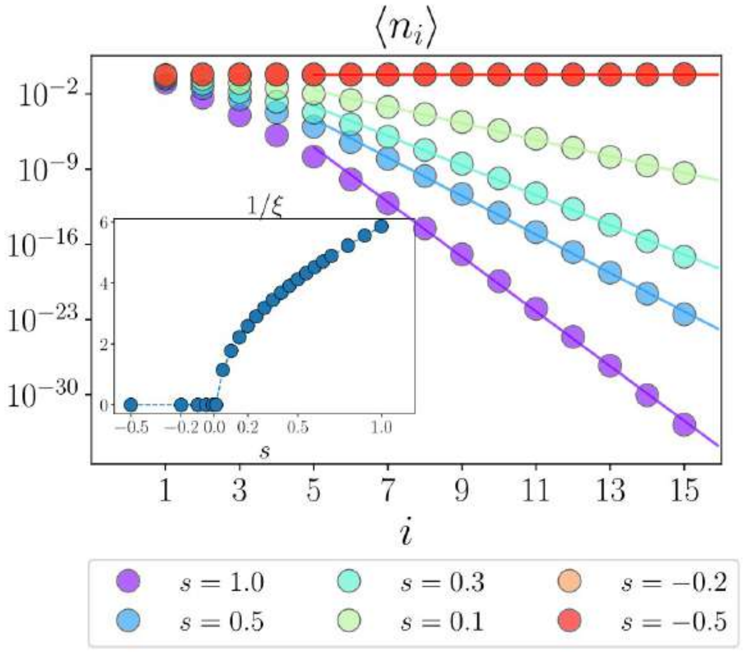
Quantum East model $\widehat{H} = \sum_j e^{-s} P_j^{\uparrow} \sigma_{j+1}^{x}$

Possible implementation with Rydberg atoms

[Valencia-Tortora *et al.*, PRL ‘24]

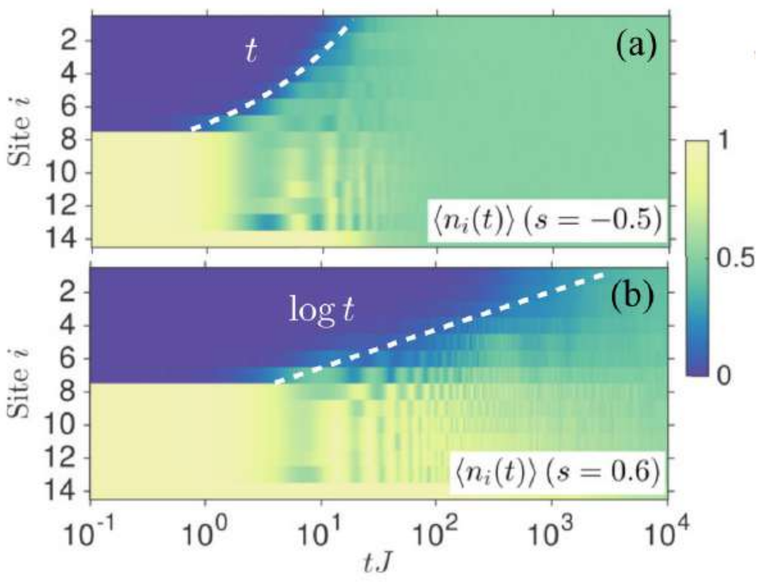


Localization in the Ground state



[Pancotti *et al.*, PRX Quantum ‘22]

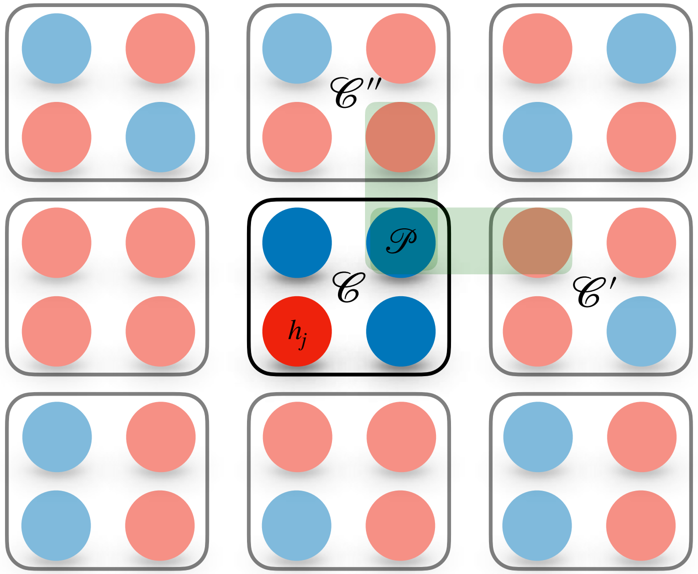
Slow Dynamics



[Van Horssen, Levi and Garrahan, PRB ‘15]

