Precision is not limited by the second law of thermodynamics

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Physical devices operating out of equilibrium are affected by thermal fluctuations, limiting their operational precision. Clocks need a thermodynamic flux towards equilibrium to measure time, resulting in a minimum entropy dissipation per clock tick. Here we present an autonomous quantum many-body clock model that achieves clock precision that scales exponentially with entropy dissipation. The result demonstrates that coherent quantum dynamics can surpass the traditional thermodynamic precision limits, potentially guiding the development of future high-precision, low-dissipation quantum devices. Additionally, by carefully-engineered coherent transport in dissipative spin chains, we achieve a scaling exponent at the precision–resolution trade-off fundamental bound, bringing this within reach of physically realistic and experimentally accessible systems.

Measuring time & Entropy productions

Time is not an observable: it has to be measured indirectly

Aristotle (IV[th] BC): "*Time is the measure of change*"

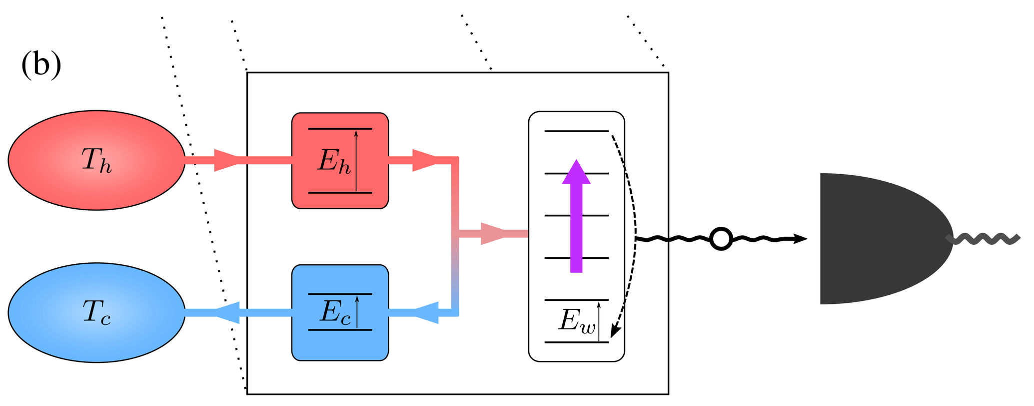
Time has a direction → the clock "ticks" forward

Crooks equation

$$\frac{p(n \to n+1)}{p(n \to n-1)} = e^{-\Sigma_{tick}}$$

Entropy per tick

Clocks are irreversible systems
thermodynamic cost is entropy dissipation



Model of an autonomous quantum clock [1] with Precision linearly upper-bounded by entropy per tick

$$\mathcal{N} \leq \frac{\Sigma_{tick}}{2}$$

Measuring time & Precision-Resolution bound

Each tick is a stochastic event: the time between two ticks is described by a probability density function (pdf)

Resolution
average number of ticks per unit of time

$$\nu = \frac{1}{E[T]}$$

Precision
number of ticks such that the uncertainty of the N[th] tick time equals the average time

$$\mathcal{N} = \frac{E[T]^2}{Var[T]}$$

Resolution and Precision can not be improved independently

Fundamental trade-off: Precision vs Resolution [2]

