Dissipation- vs chaos-induced relaxation in non-Markovian quantum many-body systems

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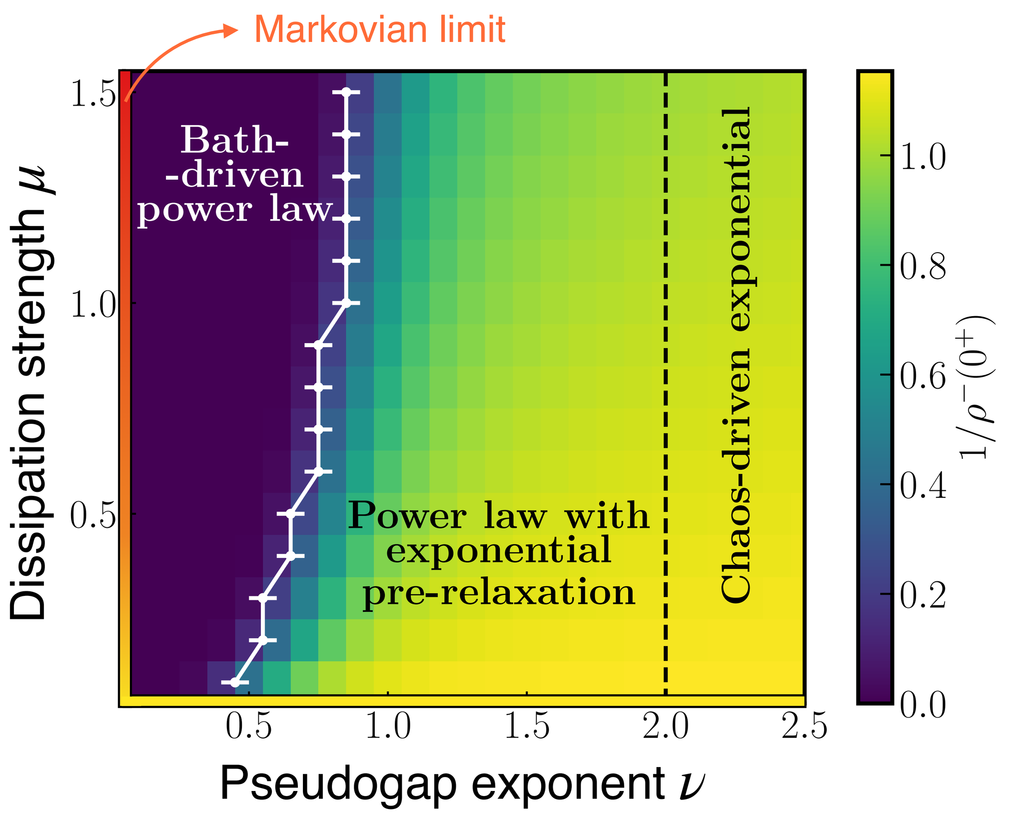
Motivation

In open quantum many-body systems, ***DISSIPATION*** and ***CHAOS*** compete to equilibrate the system

- We study this interplay in a dissipative **S**achdev-**Y**e-**K**itaev **(SYK)** model, widely used as a tractable model for strongly correlated quantum matter [1,2]
- While Markovian environments typically produce exponential relaxation [4, 5], realistic environments are intrinsically non-Markovian and can give rise to power-law relaxation



Main finding:

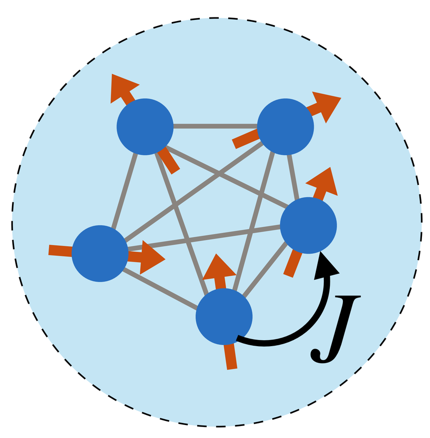
Competition between internal quantum chaotic dynamics and environment-induced non-Markovian dissipation produces a rich **relaxation phase diagram**