Interesting dynamical features, yet **no fragmentation**

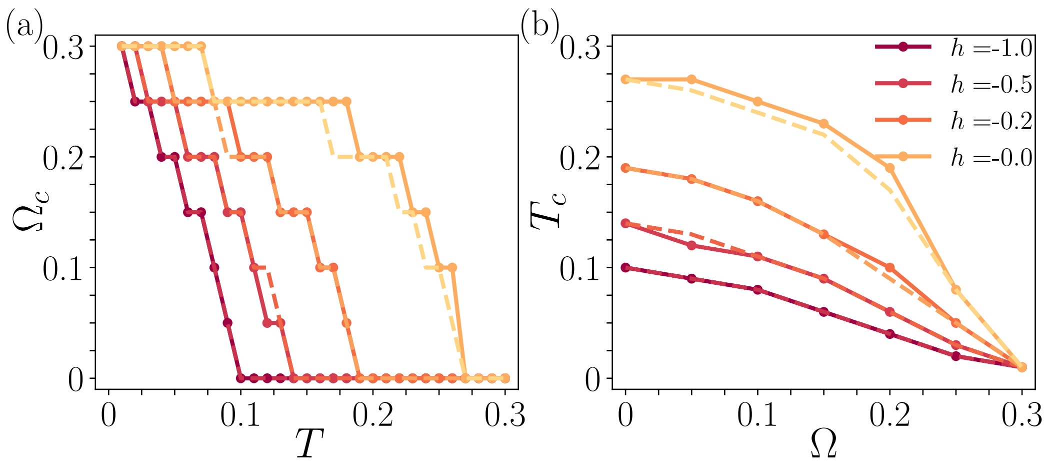
Cluster mean field



Cluster mean-field approach:

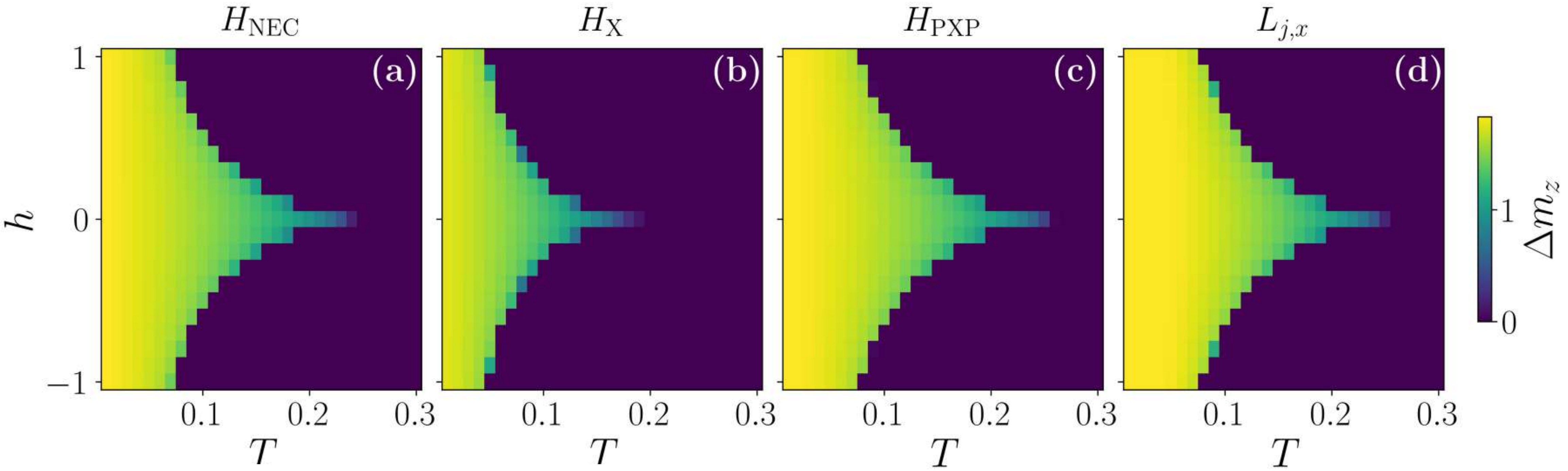
- Split the lattice in clusters of size ℓ
- Solve Lindblad **exactly** within the cluster
- Decouple operators with support on neighboring clusters
- Use their expectation value as time-dependent field/rate