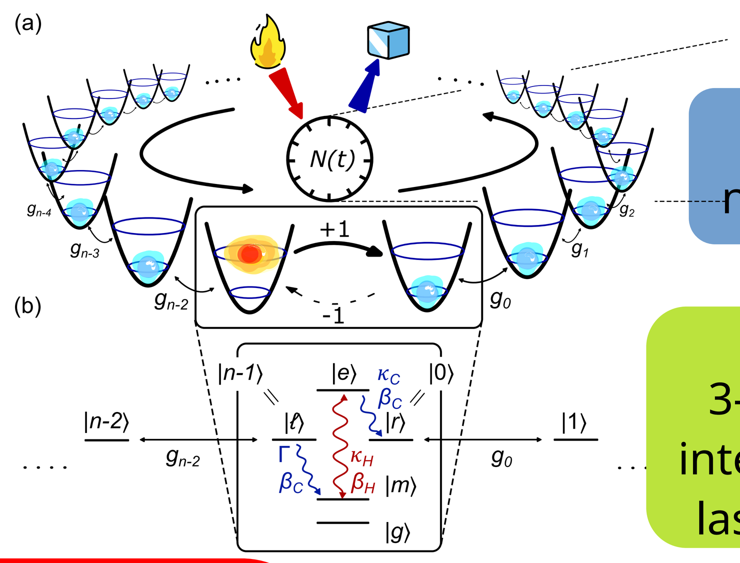
Best classical clock

$$\mathcal{N} = \frac{\Gamma}{\nu}$$

Best quantum clock

$$\mathcal{N} \leq \frac{\Gamma^2}{\nu^2}$$

The autonomous ring clock: Precision scaling exponentially with entropy dissipation [3]



The click: biased jump from the last to the first site

The clock: n qubits in one-excitation subspace

The entropy production: 3-level diagram for a directional interface between the first and the last site using a thermal gradient

Dissipative dynamics: Lindblad equation

$$\mathcal{L}\cdot = \underbrace{-i\,[H,\cdot] - \frac{1}{2}\{J^\dagger J,\cdot\}}_{\mathcal{L}_0} + \underbrace{J\cdot J^\dagger}_{\mathcal{L}_+}$$

$\mathcal{L}_0$: Non-trace preserving part of the evolution where no tick occurs

$\mathcal{L}_+$: Part generating ticks

Coherent evolution: hopping between nearest-neighbor sites

$$H = \sum_{j=1}^{n-1} g_j |j\rangle\langle j+1| + \text{h.c.}$$

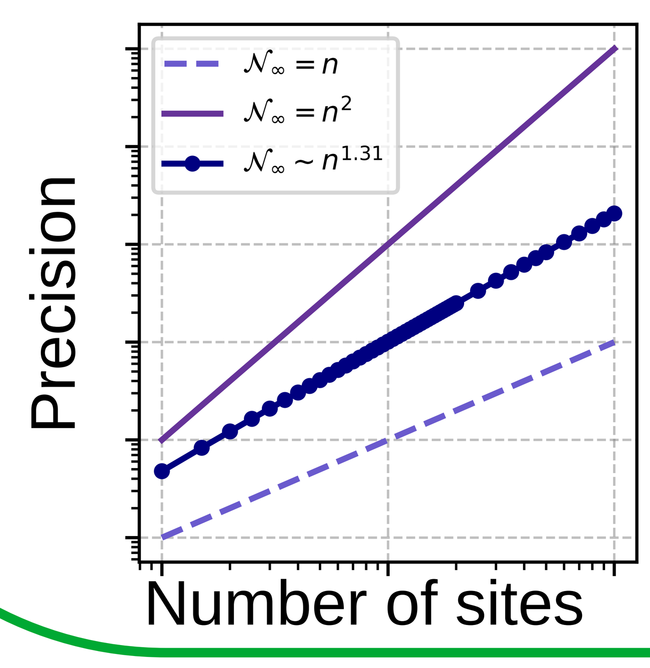
Jump operator: forward tick & backward tick