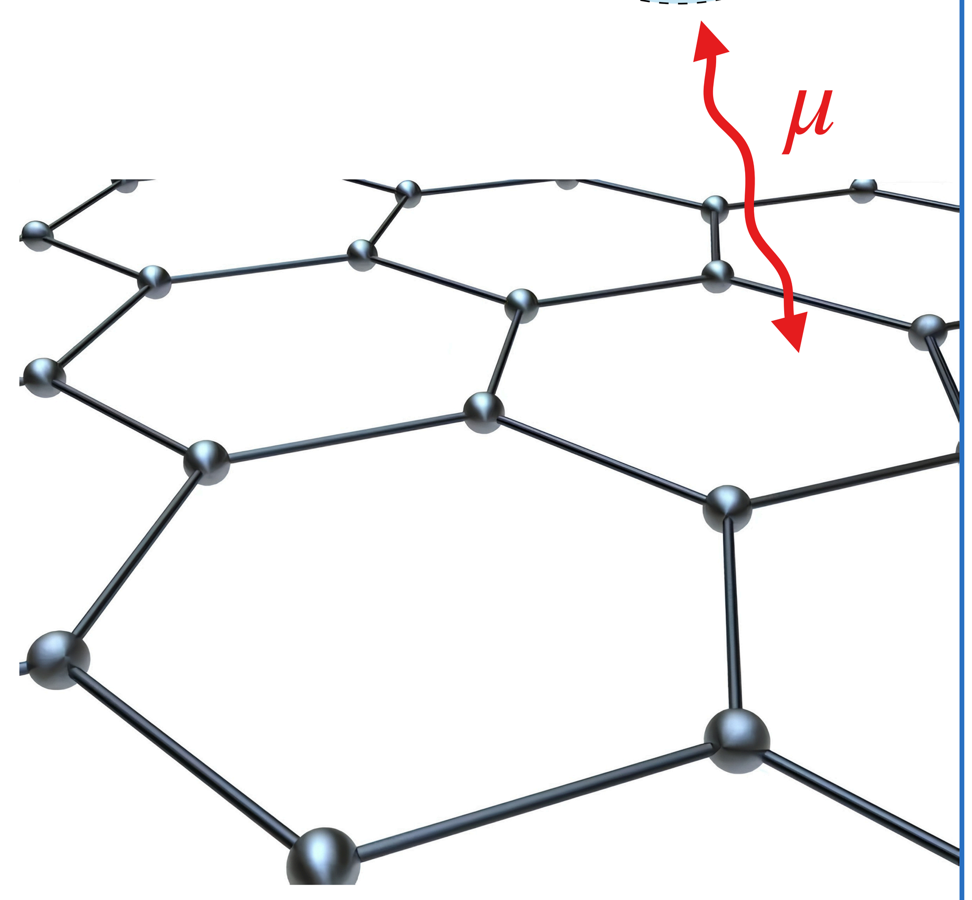
Model

$$H = \sum_{i<j<k<l} J_{ijkl}\chi_i\chi_j\chi_k\chi_l + \sqrt{\mu}\sum_i \chi_i Y_i + H_B$$

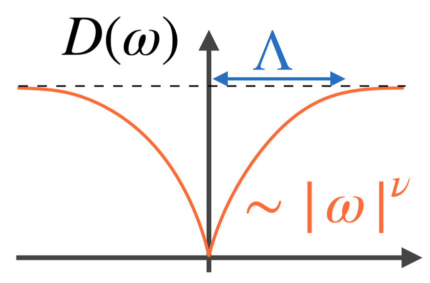
$$\langle J_{ijkl}^2\rangle = \frac{6J^2}{N^3}$$

Chaotic SYK





Pseudogapped bath



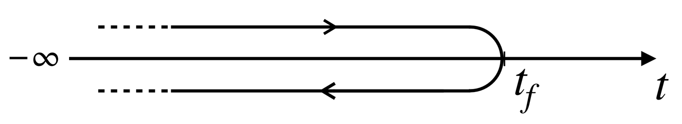
$$D(\omega) = \left(1 - e^{-\omega^2/\Lambda^2}\right)^{\nu/2}$$

$\omega \ll \Lambda$ $\qquad$ $\omega \gg \Lambda$

$$D(\omega) \approx \left|\frac{\omega}{\Lambda}\right|^{\nu} \qquad D(\omega) \approx 1$$

Structured density of states $D(\omega)$ → Non-Markovian dissipation

Keldysh formalism [3]



- The Keldysh formalism is a powerful tool for open quantum systems

$$Z = \mathrm{Tr}[\rho_{t_f}] = \int \prod_i \mathscr{D}a_i \exp\left(i\int_C dz\, \frac{1}{2}\sum_i a_i(z) i\partial_z a_i(z) - i\int_C dz \sum_{i<j<k<l} J_{ijkl} a_i(z)a_j(z)a_k(z)a_l(z) + \int dz dz' \mu K(z,z') \sum_i a_i(z)a_i(z') \right)$$

Coherent evolution

Field representation of χ_i

Non-Markovian dissipation

- We can write an effective action for the collective field $G(z,z') = -\frac{i}{N}\sum_i a_i(z)a_i(z')$
- Taking the saddle point ($N \to \infty$), we have:

Schwinger-Dyson Equations

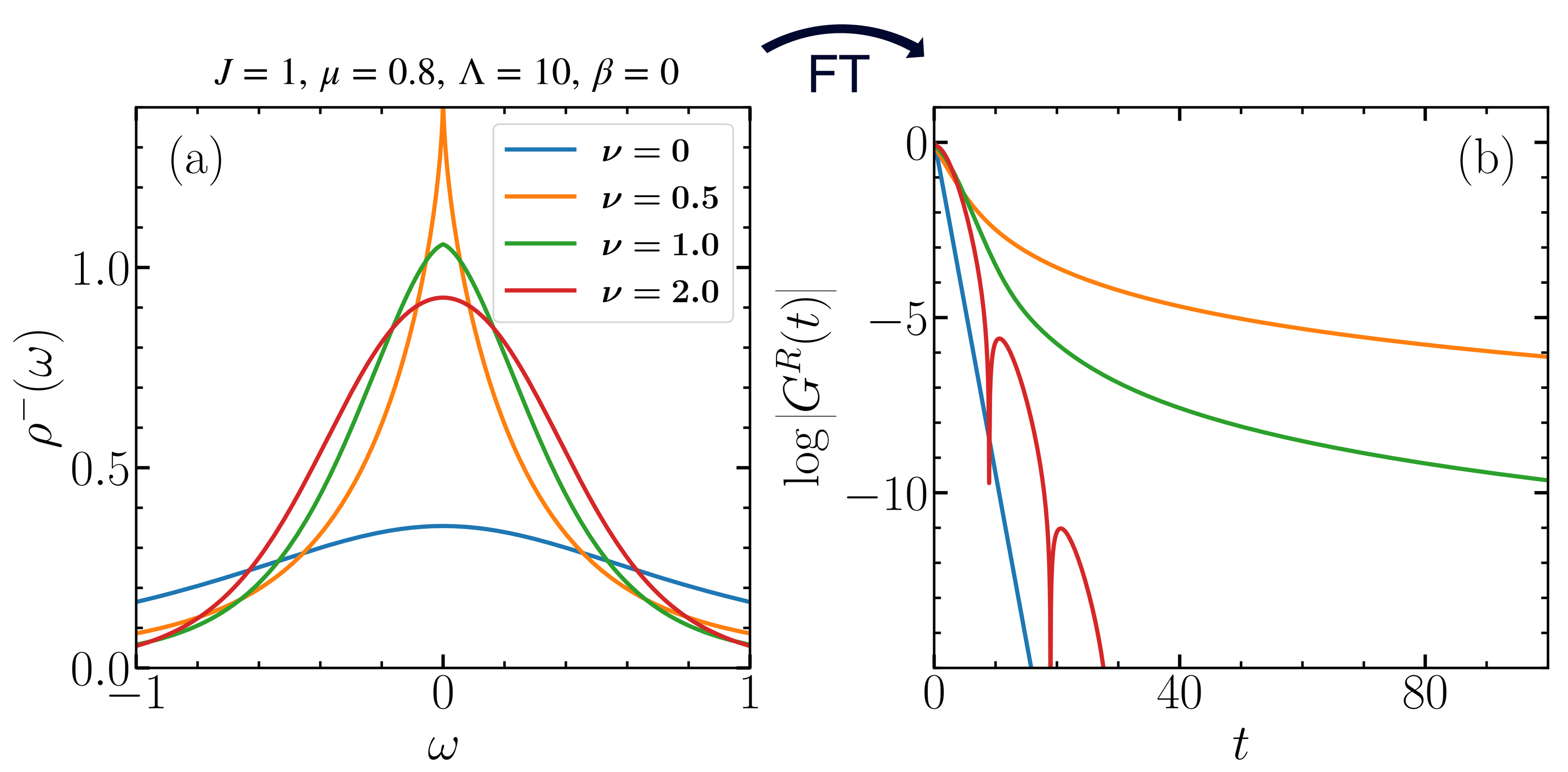
$$\sigma^-(\omega) = \frac{\mu}{\pi} D(\omega) + \frac{J^2}{4}(\rho^- * \rho^- * \rho^-)(\omega)$$

$$\rho^-(\omega) = \frac{\sigma^-(\omega)}{(\pi\sigma^H(\omega) + \omega)^2 + (\pi\sigma^-(\omega))^2} \qquad (\text{for } \beta = 0)$$

Single-particle density of states

Solutions of the SDE

- We numerically solve the Schwinger-Dyson equations on a non-uniform frequency grid, by iterating until a fixed point is reached
- Retarded Green's function $iG^R(t)$ exhibits **exponential** and/or **power-law** decay



- We can obtain analytic insight into the nature of the solutions by considering the **strong dissipation limit:** $\mu \gg \Lambda \gg J$

Strong dissipation limit

A similar structure persists at finite μ, giving rise to the phase diagram above!

- For $\nu < 1$:

$$\rho^-(\omega) \approx \frac{\cos^2(\pi\nu/2)}{\pi\mu}\left|\frac{\omega}{\Lambda}\right|^{-\nu}, \quad \omega \lesssim \Lambda$$

→ $\rho^-(\omega)$ diverges, producing power-law with exponent $p = 1 + \nu$

- For $\nu > 1$:

$$\rho^-(\omega) \approx \frac{1}{\pi^2\epsilon} - \frac{\mu}{\pi^2\epsilon^3}\left|\frac{\omega}{\Lambda}\right|^{\nu} - \frac{1}{\pi^4\epsilon^3}\omega^2, \quad \omega \lesssim J$$

→ Non-analytic contribution dominant for $\nu < 2$, producing a power-law with exponent $p = 1 + \nu$. However, the timescale at which it dominates diverges as $\nu \to 2^-$ (pre-relaxation)

Check our paper!

arXiv:2603.10815

References

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