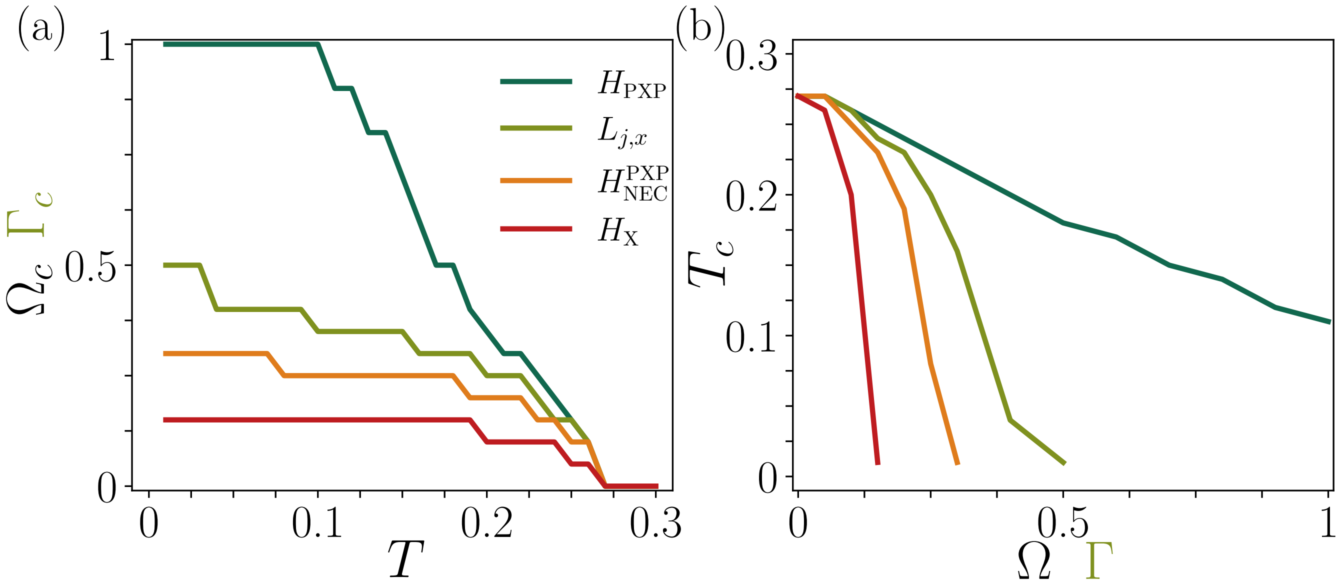
Evolve until steady state is reached



Comparison of different cluster sizes suggest **convergence** wrt cluster mean-field approximation

Universality wrt Hamiltonian





Phase boundaries for different Hamiltonians suggest a strong relation between **kinetic constraints and bistability**

Adding unconstrained dephasing along x, we show that bistability in the NEC model is stable also against experimental imperfections

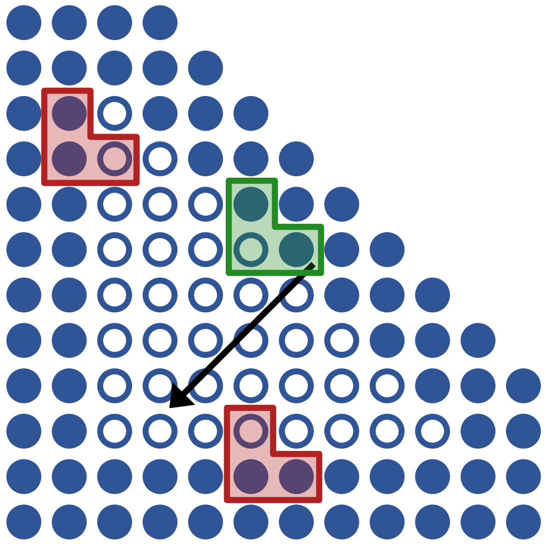
Analysis of island velocity

Define island velocity as the ratio of island size to relaxation timescale

Bistable phase:
velocity is constant with island size and decreases in magnitude as we approach the transition in T and Ω

Normal phase:
velocity increases with island size and increases in magnitude as T and Ω grow

Intuitive picture of island absorption



Surface tension:
free energy difference between configurations before and after flipping a spin

strong orientation dependence:

- On vertical and horizontal interfaces flipping the active spin does not change the free energy
- On the diagonal, instead, it reduces the free energy

This mechanism does not depend on the island size, therefore the velocity is constant!

Non-reciprocal repulsively bound pairs

Strong interaction effects $\boxed{U \gg J}$ and reservoir engineering $\boxed{\hat{L}_j = \sqrt{\Gamma_1}\left(\hat{a}_j + e^{i\theta_1}\hat{a}_{j+1}\right)}$

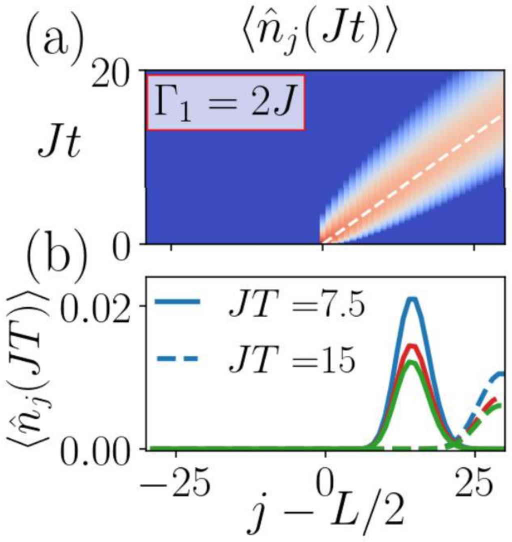
EOMs	
Single particle	Two particles
$\mathcal{J}_L = \left(J - i\frac{e^{i\theta_1}\Gamma_1}{2}\right)$	$\mathcal{J}_L^{(d)} \approx \frac{\mathcal{J}_L^2}{U + i\Gamma_1}$
$\mathcal{J}_R = \left(J - i\frac{e^{-i\theta_1}\Gamma_1}{2}\right)$	$\mathcal{J}_R^{(d)} \approx \frac{\mathcal{J}_R^2}{U + i\Gamma_1}$

Doublons **inherit** non-reciprocity!