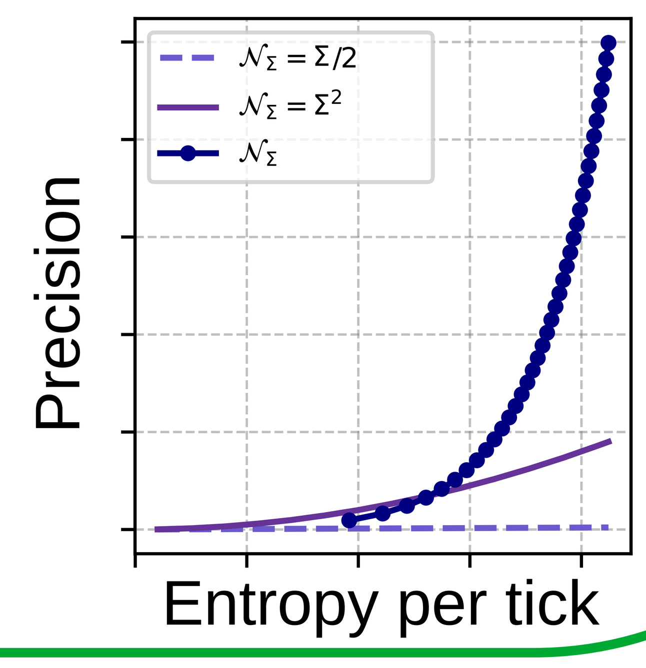
$$J = \sqrt{\Gamma}\,|1\rangle\langle n| \quad \bar{J} = e^{-\frac{\Sigma_{tick}}{2}}\sqrt{\Gamma}\,|n\rangle\langle 1|$$

Precision-Resolution scaling ← **RESULTS** → Precision-Entropy scaling

$$\left.\begin{array}{l} E[T] \sim \frac{n}{g} \\ Var[T] \sim \frac{n^{\frac{2}{3}}}{g} \end{array}\right\} \mathcal{N} \sim n^{\frac{4}{3}} \sim \left(\frac{g\Gamma}{\nu}\right)^{\frac{4}{3}}$$

$$\left.\begin{array}{l} \Sigma_{tick} \sim \log n \\ \mathcal{N} \sim n^{\frac{4}{3}} \end{array}\right\} \mathcal{N}_\Sigma = e^{\Omega(\Sigma_{tick})}$$