However, full non-reciprocity at

$$\boxed{\Gamma_1 = 2J}, \quad \theta_1 = \pm\frac{\pi}{2}$$

Strong loss



Due to fast bosons loss, the system becomes quickly effectively non-interacting

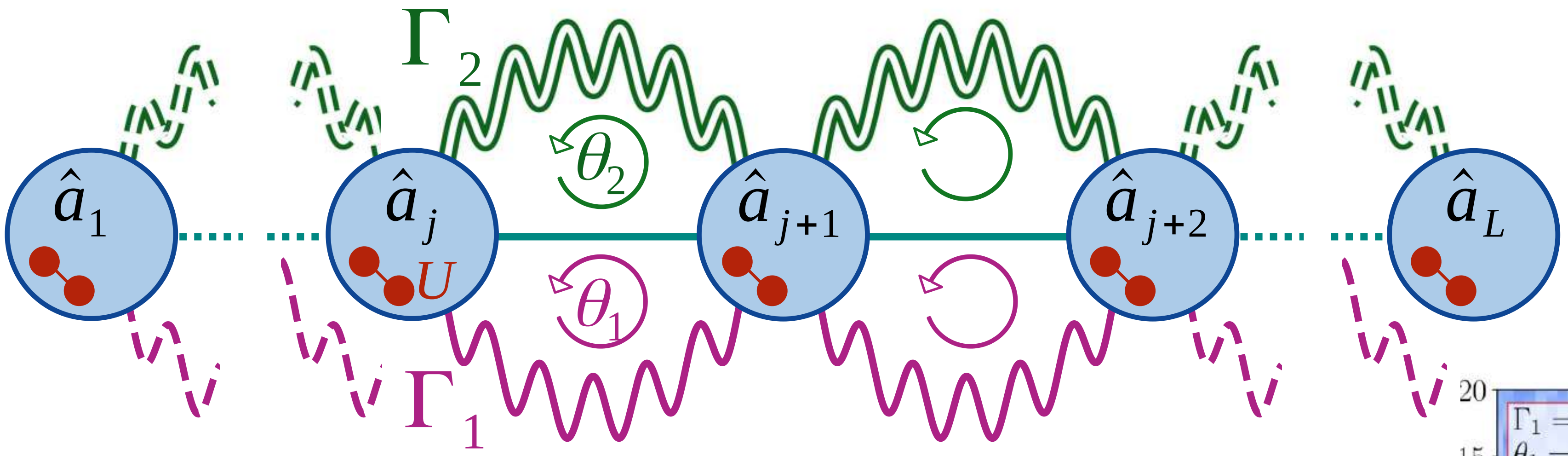
No qualitative difference between different values of U

Combining single-particle and doublon NR

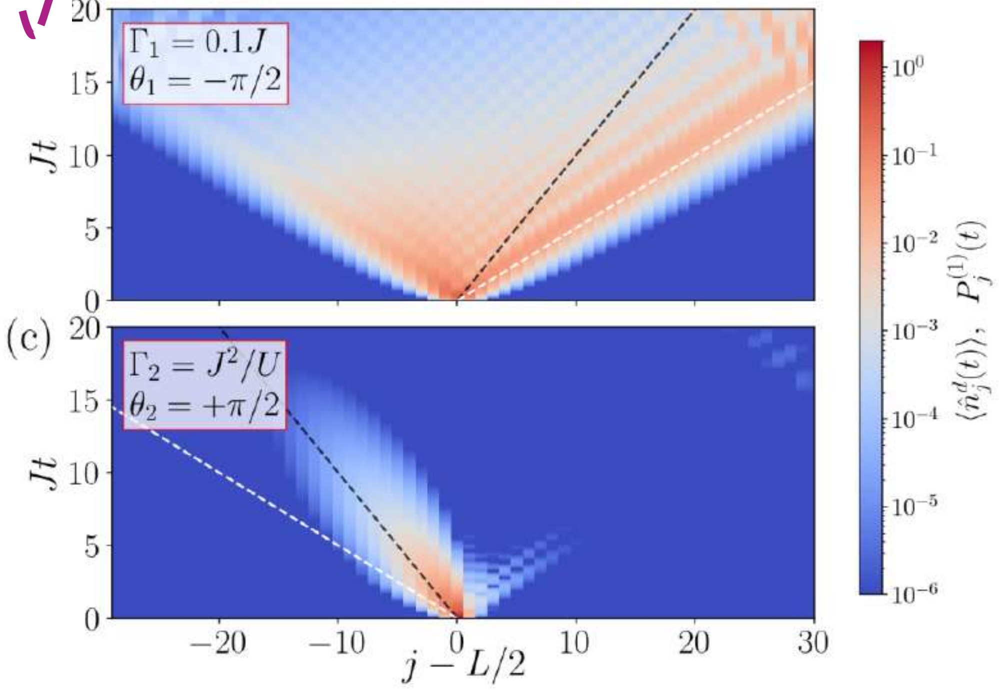
Interesting **playground** for non-reciprocal **dynamics**

$$\boxed{\hat{L}_j = \sqrt{\Gamma_1}\left(\hat{a}_j + e^{i\theta_1}\hat{a}_{j+1}\right)}$$

$$\boxed{\hat{L}_j = \sqrt{\Gamma_2}\left(\hat{a}_j^2 + e^{i\theta_2}\hat{a}_{j+1}^2\right)}$$



Single-particles and doublons move with **opposite directionality!**

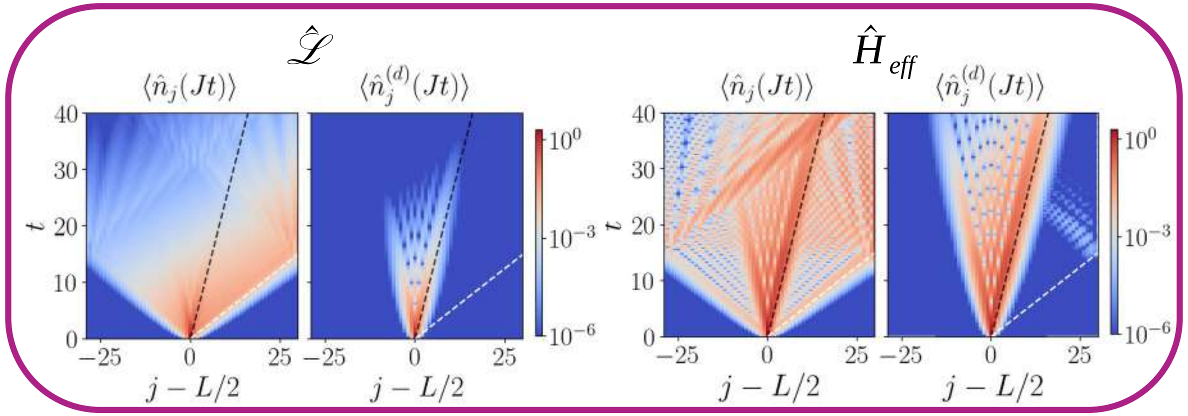


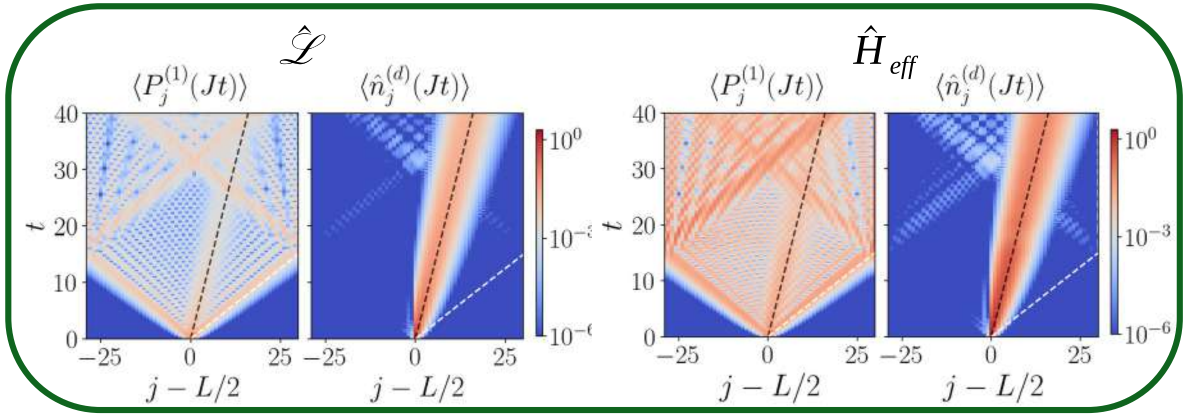
Comparison to non-Hermitian Hamiltonian

In case of pure loss and dilute systems Lindblad and nHH are qualitatively similar