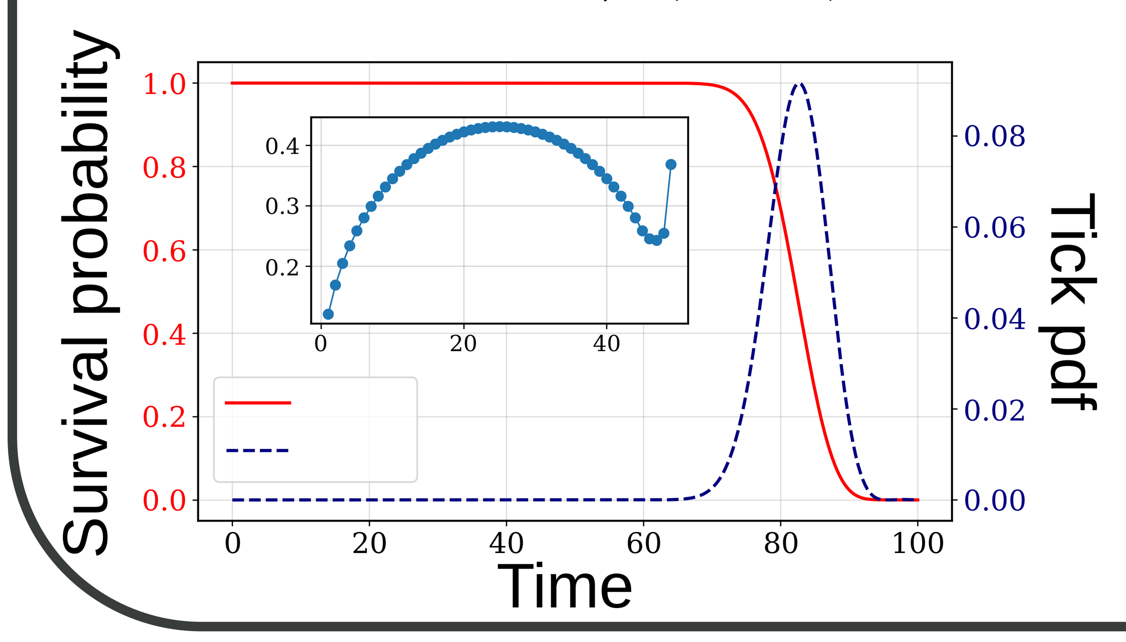


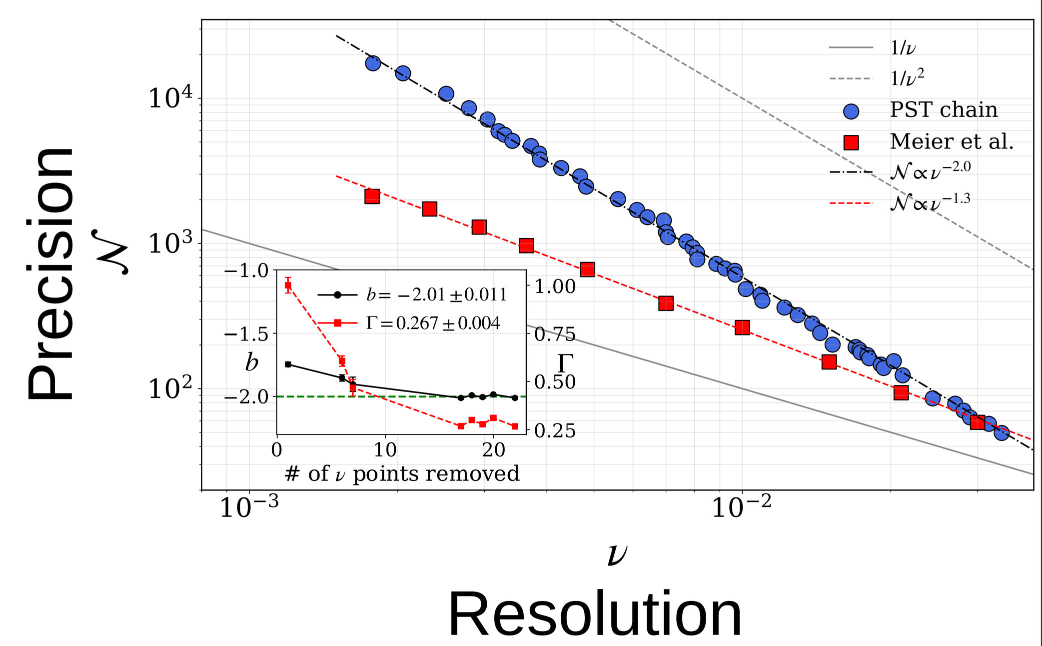


Exploiting Perfect State Transfer in non-Hermitian dynamics: Achieving the Precision-Resolution bound [4]

$$\hat{H}_{XX} = \sum_{i=1}^{N-1} \frac{J_i}{2}\left(\hat{\sigma}_i^x\hat{\sigma}_{i+1}^x + \hat{\sigma}_i^y\hat{\sigma}_{i+1}^y\right)$$

$$J_i = J_0\sqrt{i(N-i)}$$





$$\mathcal{N} \sim \nu^{-2}$$

[1] P. Erker, M. T. Mitchison, R. Silva, M. P. Woods, N. Brunner, and M. Huber, *Autonomous quantum clocks: Does thermodynamics limit our ability to measure time?*, Phys. Rev. X **7**, 031022 (2017).

[2] F. Meier, E. Schwarzhans, P. Erker, and M. Huber, *Fundamental accuracy-resolution trade-off for timekeeping devices*, Phys. Rev. Lett. **131**, 220201 (2023).

[3] Florian Meier, Yuri Minoguchi, Simon Sundelin, Tony J. G. Apollaro, Paul Erker, Simone Gasparinetti, Marcus Huber, *Precision is not limited by the second law of thermodynamics*, Nature Physics **21**, 1147–1152(2025)

[4] Chad Nelmes, Emanuel Schwarzhans, Tony J. G. Apollaro, Timothy Spiller, and Irene D'Amico, *Approaching the Limit of Quantum Clock Precision*, arXiv:2604.22704