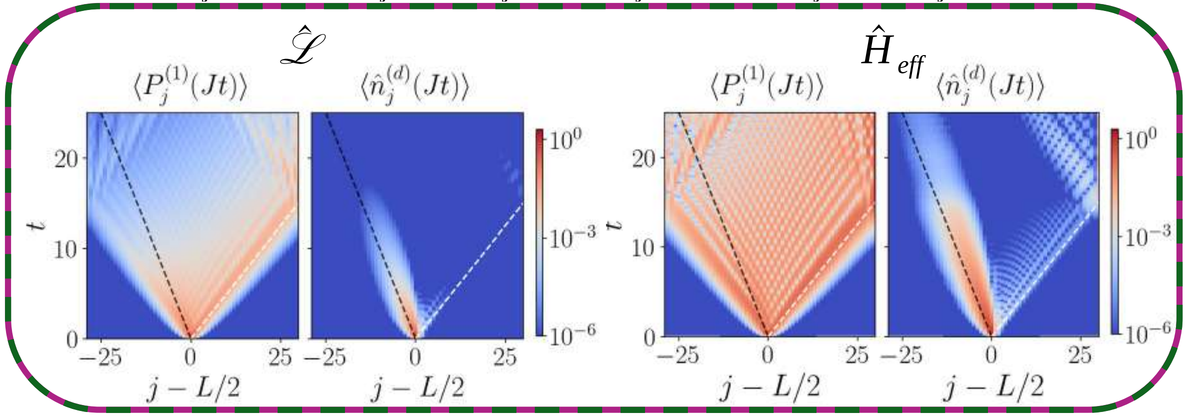
$$\hat{H}_{eff} = \hat{H} - \frac{i}{2}\sum_j \hat{L}_j^\dagger \hat{L}_j \qquad \hat{\mathcal{L}}\rho = -i[\hat{H},\rho] + \Gamma\sum_j \hat{L}_j \rho \hat{L}_j^\dagger - \frac{1}{2}\{\hat{L}_j^\dagger \hat{L}_j, \rho\}$$

$$\hat{L}_j = \sqrt{\Gamma_1}\left(\hat{a}_j + e^{i\theta_1}\hat{a}_{j+1}\right) \qquad \hat{L}_j = \sqrt{\Gamma_2}\left(\hat{a}_j^2 + e^{i\theta_2}\hat{a}_{j+1}^2\right)$$



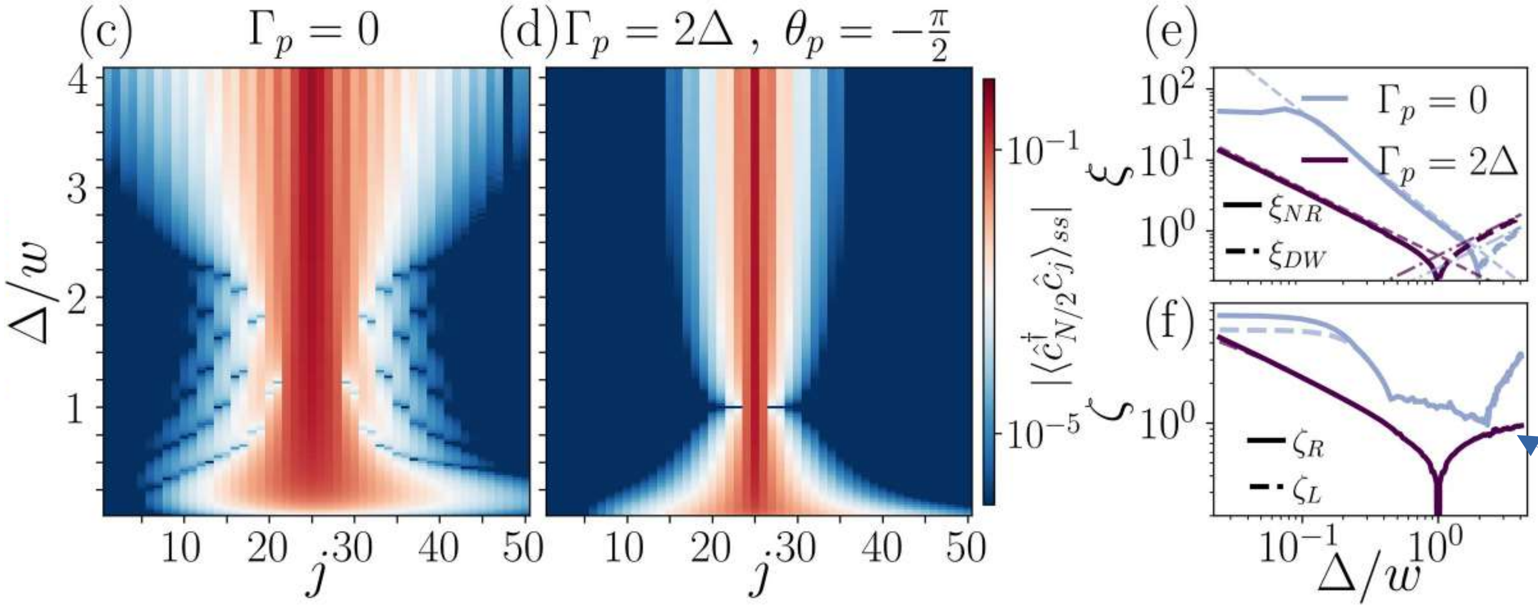


$$\hat{L}_j^{(1)} = \sqrt{\Gamma_1}\left(\hat{a}_j + e^{i\theta_1}\hat{a}_{j+1}\right) \qquad \hat{L}_j^{(2)} = \sqrt{\Gamma_2}\left(\hat{a}_j^2 + e^{i\theta_2}\hat{a}_{j+1}^2\right)$$



Correlations in the non-reciprocal Kitaev chain

The correlations in the steady state are also affected by the phase transition!



Non-reciprocal phase

- Asymmetric decay
- Shrinking range approaching critical point

$$\langle c_i^+ c_j \rangle \propto e^{-|i-j|/\zeta}$$

Density wave phase

$\Gamma_p = 0$

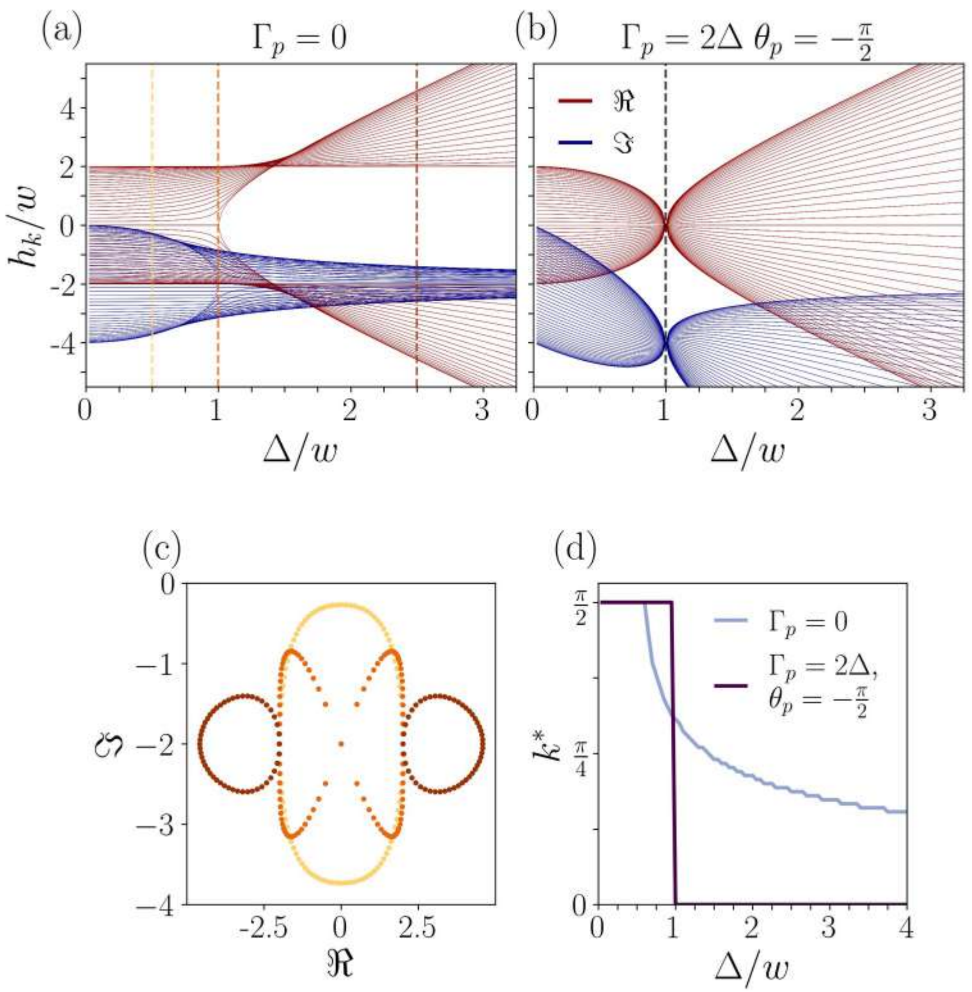
- Long-range correlations

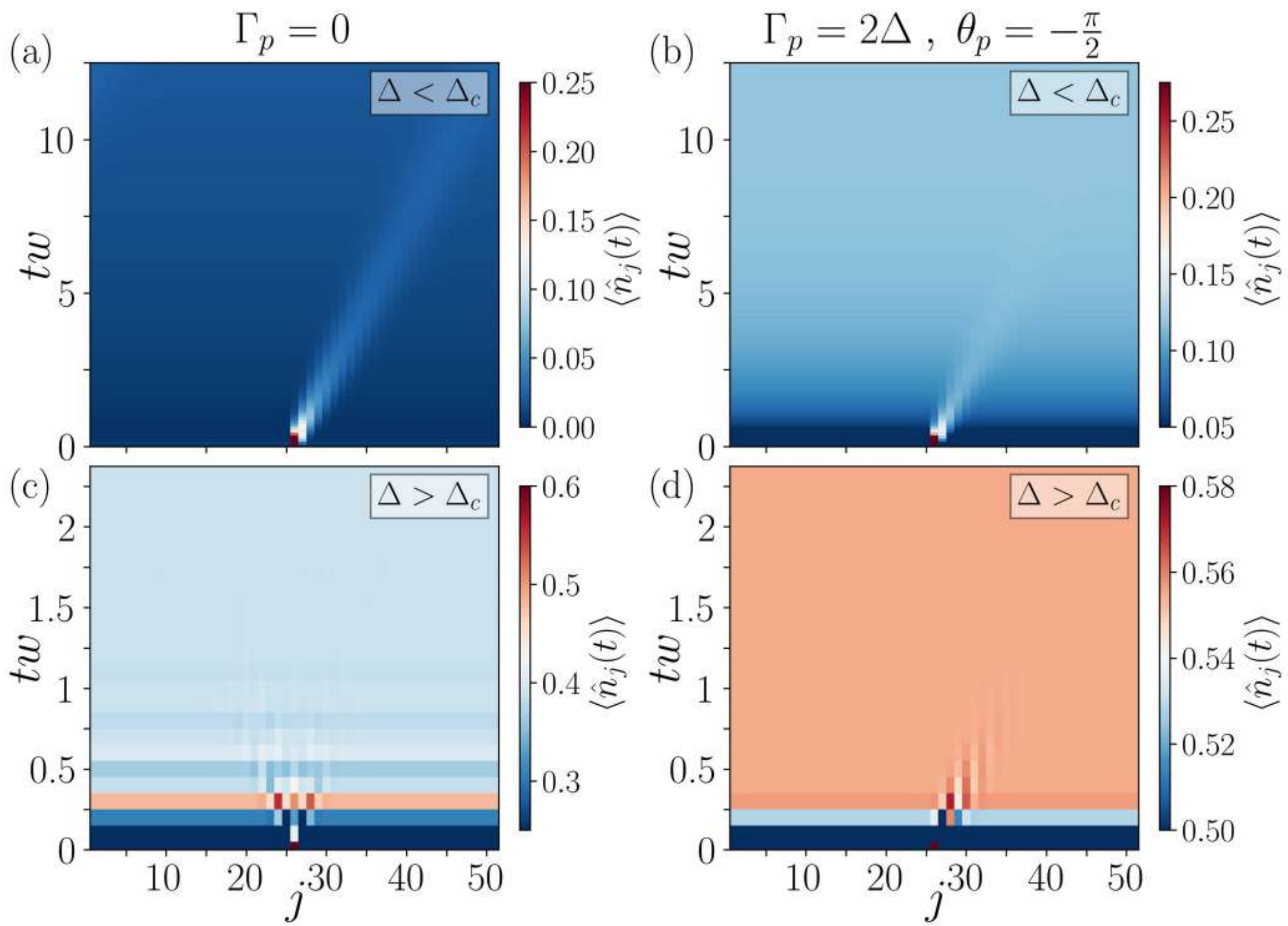
$\Gamma_p = 2\Delta$

- Extremely short-range correlations

Non-reciprocal Kitaev chain: PBC

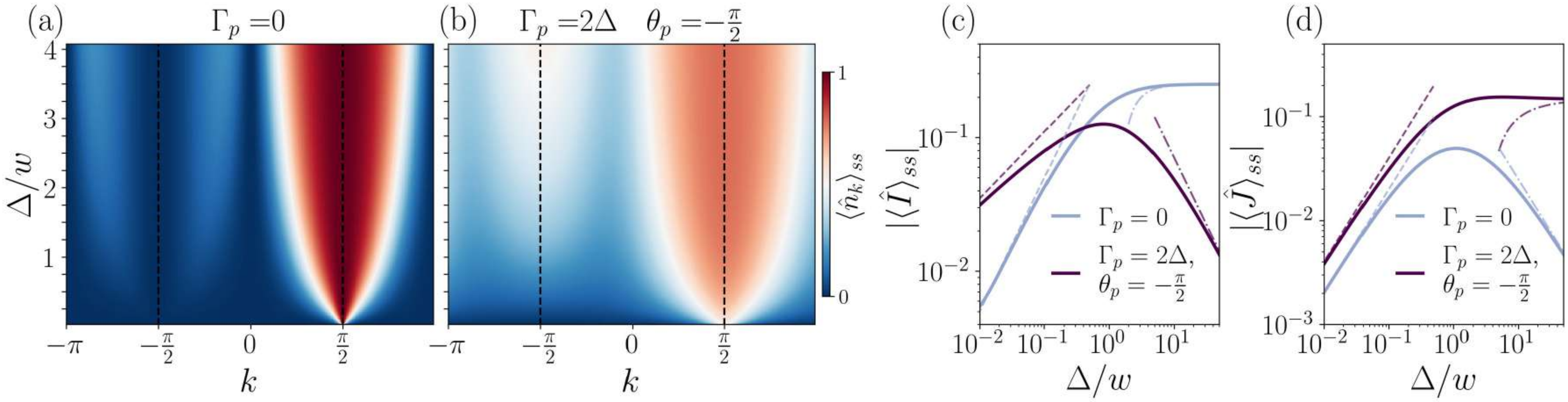
In periodic boundary conditions, only the dynamics are clearly affected by the phase transition





Non-reciprocal Kitaev chain: PBC

For the steady state, one needs to check more complicated and less intuitive quantities



Momentum occupation nk is not really useful...

Currents defined via continuity equation

I: usual current

J: pairing current